

# High Frequency Lumped Element Models for Substrate Noise Coupling

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## Abstract

Lumped element models for the frequency dependence of substrate parasitics in integrated circuits are evaluated. The frequency ranges for which these models are applicable have been identified. Two new models that are suitable for very high frequencies are also presented.

## 1. Introduction

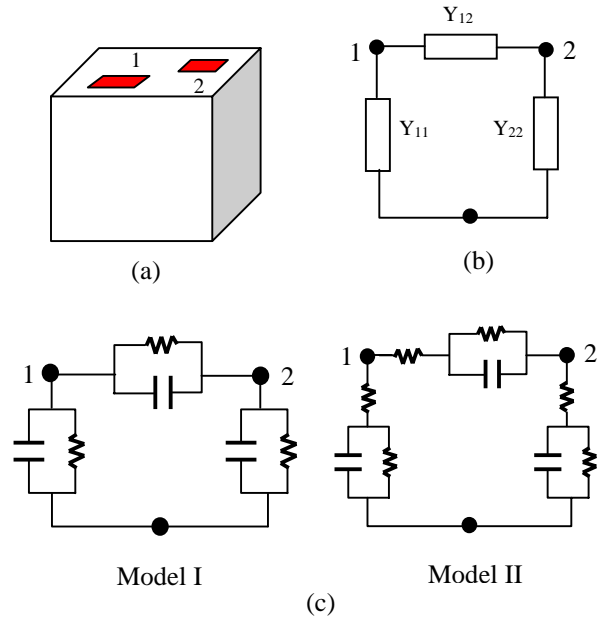
With the increasing integration of analog and digital circuits on a chip, circuit isolation has become a serious problem. The problem has been exacerbated by increasing operating frequencies. Up to now, most of the substrate coupling publications have focused on relatively low frequencies, where the substrates can be assumed to be purely resistive [1–4]. This assumption has been considered to be valid up to a few GHz. At higher frequencies, however, the dielectric behavior of the substrate becomes important and the intrinsic substrate capacitances must also be taken into account.

Several papers have been published on the high frequency behavior, simulation techniques, equivalent circuit models and experimental data [5–8]. However, a systematic study of the high frequency behavior is still not available. This study is essential to validate and improve existing models, or develop new models.

Substrate noise coupling is included in circuit design by equivalent circuit models. For a two-contact example shown in Fig. 1(a), a pi-equivalent lumped element model, as shown in Fig. 1(b), is used. For low frequency applications, the admittances are modeled by resistors. At higher frequencies, the dielectric behavior of the substrate is commonly reflected in equivalent circuits including circuit components such as capacitors. Two high frequency equivalent circuit models proposed in [5–7] are shown in Fig. 1(c).

In this paper, a comprehensive study of the frequency dependence of substrate parasitics is presented using the method of [6]. Various lumped element models are evaluated in terms of accuracy and the frequency range of application. Furthermore, new equivalent circuit models

are proposed. In Section 2, a study of the dependence of the substrate parasitics on contact configuration and substrate properties is presented. New lumped element models are then proposed in Section 3, and conclusions are provided in Section 4.



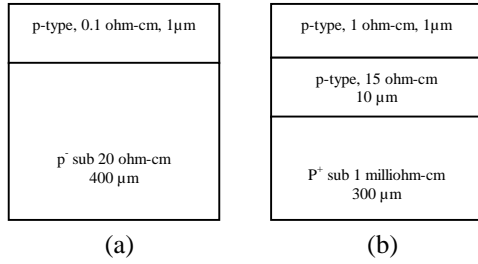
**Fig. 1:** Contact configuration and corresponding circuit models for substrate noise coupling. (a) Two contacts on a substrate. (b) Generic lumped element model.  $Y_{11}$  and  $Y_{22}$  are the self admittances and  $Y_{12}$  is the mutual admittance. (c) Model I and Model-II are the high frequency models proposed in [5–7].

