Automatic Generation of Geometrically Parameterized Reduced Order Models for Integrated Spiral RF-Inductors

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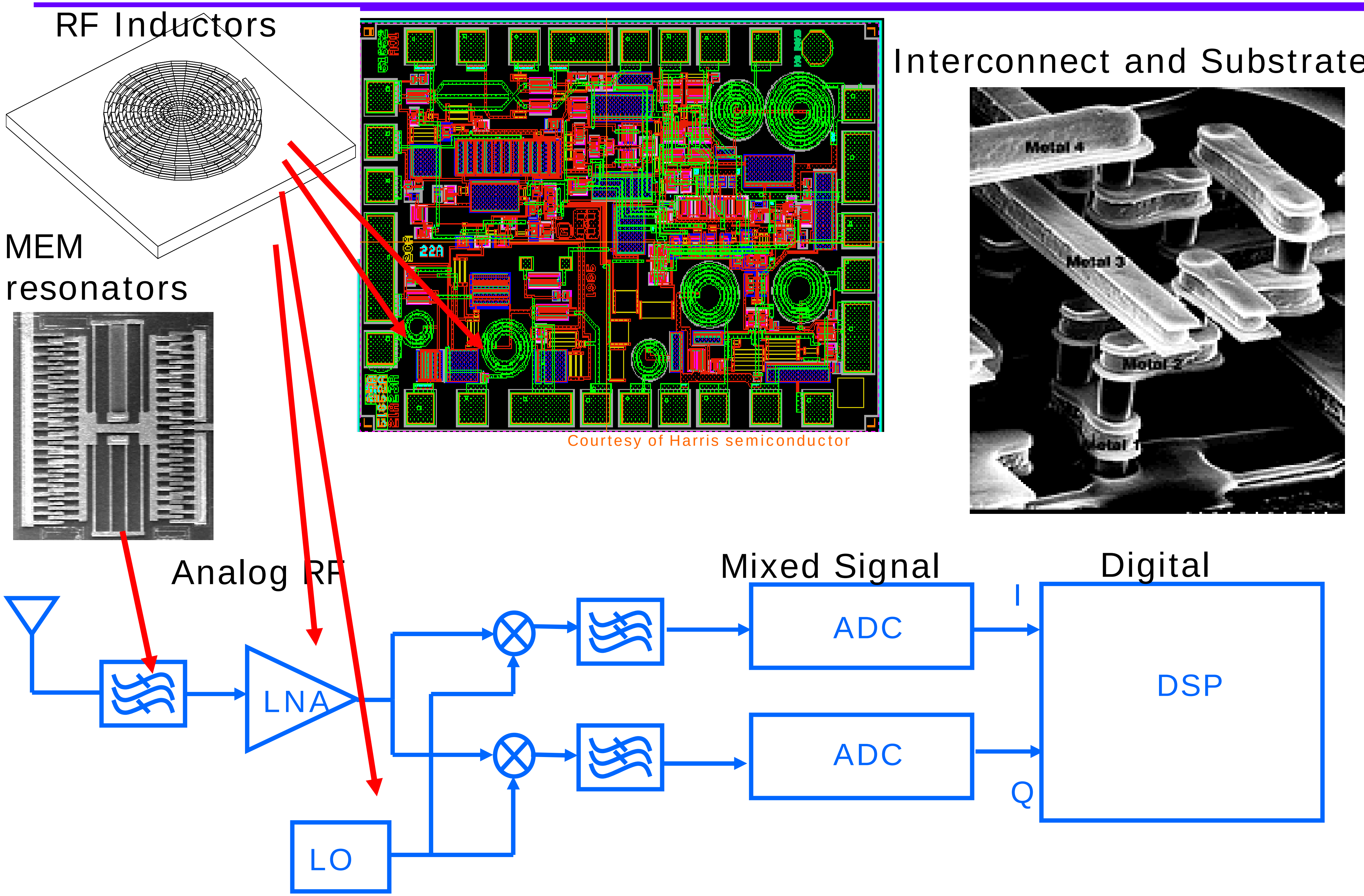
Systems on Chip or Package: heterogeneous components analog RF + MEM + mixed signal + digital



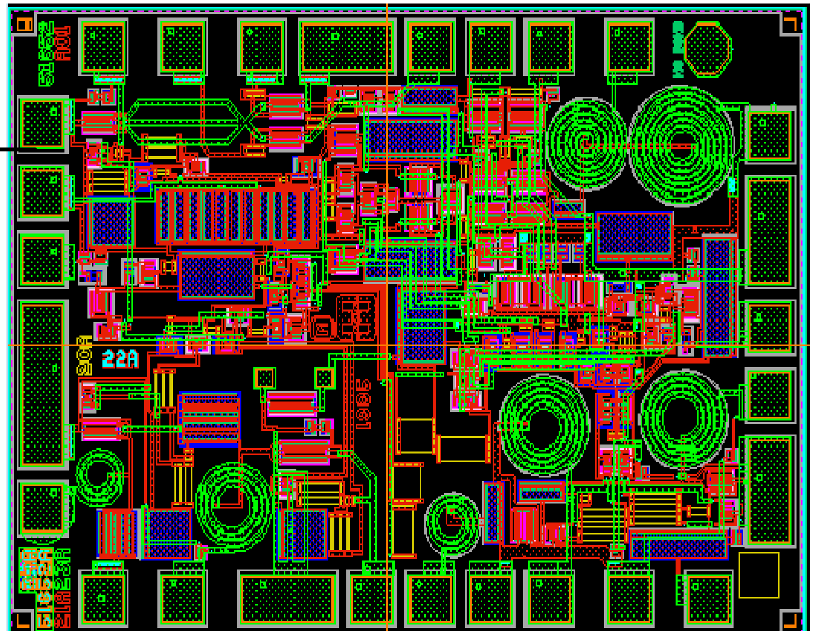
Verification of the overall system performance is a challenge

RF Inductors

$$\nabla \times E = -\mu \frac{dH}{dt}$$

$$\nabla \times H = \varepsilon \frac{dE}{dt}$$

MEM resonators



Courtesy of Harris semiconductor

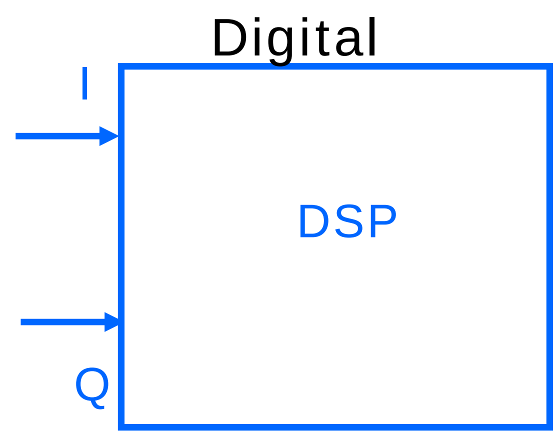
Interconnect and Substrate

$$\nabla \times E = -\mu \frac{dH}{dt}$$

$$\nabla \times H = \varepsilon \frac{dE}{dt}$$

$$EI\frac{\partial^4 u}{\partial x^4} - S\frac{\partial^2 u}{\partial x^2} = F_{elec} + \int_0^w (p - p_a)dy - \rho\frac{\partial^2 u}{\partial t^2}$$

$$\nabla \cdot ((1 + 6K)u^3 p \nabla p) = 12\mu \frac{\partial(pu)}{\partial t}$$



