

# Modeling Memory Effects in Nonlinear Subsystems by Dynamic Volterra Series

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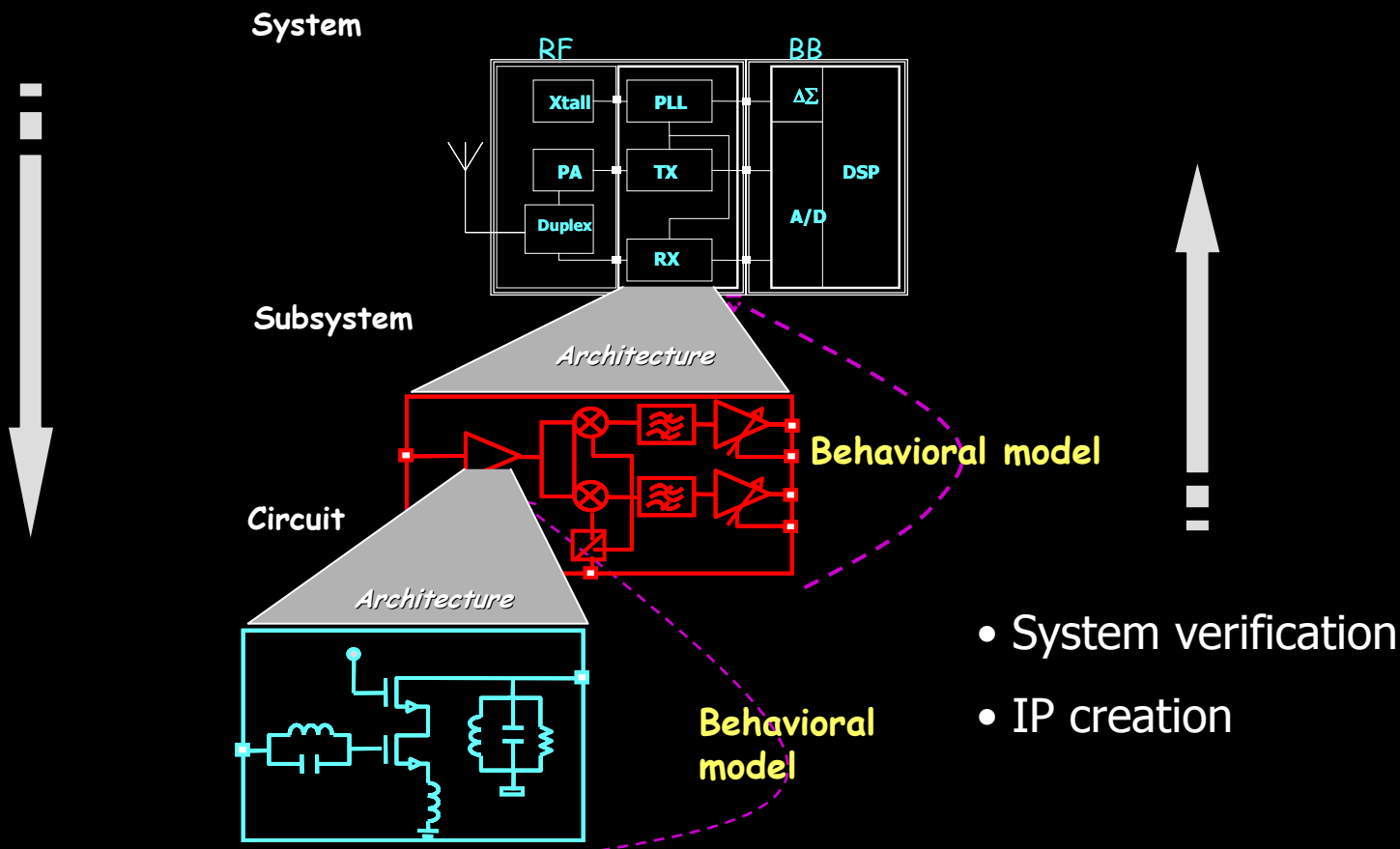
## Outline