## 2. Extraction of Substrate Parasitics

A software tool, *EPIC* (*Extraction of Parasitics for IC's*), was developed and used in this study to extract the substrate coupling parasitics. The numerical method is based on the Green's function method developed in [6]. In this method, the substrate is regarded as a lossy dielectric with a complex conductivity  $\sigma_c = \sigma + j\omega\epsilon$ , where  $\omega$  is the radian frequency and  $\sigma$  and  $\epsilon$  are the substrate conductivity and permittivity, respectively.

Consider two contacts, and assume the two contacts are symmetrically located on a substrate with a separation

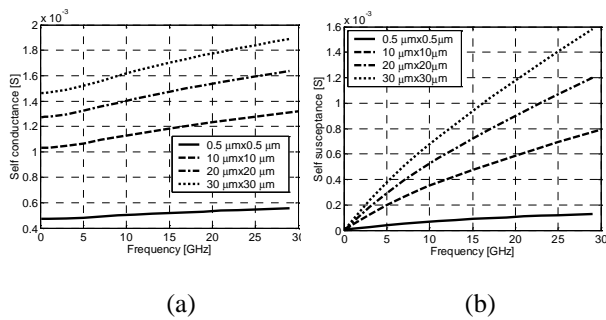
of  $10\mu\text{m}$ . Due to the symmetry, the two self admittances,  $Y_{11}$  and  $Y_{22}$  are equal.



**Fig. 2:** Typical heavily doped and lightly doped substrates [1]. (a) Lightly doped substrate, (b) heavily doped substrate.

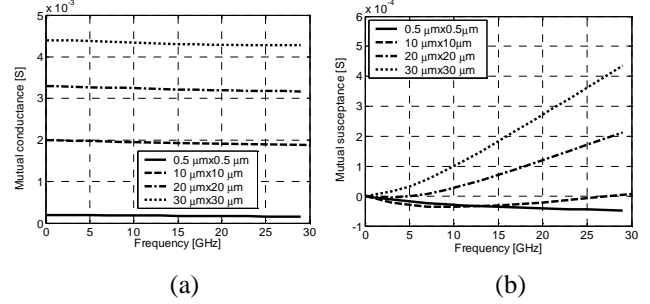
Fig. 2 shows two kinds of substrates, i.e., the lightly doped and the heavily doped substrates. Fig. 3 shows the self admittance for different contact sizes as a function of frequency for the lightly doped substrate. Both the self conductance (Fig. 3(a)) and the self susceptance (Fig. 3(b)) increase with frequency. The mutual conductance, as shown in Fig. 4(a), decreases as the frequency increases, and the variation is smaller than that of the self conductance. The mutual susceptance (Fig. 4(b)) can be inductive (negative) or capacitive (positive). When the contact separation is fixed, the susceptance switches from an inductive behavior to a capacitive behavior as the contact size increases.

The inductive behavior of mutual admittance was also reported in [8], but only for a heavily doped substrate. We have observed that both lightly doped and heavily doped substrates can show an inductive behavior.



**Fig. 3:** Frequency dependence of self admittance for two contacts in a lightly doped substrate. (a) Self conductance, (b) self susceptance.

In general, depending on the contact size, separation, substrate resistivity, and frequency, the mutual susceptance can be inductive or capacitive, and the mutual conductance can increase or decrease with frequency.



**Fig. 4:** Frequency dependence of mutual admittance for two contacts in a lightly doped substrate. (a) Mutual conductance, (b) mutual susceptance.

### 3. New Equivalent Circuit Models

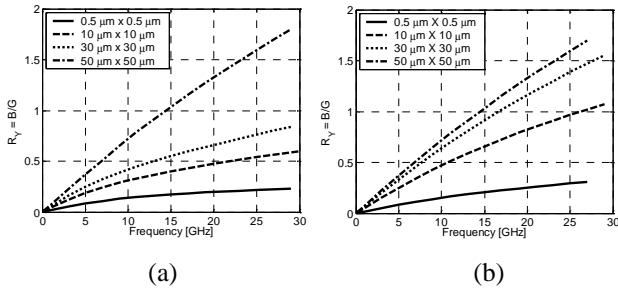
The observation that the frequency behavior of the self and mutual admittances is fundamentally different suggests different equivalent circuits should be used for these admittances. This is different from what is done in existing approaches (Fig. 1(c)).