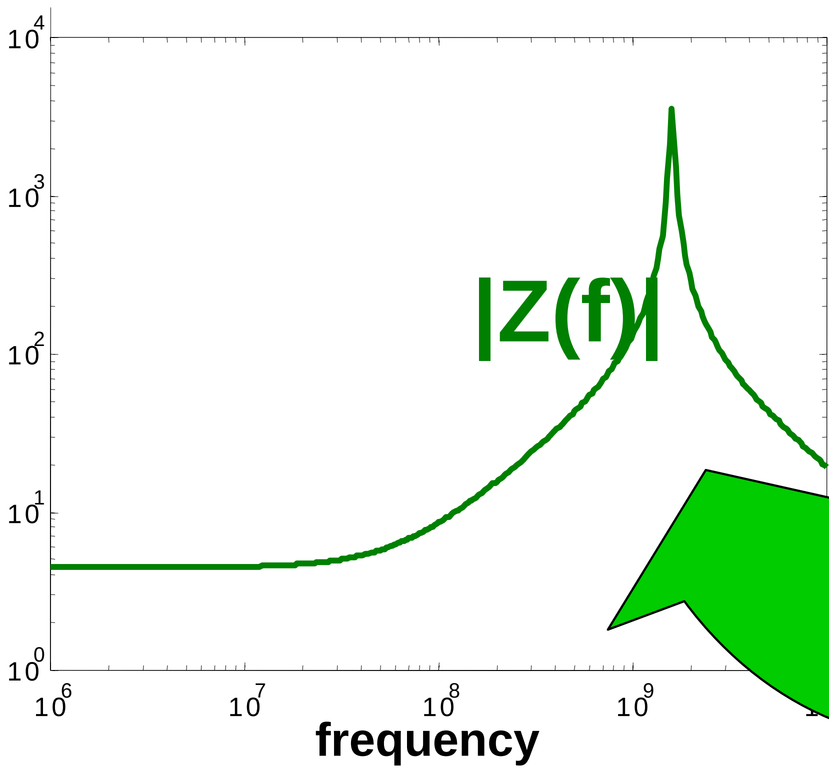
Analog RF

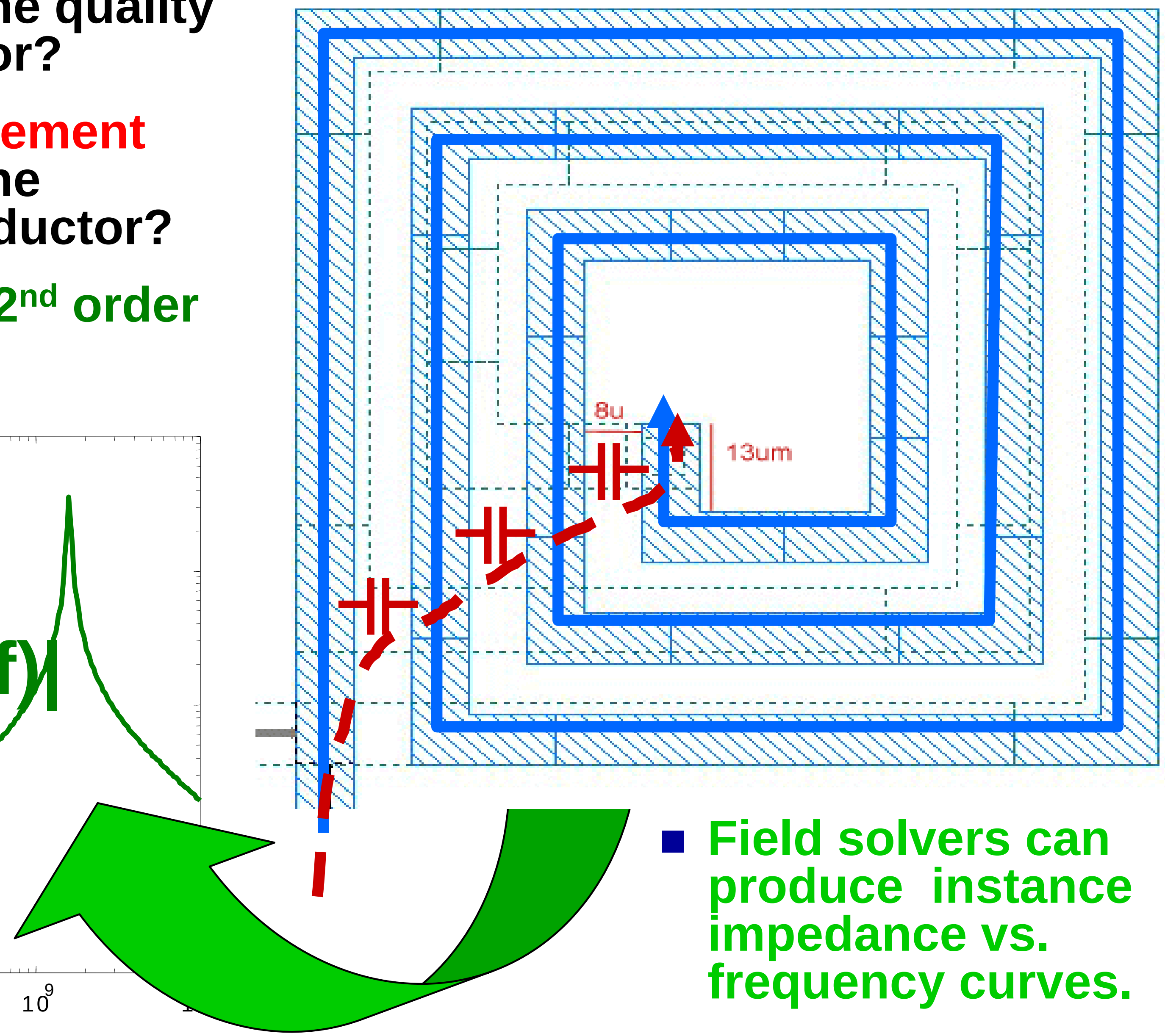
Mixed Signal

$$C(v)\frac{dv}{dt} = -G(v) + Bv_{in}$$

Motivation.
Example: RF micro-inductor

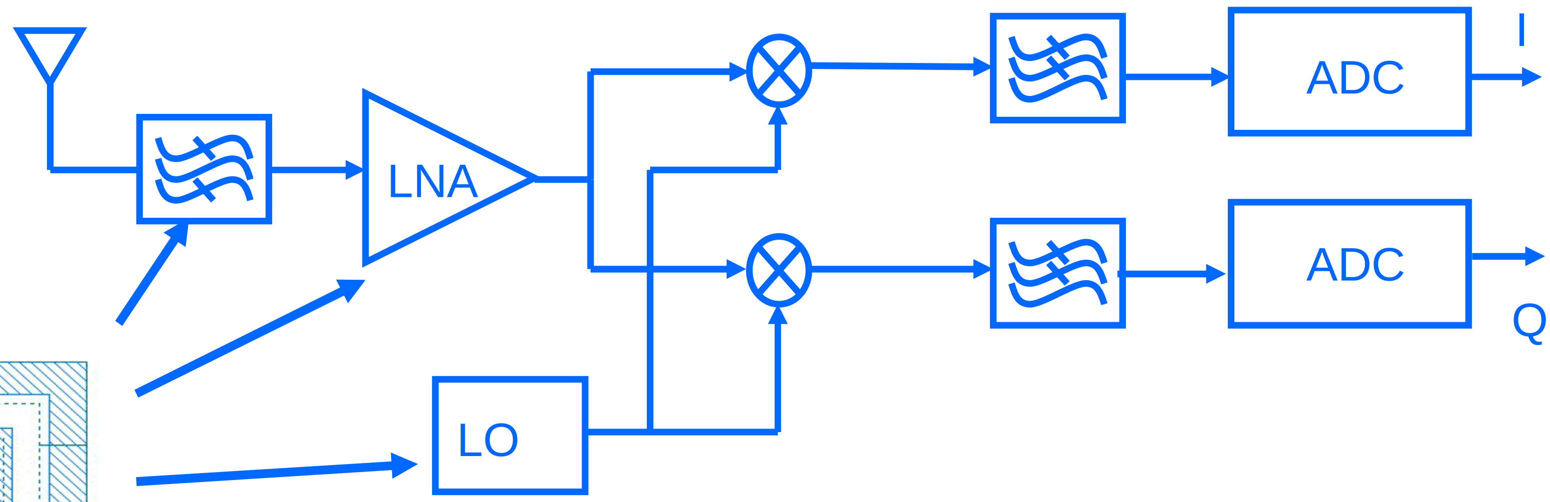
- How are the **substrate eddy currents** affecting the quality factor of the inductor?
- How are the **displacement currents** affecting the resonance of the inductor?
- Need to capture **all 2nd order effects**

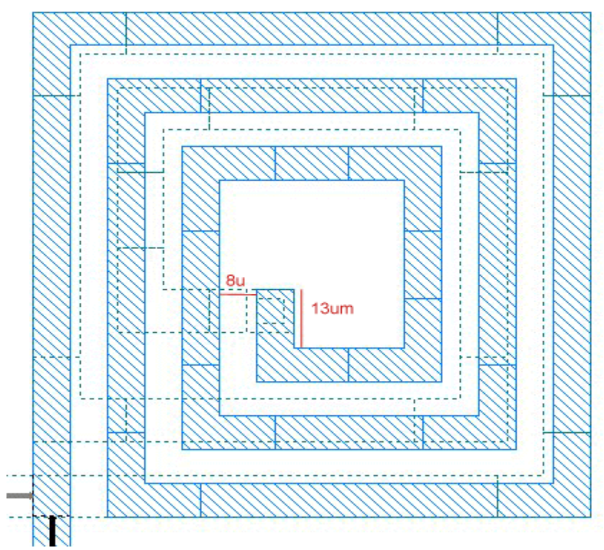




- Field solvers can produce instance impedance vs. frequency curves.

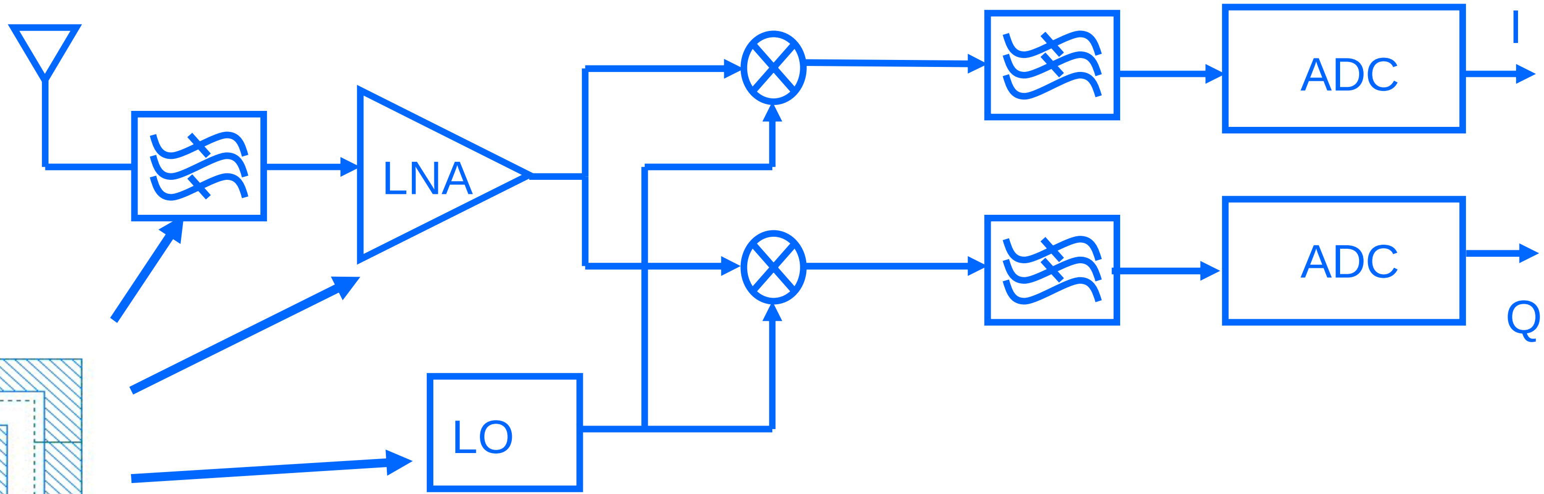
Motivation
Example: RF micro-inductor

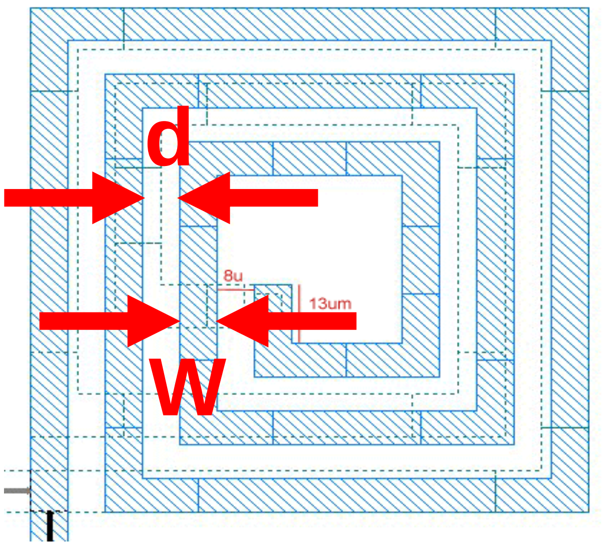




- ***<u>"Model Order Reduction"</u>*** **can help verify how the inductor performance (Q, resonance position, etc) affect the transceiver performance (distortion, interference rejection etc.)**
- **Modeling Requirements:**
  - **as accurate as a field solver**
  - **automatic and robust**
  - **model compatible with circuit simulators**

Motivation. An attempt to filling Jaijeet's "Gap" between bottom-up verification and system design





- ***<u>"Parameterized Model Order Reduction"</u>*** **can help verify quickly and with a field solver accuracy how the transceiver performance changes when I change wire widths W and wire separations d**
- **Hence it can be used within an optimization loop for automatic system design**

Previous work

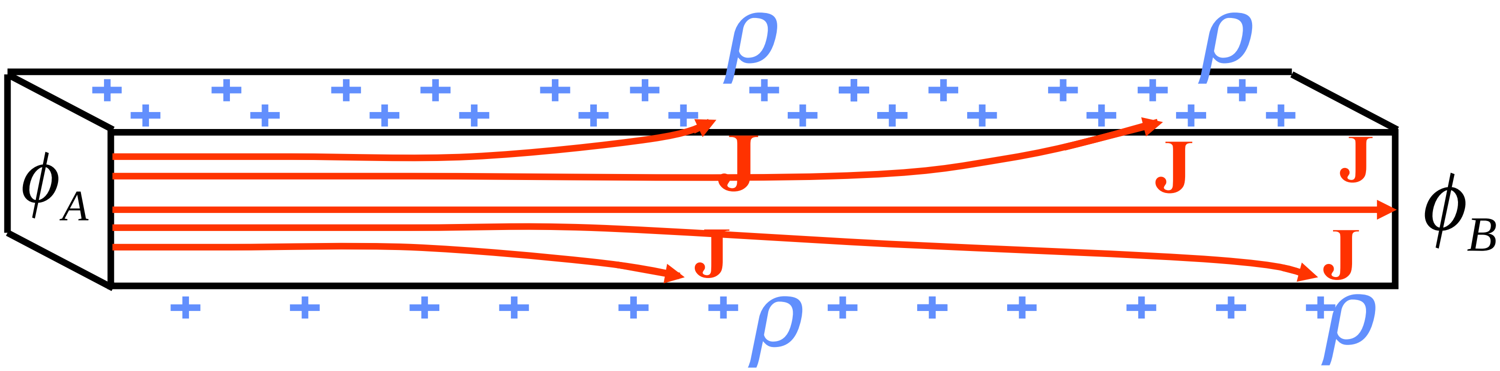
- Parameterized reduced-order models focusing on statistical data mining performance evaluation
  - for interconnect [Liu DAC99, Heydari ICCAD01]
  - and for analog circuit blocks [Rutenbar DAC02]

- parameterized moment-matching model reduction
  - Moment sensitivity based wire sizing for clock skew minimization [Pullela TCAD97]
  - with two parameters [Weile Applied Math. Letters 1999]
  - and with p-parameters applied to RC models of interconnect [Daniel ISPD02].
  - reducing a discretized linear parameterized PDE [Prud'homme Journal Fluids Eng. 2002].
  - ALL FOR LINEARLY PARAMETERIZED SYSTEMS

Outline

- Motivations
- Background
  - A Volume Integral Equation (VIE) formulation
  - From VIE to dynamical linear systems
  - Linearly Parameterized Projection-based Model Reduction
- An interpolation approach for non-linearly parameterized model order reduction
- RF-inductor Results
- Future Work and Conclusions

Background
Volume Integral Equation Method



$$\frac{\mathbf{J}(\mathbf{r})}{\sigma} + j\omega \frac{\mu}{4\pi} \int_V \mathbf{J}(\mathbf{r}') \frac{1}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r}' = -\nabla \phi$$

resistive effect **magnetic coupling**

$$\frac{1}{4\pi\varepsilon} \int_S \rho(\mathbf{r}_s') \frac{1}{|\mathbf{r}_s - \mathbf{r}_s'|} d\mathbf{r}_s' = \phi(\mathbf{r}_s)$$

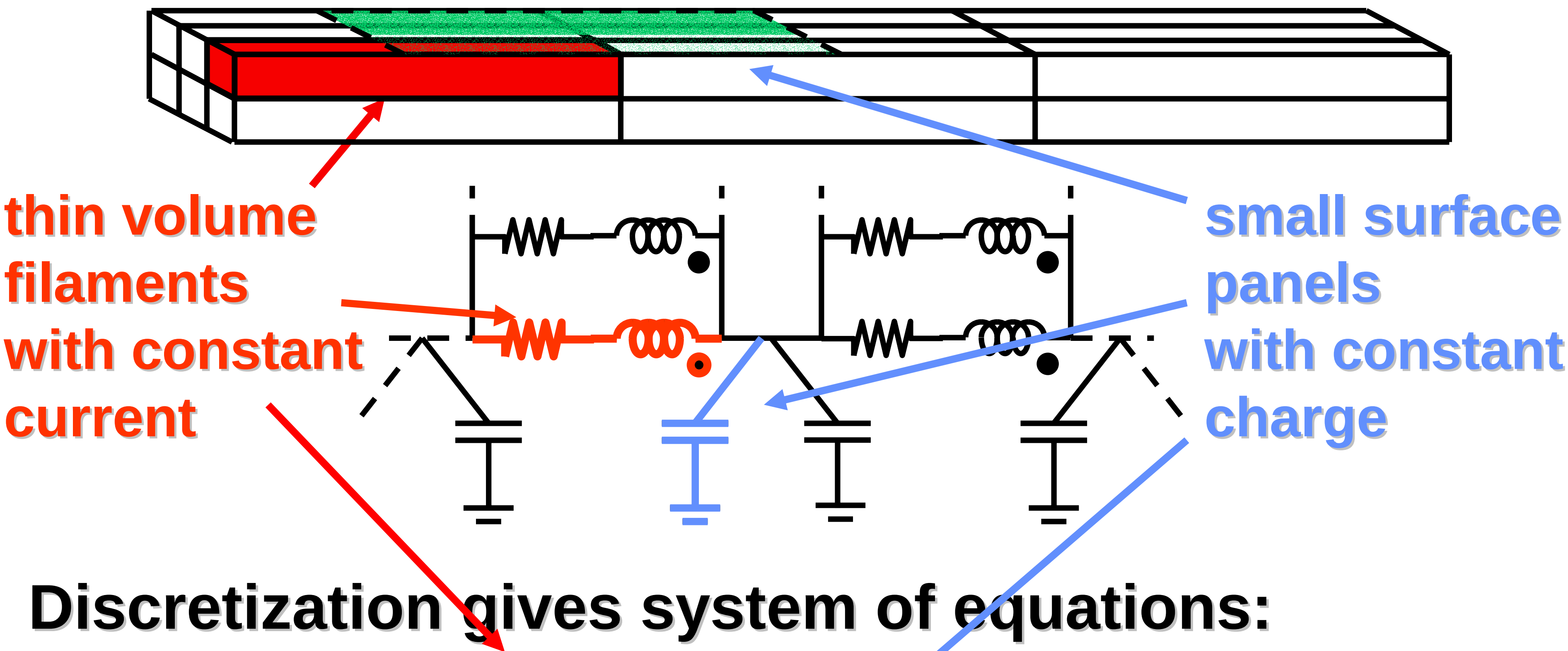
charge-voltage relation

$$\nabla \cdot \mathbf{J}(\mathbf{r}) = 0 \qquad \hat{\mathbf{n}} \cdot \mathbf{J}(\mathbf{r}) = j\omega\rho(\mathbf{r}_s)$$

current and charge conservation

Background [PEEC Ruehli74]
Discretization Basis Functions

- Standard numerical procedure: discretize volume currents and surface charges.

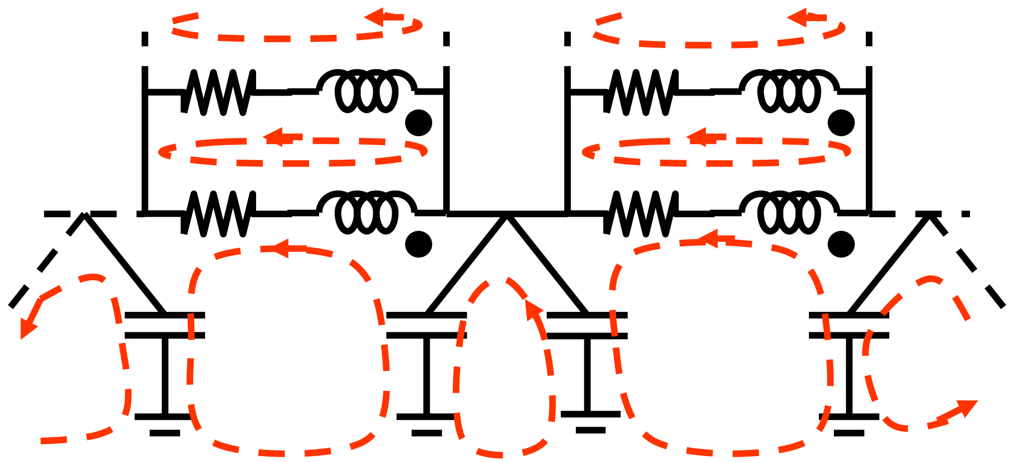


- Discretization gives system of equations:

$$\begin{bmatrix} R_c + j\omega L_c & 0 \\ 0 & P_c \end{bmatrix} \begin{bmatrix} I_c \\ q_c \end{bmatrix} = \begin{bmatrix} V_c \\ \phi_c \end{bmatrix}$$

Background: Mesh Analysis [Kamon et al. Trans Packaging98]

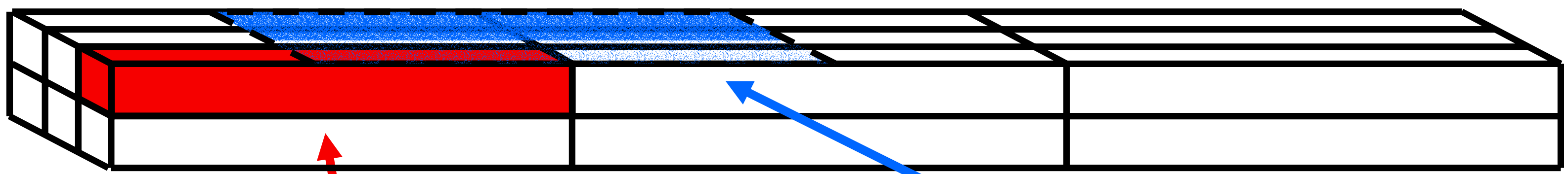
Imposing current conservation with **mesh analysis (KVL)**



$$M\begin{bmatrix} R_c + j\omega L_c & 0 \\ 0 & \frac{P_c}{j\omega} \end{bmatrix} M^T I_m = V_{ms}$$

Background (Fastpep ICCAD97) Volume Integral Equation Method (cont.)