#### 3.1. Self-coupling admittance model

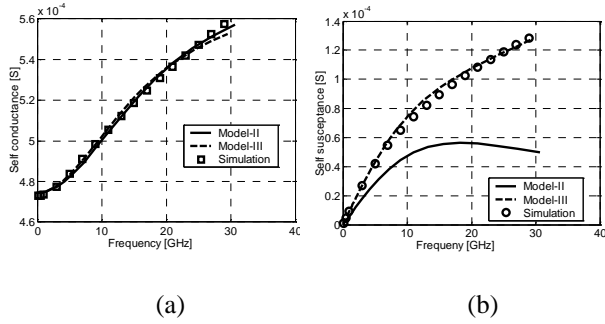
Table 1 lists all the self-coupling admittance models, Model 0 – Model III. Each model has a frequency range for which they can be used. Model 0 is suitable for low frequencies, i.e., less than 1 GHz. At these frequencies, the susceptance is negligibly small compared to the conductance. Model I implies a constant conductance and a susceptance that increases linearly with frequency. However, as shown in Fig. 3, the conductance is not a constant. Furthermore, the susceptance, does not linearly increase with frequency. Thus, Model I does not have the proper frequency dependence. However, for low frequencies, the conductance variation is small and the susceptance increases nearly linearly with frequency so Model-I can be used. The upper frequency limit for this model is defined as the frequency at which the susceptance to conductance ratio exhibits a nonlinear variation with frequency. This upper limit (shown in Fig. 5) is about 5GHz for both types of substrates.

When the frequency is above 5GHz, Model-I is no longer accurate. For higher frequencies, models that provide a conductance and susceptance variation with frequency are needed. Model-II and Model-III in Table 1 possess this desired feature. For Model-II, although the conductance and susceptance can fit data individually, they may not fit the data simultaneously. Model-III is found to have a much better fit. As an example, Fig. 6 shows a comparison between Model-II and Model-III for two  $0.5\mu\text{m}\times 0.5\mu\text{m}$  contacts  $10\mu\text{m}$  apart in the lightly doped substrate. We see that for frequencies larger than 15GHz, Model-II does not show the correct trend for self susceptance, although the conductance is in good

agreement with simulation. This indicates the need for Model III as proposed in this paper.



**Fig. 5:** Susceptance-to-conductance ratio as a function of frequency. (a) Lightly doped substrate, (b) heavily doped substrate.



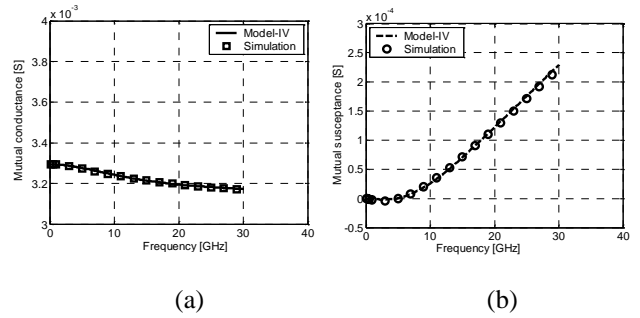
**Fig. 6:** Comparison of self-admittance models for two  $0.5\mu\text{m} \times 0.5\mu\text{m}$  contacts  $10\mu\text{m}$  apart in the lightly doped substrate. (a) Self conductance, (b) self susceptance.

### 3.2. Mutual coupling admittance model

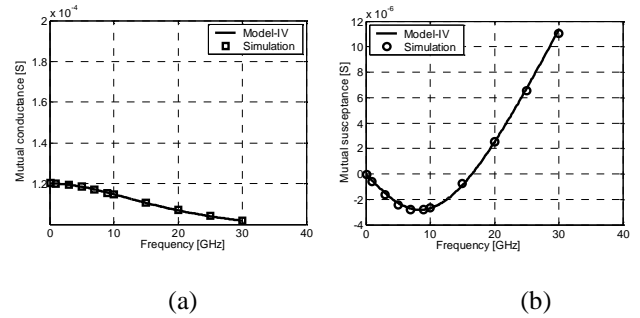
Table 2 lists all the available mutual coupling models. Model 0 is for very low frequency applications. Model I through Model III correspond to the self admittance models in Table 1. These models can only be used when the susceptance is capacitive and the conductance increases with frequency. Since the self admittance is more frequency sensitive, the suitable frequency range for these models should be determined by the corresponding frequency range for the self admittance model. For other conditions, new equivalent models need to be established.