- **Piece-wise constant discretization basis functions: volume filaments for currents, surface panel for charges.**



thin volume filaments with constant current

small surface panels with constant charge

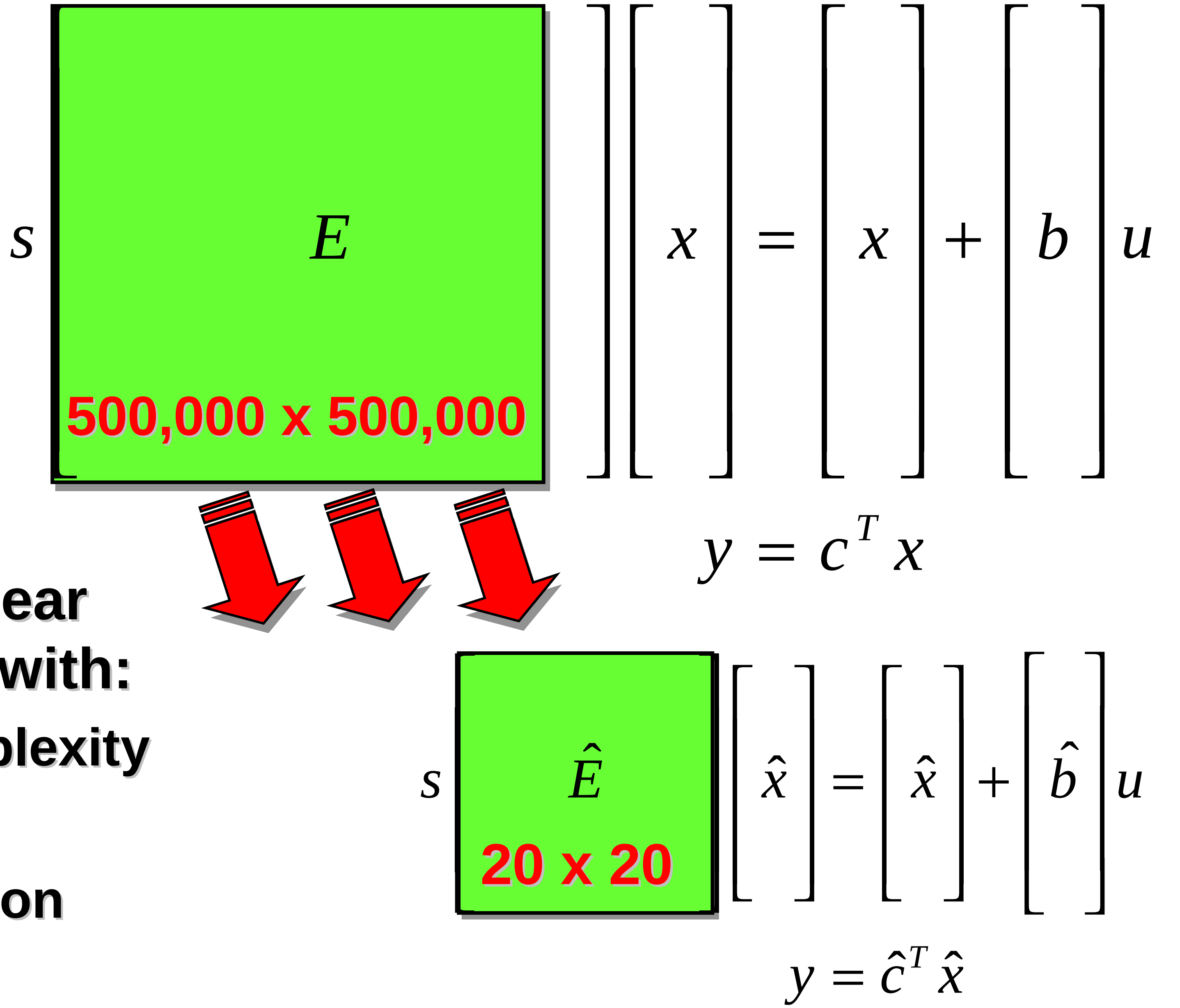
- **Discretization produces a huge state space dynamical linear system:**

$$s\begin{bmatrix} M_f L_f M_f^T & 0 \\ 0 & P^{-1} \end{bmatrix} x = -\begin{bmatrix} M_f R_f M_f^T & M_p \\ -M_p^T & 0 \end{bmatrix} x + \begin{bmatrix} V_m \\ 0 \end{bmatrix} u$$

L R

Background
Non-parameterized Model order reduction

- **Given a large linear system model:**
- **Construct a linear system model with:**
  - **smaller complexity**
  - **same fidelity**
  - **small reduction cost**



Background
Model order reduction (cont.)

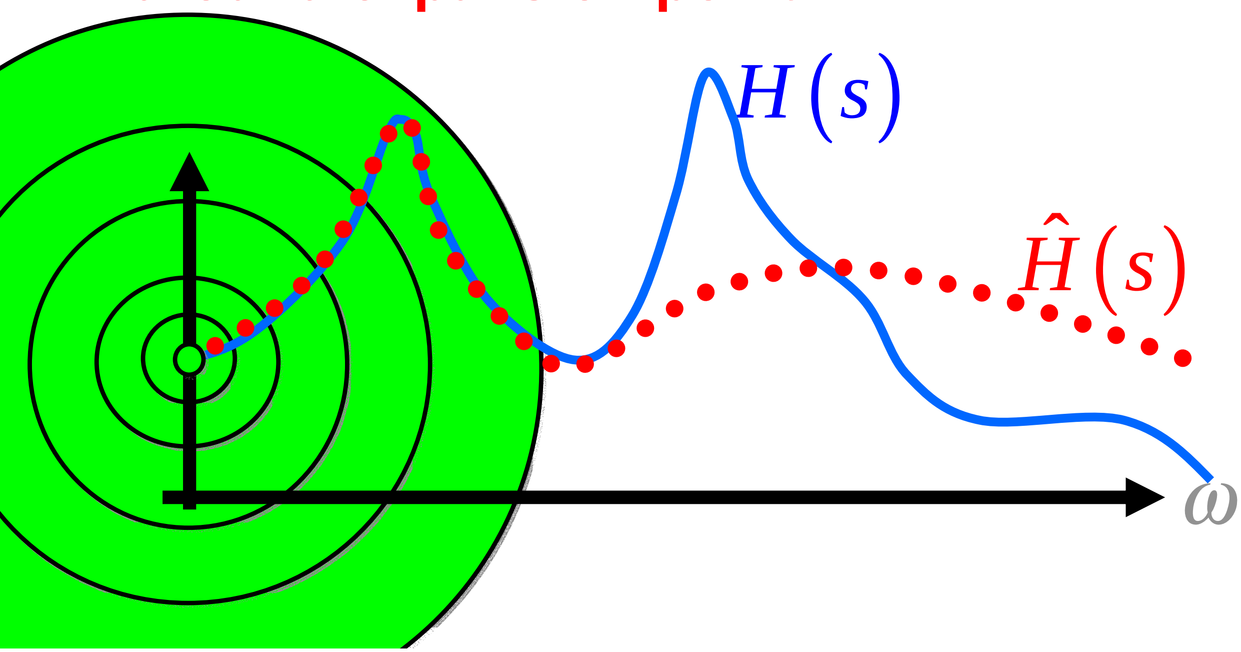
$$sEx = x + bu \Rightarrow x = -(I - sE)^{-1} bu$$

Taylor series expansion:

$$x = -\sum_{k=0}^{\infty} s^k \; E^k b \quad u \Rightarrow x \in \text{span}\{b, Eb, E^2 b, \cdots\}$$

- **change basis and use only the first few vectors of the Taylor series expansion: equivalent to match first derivatives around expansion point**

$$\begin{bmatrix} x \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} b \end{bmatrix} & \begin{bmatrix} Eb \end{bmatrix} & \begin{bmatrix} E^2 b \end{bmatrix} \end{bmatrix} \begin{bmatrix} \hat{x} \end{bmatrix}$$



Background
Projection Framework (cont.)

$$sEx = x + bu \quad \Rightarrow \quad s\underbrace{U_q^T E U_q}_{\hat{E}}\hat{x} = \hat{x} + \underbrace{U_q^T b}_{\hat{b}}u$$

$$y = c^T x \quad \Rightarrow \quad y = \underbrace{c^T U_q}_{\hat{c}^T}\hat{x}$$

Congruence transformation

U_q^T (**qxn**) $\quad E$ (**nxn**) $\quad U_q$ (**nxq**) $\Rightarrow$ $\hat{E}$ (**qxq**)

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Background: Model Order Reduction for LINEARLY Parameterized Systems

- **Given a large parameterized linear system:**

$$\left(E_0 + s_1 E_1 + \ldots + s_p E_p \right) x = b\,u$$

$$y = c^T x$$

$$\left(\hat{E}_0 + s_1 \hat{E}_1 + \ldots + s_p \hat{E}_p \right) \hat{x} = \hat{b}\,u$$

$$\hat{y} = \hat{c}^T \hat{x}$$

- **construct a reduced order system with similar frequency response**

Background: Model Order Reduction for LINEARLY Parameterized Systems [Weile99, Daniel ISPD02]

two parameters

p-parameters

$$\left[s_1 E_1 + \ldots + s_p E_p - I \right] x = bu$$

$$y = c^T x$$

$$x = -\left[I - \left(s_1 E_1 + \cdots + s_p E_p \right) \right]^{-1} bu = \sum_{m=0}^{\infty} \left(s_1 E_1 + \cdots + s_p E_p \right)^m b \quad u$$

- **It is a p-variables Taylor series expansion**

$$x \in span\{\underbrace{b, E_1 b, E_2 b, \cdots, E_p b}, E_1^2 b, (E_1 E_2 + E_2 E_1) b, \cdots\}$$

$$[x] = [U_q][\hat{x}]$$

Once again change basis and project state onto the first few vectors of the Taylor series expansion, in order to match the first derivatives with respect to all parameters

Background: Model Order Reduction for LINEARLY Parameterized Systems [Daniel at al. ISPD02]

$$[E_0 + s_1 E_1 + \ldots + s_p E_p]\, x = b\, u, \qquad y = c^T x$$

$$[\underbrace{U^T E_0 U}_{\hat{E}_0} + s_1 \underbrace{U^T E_1 U}_{\hat{E}_1} + \ldots + s_p \underbrace{U^T E_p U}_{\hat{E}_p}]\, x = \underbrace{U^T b}_{\hat{b}}\, u, \qquad y = \underbrace{c^T U}_{\hat{c}}\, x$$

$$U^T \; E_k \; U = \hat{E}_k$$

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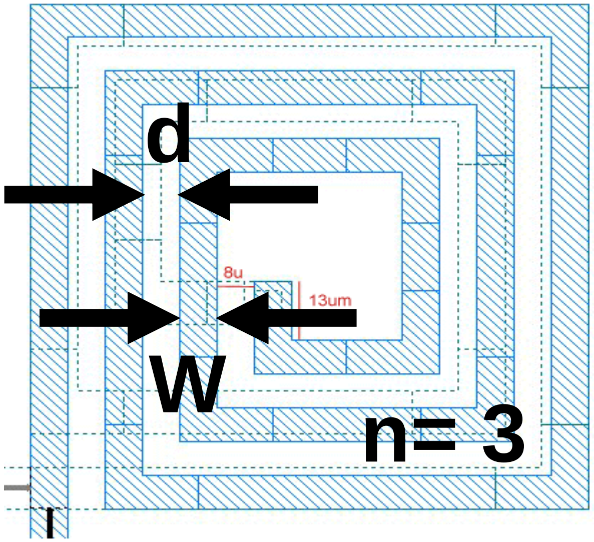
Hypothesis

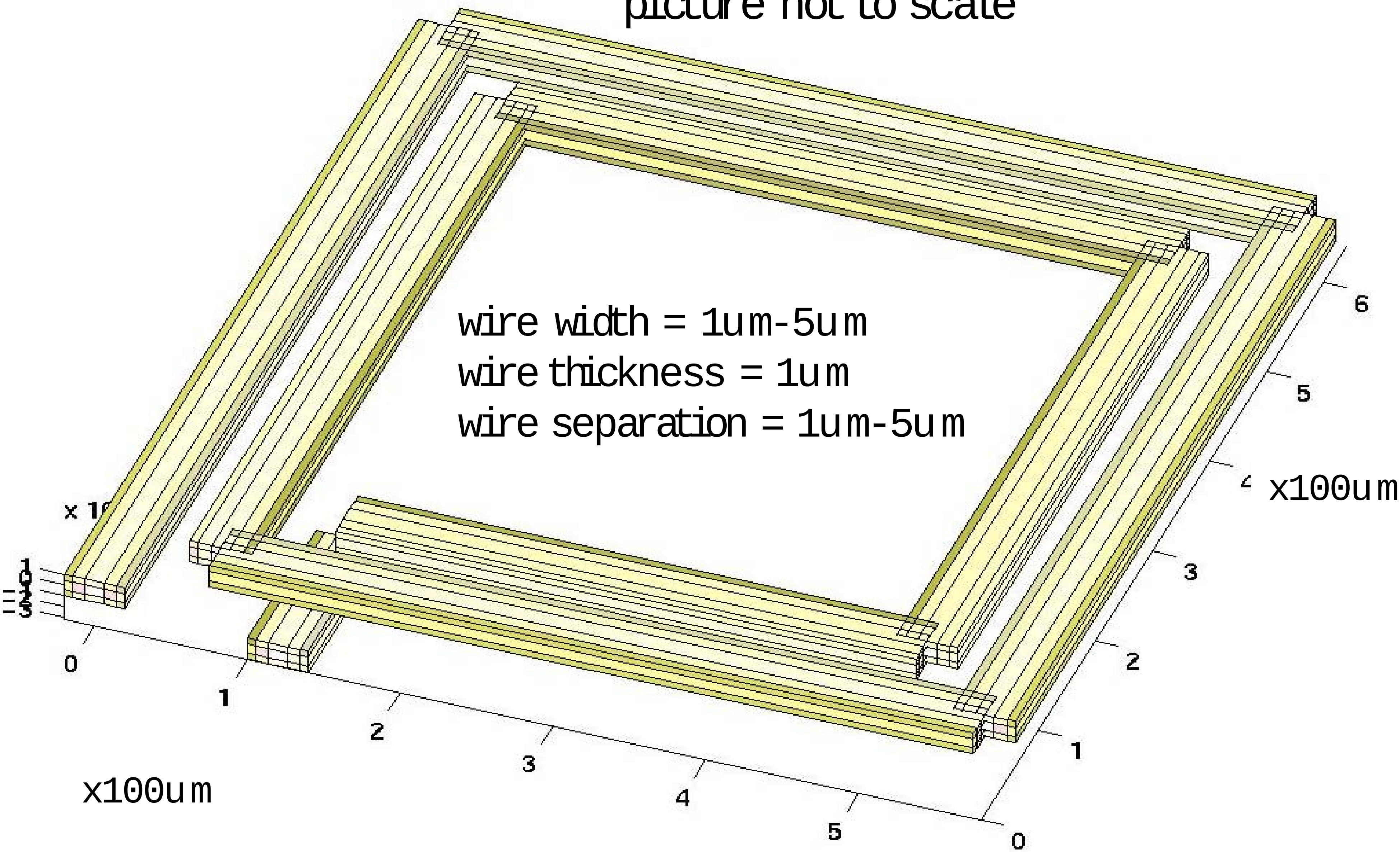
- Design parameters:
  - wire dimensions and separations,
  - number of turns
  - type of substrate and distance from wires
- Performance parameters:
  - inductance
  - quality factor Q
  - resonance frequency
  - power
  - area
- Effects captured:
  - displacement currents — affect resonance
  - skin effect in wires — affect Q
  - proximity effect in wires — affect Q
  - dielectrics — affect Q and resonance
  - substrate eddy currents — affect Q and resonance
  - interference with other devices



A simple example:
Parameterized model reduction of a two-turns RF-Inductor

overall dimensions = 600um x 600um
picture not to scale

wire width = 1um-5um
wire thickness = 1um
wire separation = 1um-5um



An Interpolation Approach for Model Order Reduction of NON-LINEARLY Parameterized Systems

$$[R(W,d) + s\,L\,(W,d)]\;x = b\;u$$

1. **Fit a low order polynomial (e.g. quadratic) to the evaluated matrices R and L**

$$R(W,d) \approx R_{0,0} + W R_{1,0} + d\,R_{0,1} + W^2 R_{2,0} + Wd\,R_{1,1} + d^2\,R_{0,2}$$

$$L(W,d) \approx L_{0,0} + W L_{1,0} + d\,L_{0,1} + W^2 L_{2,0} + Wd\,L_{1,1} + d^2\,L_{0,2}$$

$$[R_{0,0} + W R_{1,0} + d\,R_{0,1} + W^2 R_{2,0} + Wd\,R_{1,1} + d^2\,R_{0,2} + \ldots$$

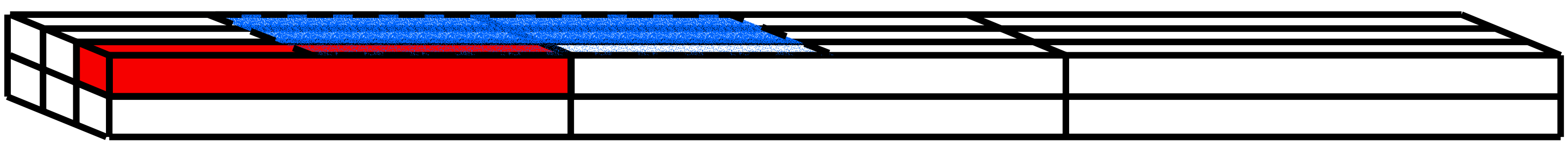
$$s L_{0,0} + sW L_{1,0} + sd\,L_{0,1} + sW^2 L_{2,0} + sWd\,L_{1,1} + sd^2\,L_{0,2}\,]\;x = b\;u$$

An Interpolation Approach for Model Order Reduction of NON-LINEARLY Parameterized Systems

2. **Selected a grid of 9 evaluation points for different combination of parameters**

$$(W,d) = (1um,1um), (1um,3um), (1um,5um), (3um,1um), (3um,3um), (3um,5um), (5um,1um), (5um,3um), (5um,5um)$$

3. **Used the Volume Integral Equation code to generate system matrices $L_k = L(W_k, d_k)$ and $R_k = R(W_k, d_k)$ for each combination of parameters**



$$\underbrace{\begin{bmatrix} M_f R_f M_f^T & M_p \\ -M_p^T & 0 \end{bmatrix}}_{L_k} + s \underbrace{\begin{bmatrix} M_f L_f M_f^T & 0 \\ 0 & P^{-1} \end{bmatrix}}_{R_k} x = bu$$

Calculating Interpolation coefficients

- **Need to calculate 6 polynomial coefficients**
- **Hence need at least 6 equations imposing fit in 6 evaluation points**
- **However in general it is better to use more evaluation points than the minimum.**
- **For instance here we used the 9 evaluation points above and solved with a least square method**

$$\begin{bmatrix} 1 & W_1 & d_1 & W_1^2 & W_1 d_1 & d_1^2 \\ 1 & W_2 & d_2 & W_2^2 & W_2 d_2 & d_2^2 \\ 1 & W_3 & d_3 & W_3^2 & W_3 d_3 & d_3^2 \\ 1 & W_4 & d_4 & W_4^2 & W_4 d_4 & d_4^2 \\ 1 & W_5 & d_5 & W_5^2 & W_5 d_5 & d_5^2 \\ 1 & W_6 & d_6 & W_6^2 & W_6 d_6 & d_6^2 \\ 1 & W_7 & d_7 & W_7^2 & W_7 d_7 & d_7^2 \\ 1 & W_8 & d_8 & W_8^2 & W_8 d_8 & d_8^2 \\ 1 & W_9 & d_9 & W_9^2 & W_9 d_9 & d_9^2 \end{bmatrix} \begin{bmatrix} \mathrm{R}_{0,0}^{i,j} \\ \mathrm{R}_{1,0}^{i,j} \\ \mathrm{R}_{0,1}^{i,j} \\ \mathrm{R}_{2,0}^{i,j} \\ \mathrm{R}_{1,1}^{i,j} \\ \mathrm{R}_{0,2}^{i,j} \end{bmatrix} = \begin{bmatrix} \mathrm{R}_1^{i,j} \\ \mathrm{R}_2^{i,j} \\ \mathrm{R}_3^{i,j} \\ \mathrm{R}_4^{i,j} \\ \mathrm{R}_5^{i,j} \\ \mathrm{R}_6^{i,j} \\ \mathrm{R}_7^{i,j} \\ \mathrm{R}_8^{i,j} \\ \mathrm{R}_9^{i,j} \end{bmatrix}$$

An Interpolation Approach for Model Order Reduction of NON-LINEARLY Parameterized Systems (cont.)