The existence of an inductive susceptance suggests that inductors should be included in the equivalent circuit for the mutual admittance. A possible model is shown in Table 2 as Model IV. Fig. 7 shows a comparison between the model and the simulation for two  $20\mu\text{m} \times 20\mu\text{m}$  contacts  $10\mu\text{m}$  apart in a lightly doped substrate, and Fig. 8 shows the comparison for  $10\mu\text{m} \times 10\mu\text{m}$  contacts  $10\mu\text{m}$  apart in a heavily doped substrate. Very good agreement is obtained for a large frequency range with this model for both cases. Model II and Model III are not applicable to

these two cases because of the inductive susceptance and a conductance that decreases with frequency.



**Fig. 7:** Comparison of Model IV and simulations for two  $20\mu\text{m} \times 20\mu\text{m}$  contacts in the lightly doped substrate. (a) Mutual conductance, and (b) mutual susceptance.



**Fig. 8:** Comparison of Model IV and simulations for two  $10\mu\text{m} \times 10\mu\text{m}$  contacts  $10\mu\text{m}$  apart in the heavily doped substrate. (a) Mutual conductance, and (b) mutual susceptance.

## 4. Conclusions

In this paper, a comprehensive simulation of the frequency dependence of substrate parasitics has been presented. The frequency behavior of the self admittance is found to be fundamentally different from that of the mutual admittance. This suggests that different equivalent circuit models should be used for the self and mutual admittances. Two new models have been proposed that show good agreement with the simulation results.

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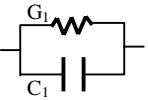
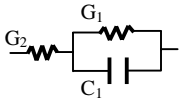
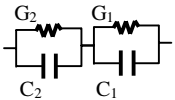
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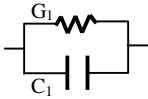
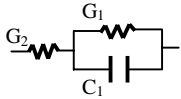
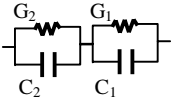
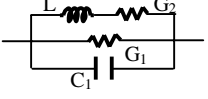
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**Table 1:** Self-coupling admittance models. Model 0, Model I, and Model II are existing lumped element models. Model III is a new model that is proposed in this paper. Model III accurately models the high frequency behavior of the conductance and susceptance.

Model Name	Model 0	Model I	Model II	Model III
Circuit Configuration				
General suitable frequency range	$f < 1\text{GHz}$	$1\text{GHz} < f < 5\text{GHz}$	$5\text{GHz} < f < 10\text{GHz}$	$f > 5\text{GHz}$
Comments	The simplest low frequency model.	Not able to model the frequency dependence of G.	Capable of modeling the frequency dependence of G and B.	Similar to Model II but with better agreement with simulations.

**Table 2:** Mutual coupling admittance models. Model 0, Model I, and Model II are existing lumped element models. Model III is derived from the self-coupling model in Table 1. Model IV is a new model that is proposed in this paper. Model IV accurately models the inductive behavior at high frequencies.

Model Name	Model 0	Model I	Model II	Model III	Model IV
Circuit Configuration					
General suitable frequency range	$f < 1\text{GHz}$	$1\text{GHz} < f < 5\text{GHz}$	$5\text{GHz} < f < 10\text{GHz}$	$f > 5\text{GHz}$	$f > 1\text{GHz}$
Comments	The simplest low frequency model.	Not able to model the frequency dependence of G. Capacitive coupling only.	Capable of modeling the frequency dependence of G and B. Capacitive susceptance and conductance that increases with frequency.	Similar to Model II but with better agreement with simulations.	Suitable for both capacitive and inductive coupling. Very good agreement with simulations.