4. **Transformed the polynomial parameterized system into a linearly parameterized system introducing new parameters**

$E_0 \quad s_1\ E_1 \quad s_2\ E_2 \quad s_3\ E_3 \quad s_4\ E_4 \quad s_5\ E_5$

$$[R_{0,0} + W R_{1,0} + d\ R_{0,1} + W^2 R_{2,0} + Wd\ R_{1,1} + d^2\ R_{0,2} + \dots$$

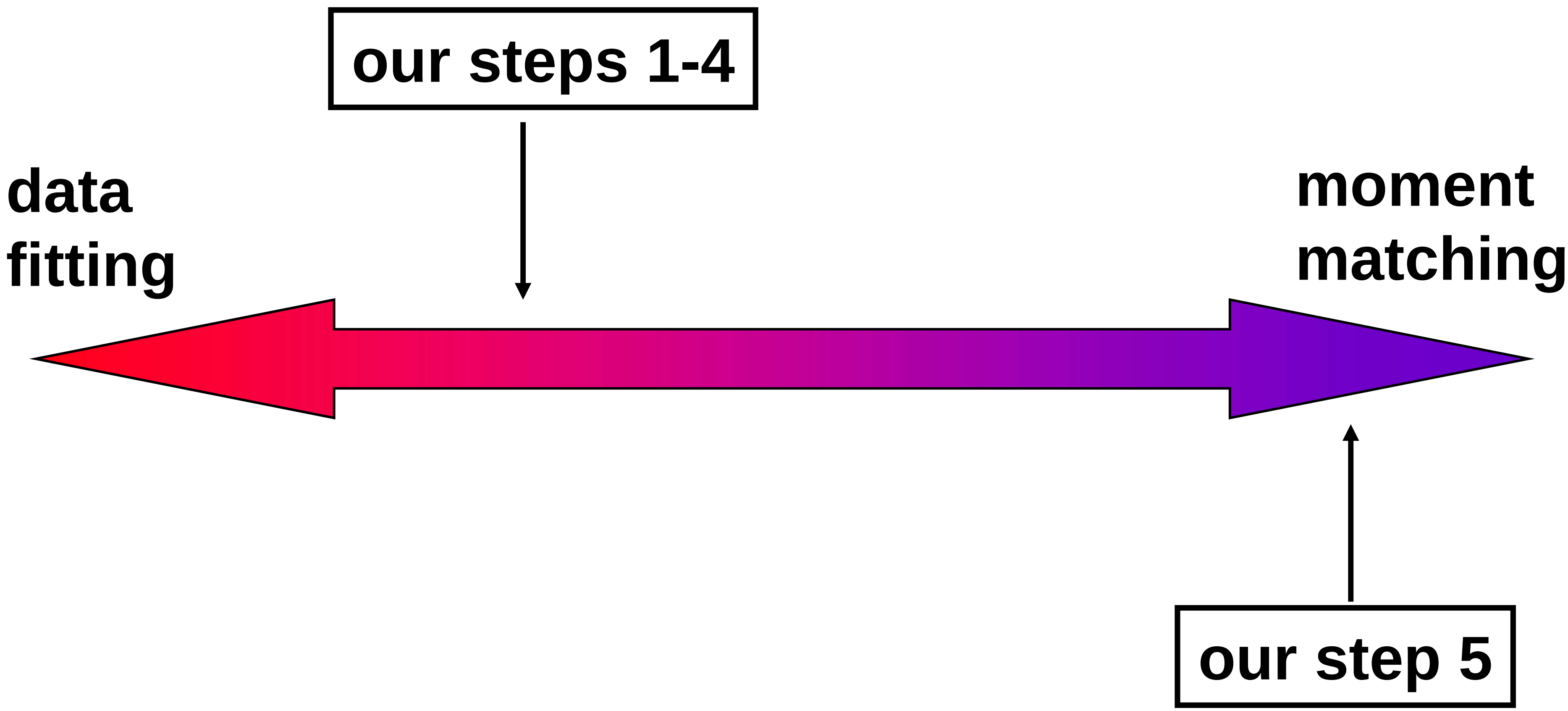
$$s L_{0,0} + sW L_{1,0} + sd\ L_{0,1} + sW^2 L_{2,0} + sWd\ L_{1,1} + sd^2\ L_{0,2}\ ]\ x = b\ u$$

$s_6\ E_6 \quad s_7\ E_7 \quad s_8\ E_8 \quad s_9\ E_9 \quad s_{10}\ E_{10} \quad s_{11}\ E_{11}$

$$[E_0 + s_1 E_1 + s_2 E_2 \dots + s_{11} E_{11}]\ x = b\ u$$

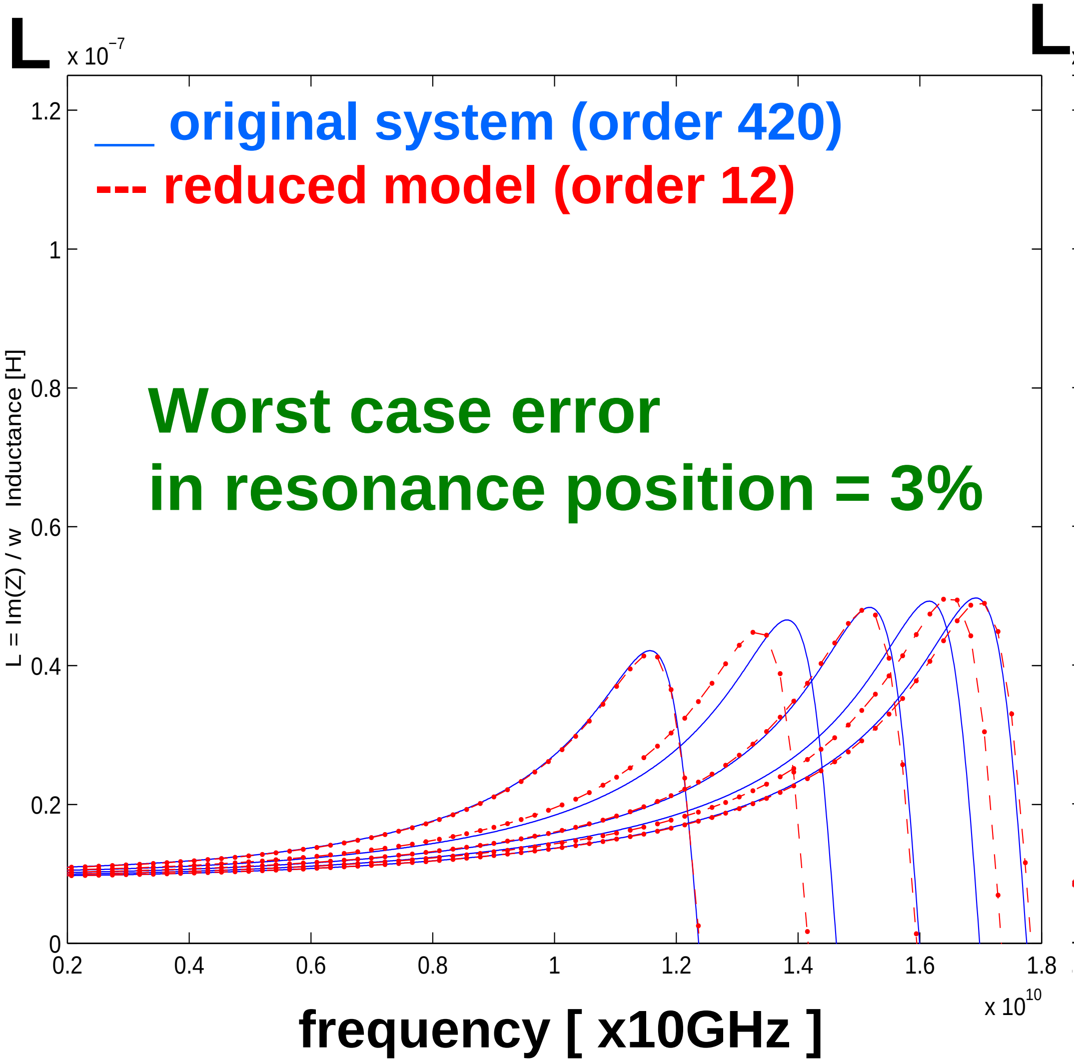
5. **Used the previously developed model reduction for linearly parameterized systems (ISPD02)**

Where is this in Rutenbar's plot?

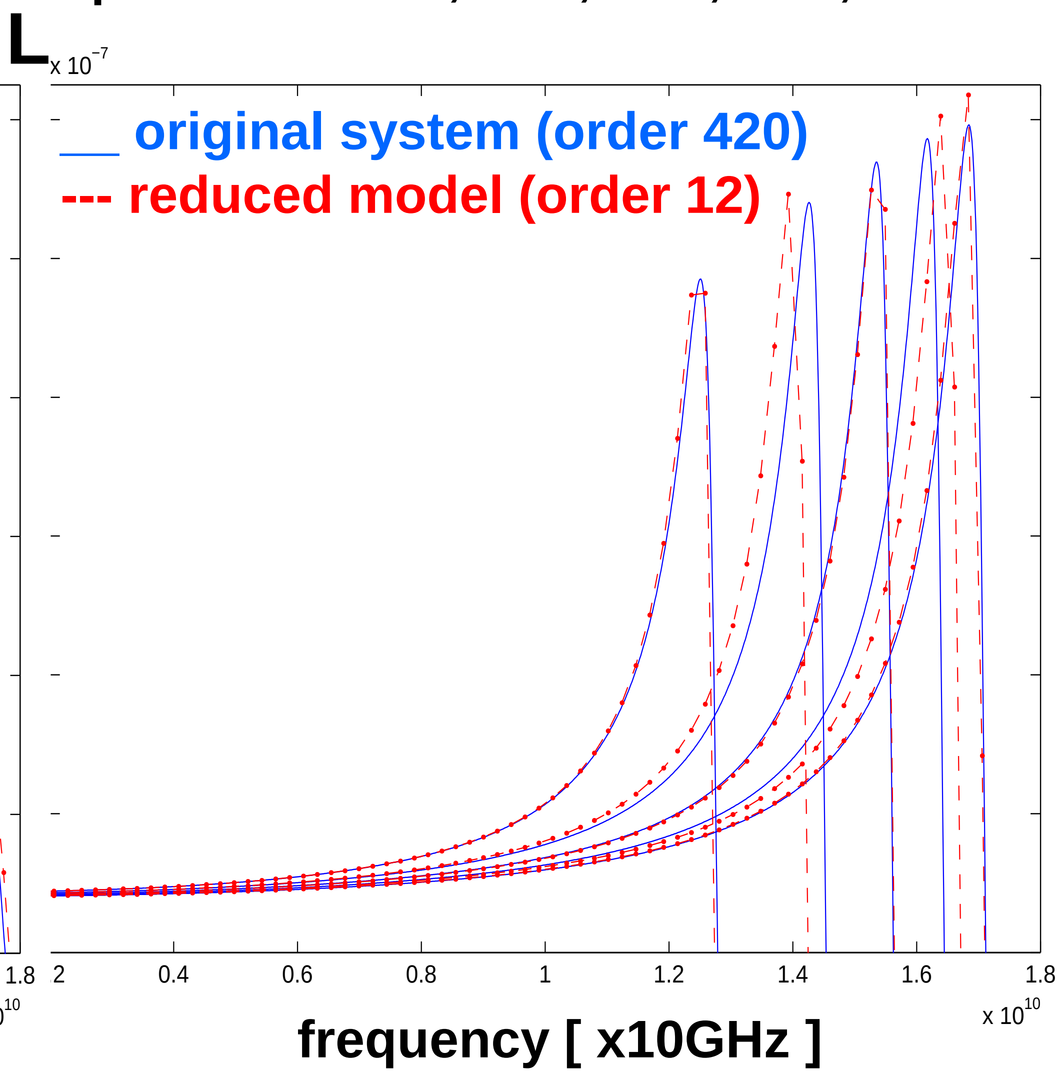


Results: Inductance vs. frequency

Wire width = 1um
separation = 1um, 2um, 3um, 4um, 5um

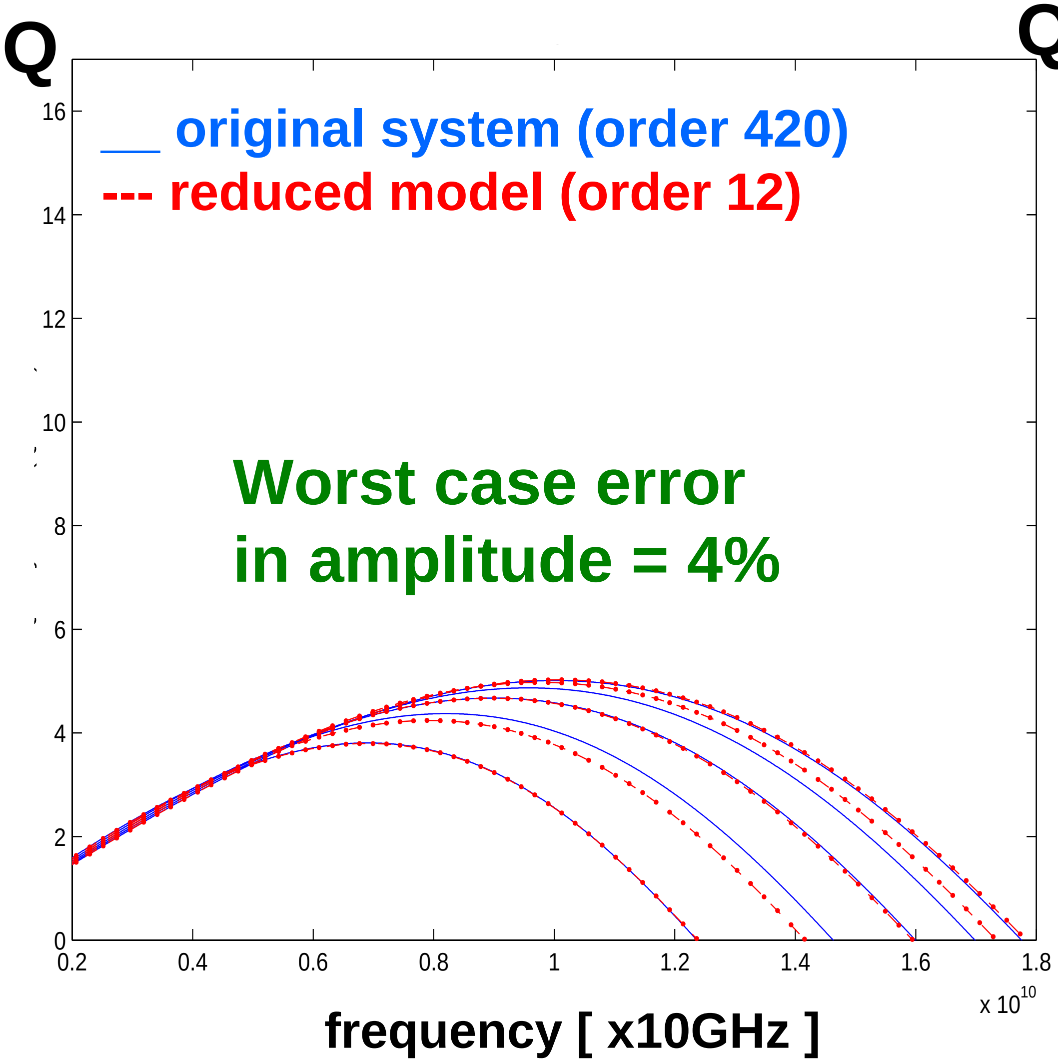


Wire width = 5um
separation = 1um, 2um, 3um, 4um, 5um

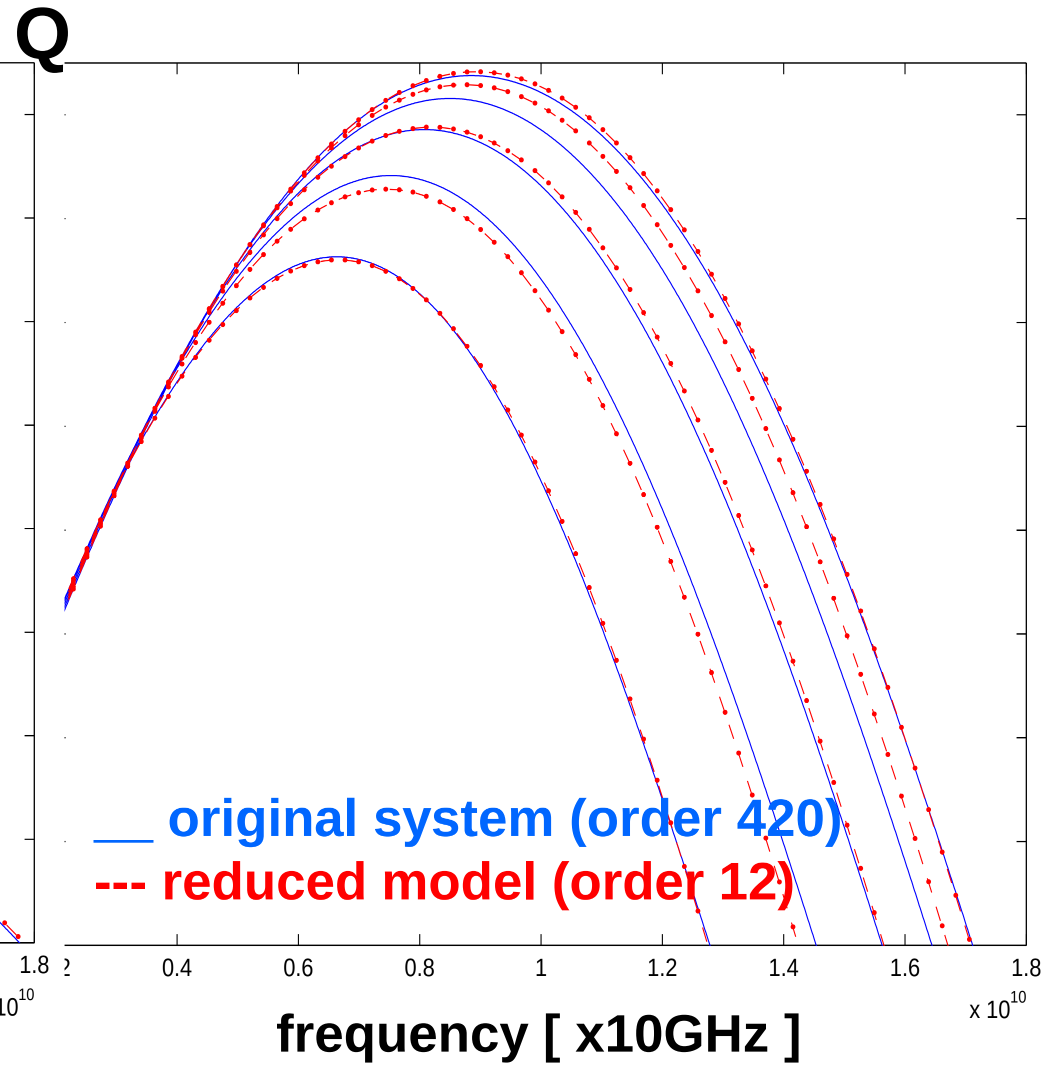


Results: Quality factor (Q=wL/R) vs. frequency

Wire width = 1um
separation = 1um, 2um, 3um, 4um, 5um

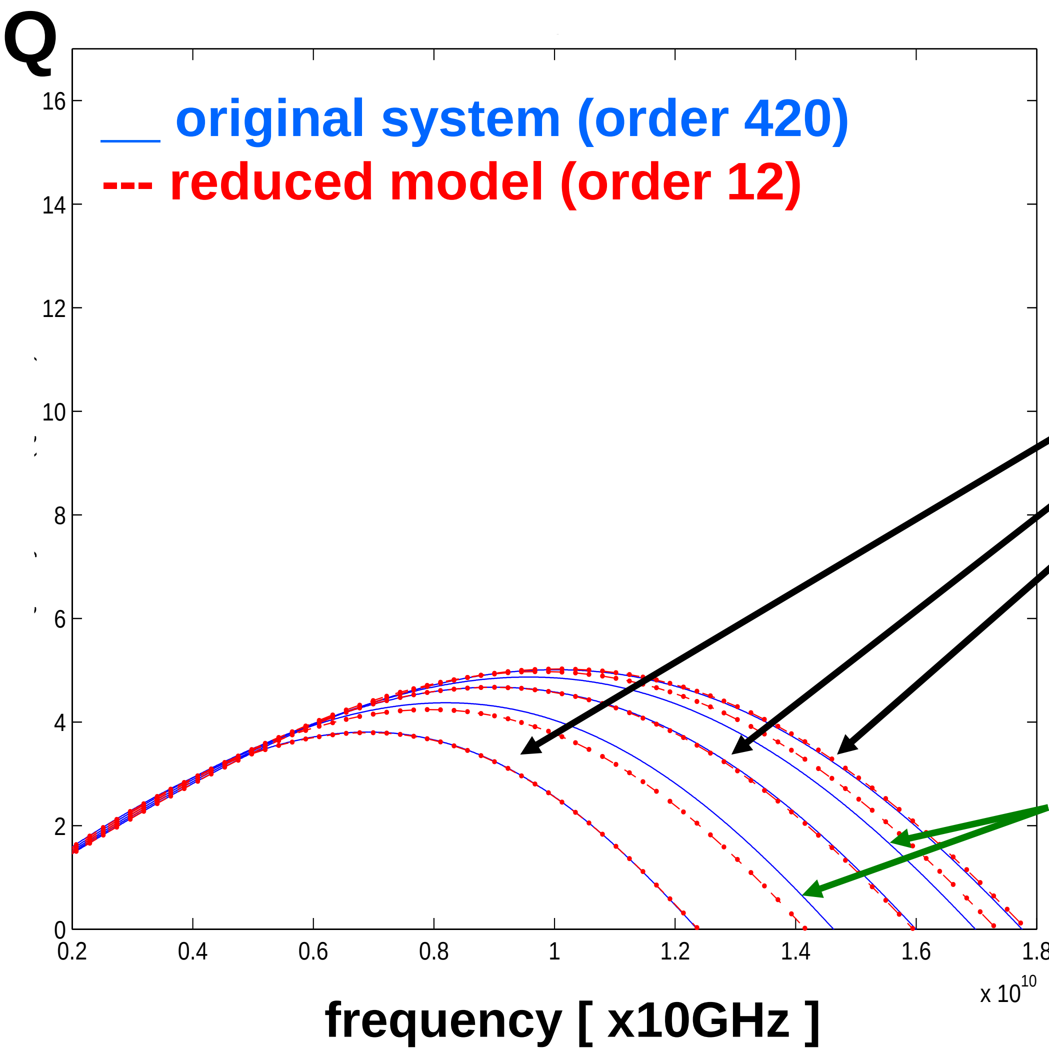


Wire width = 5um
separation = 1um, 2um, 3um, 4um, 5um



Limitations and future work
use better interpolation!

Wire width = 1um
separation = 1um, 2um, 3um, 4um, 5um



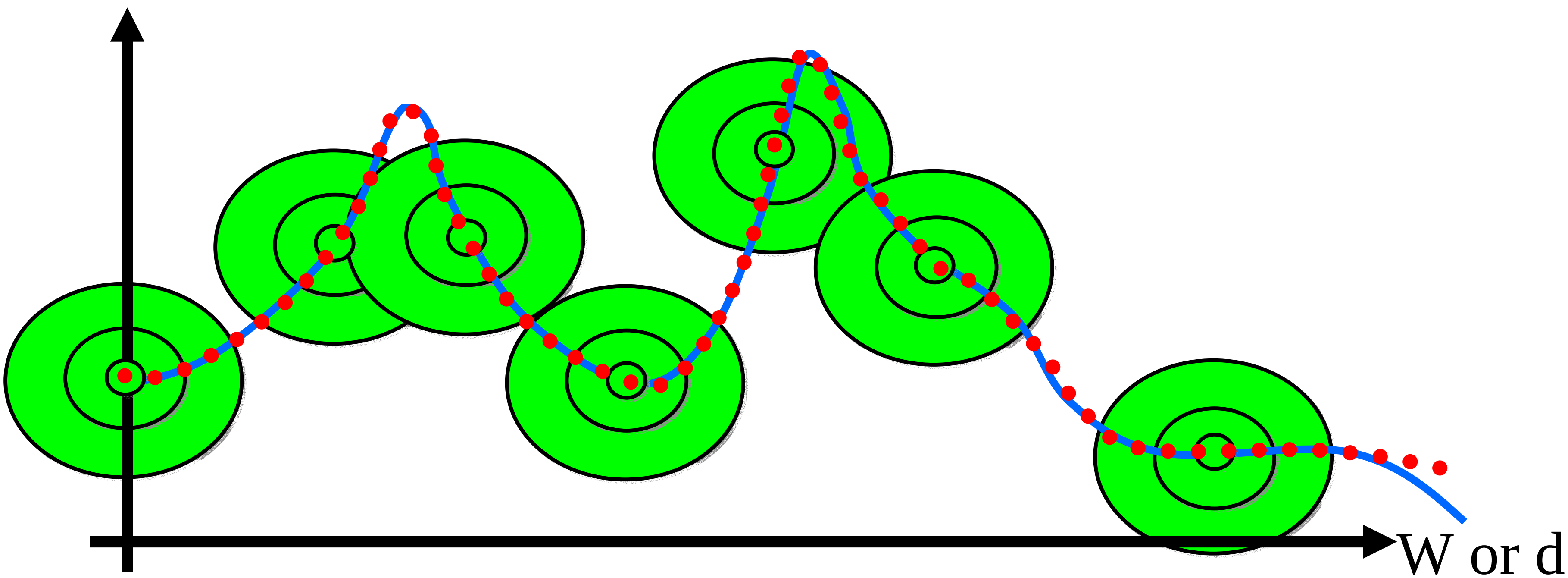
very little error in the points used for fitting 1um, 3um, 5um

so the critical step is the fitting!!

Worst case errors far from fitting points 2um, 4um (3% in resonance position)

Limitations and future work in parameterized reduced order modeling

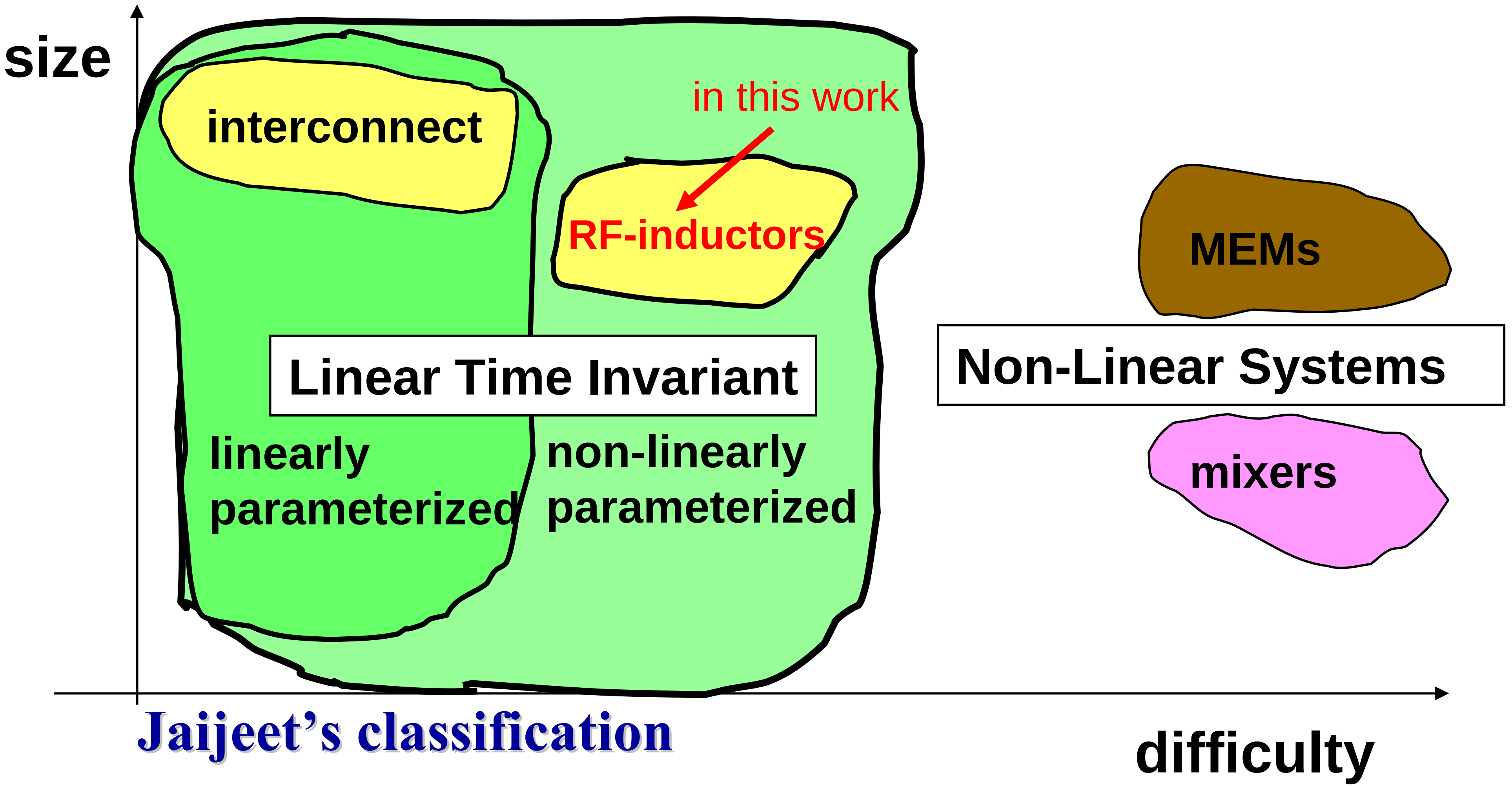
- As for linearly parameterized MOR in ISPD02, Model order grows as $O(p^m)$ where p = # parameters and m = # derivatives matched for each parameter
  - however is linear in # of parameters when matching only one derivative (m = 1) per parameter s_j and still produces good accuracy in our experiments.
  - furthermore can try multiple points moment matching in parameterized reduction



Limitations and future work.
include other second order effects

- Need to include effects of IC substrate and Full-wave effects
  - important for RF inductors, packages and other applications
  - e.g. use green functions (most computationally effective for field solver)
  - but most challenging from model reduction standpoint

Future work. Need parameterized model reduction for non-linear systems



Jaijeet's classification

- Need parameterized model order reduction for non-linear systems
  - much more complicated than non-linearly parameterized system response: cannot even talk about a "frequency" response

Conclusions

- **Systems on Chip and on Package are becoming a collection of very heterogeneous components**
- **Parameterized Reduced Order Modeling can enable system optimization by producing component models:**
  - **automatically and robustly**
  - **as accurate as a field solver**
  - **small for evaluation within an optimization loop**
- **Non-linear dependency on geometric parameters can be captured by simple polynomial interpolation**
- **Introducing new parameters for each polynomial term transforms system into a linearly parameterized reduction problem (can use e.g. Daniel ISPD02)**
- **Example: simple two turns RF-inductor neglecting substrate, parameterized models are 3-4% accurate**