- RF subsystem modeling challenge
- Envelope domain modeling basics
- Modified Volterra series approaches for highly nonlinear systems with memory
  - Examples
  - Summary

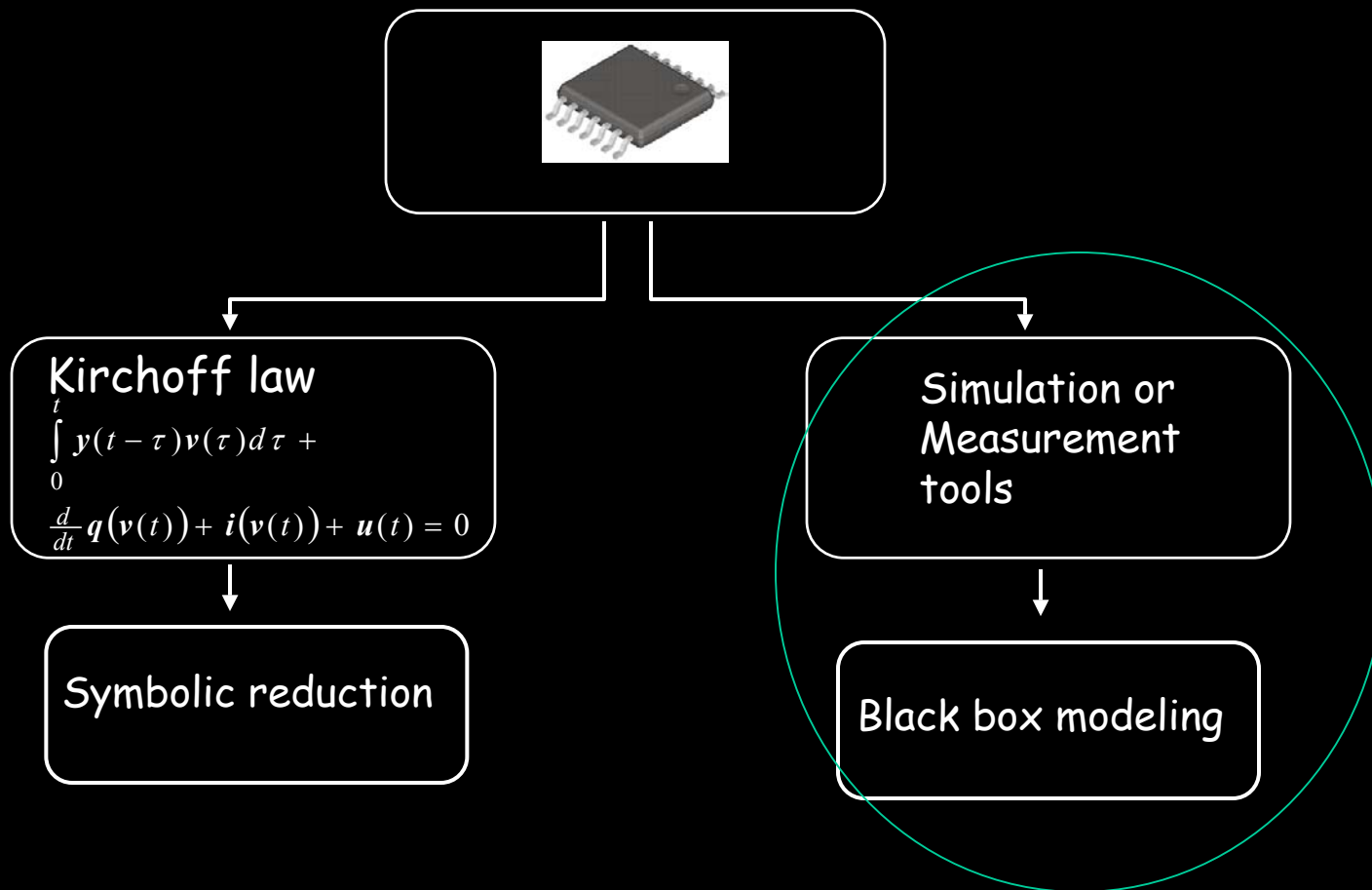


# Behavioral modeling for system verification and IP creation

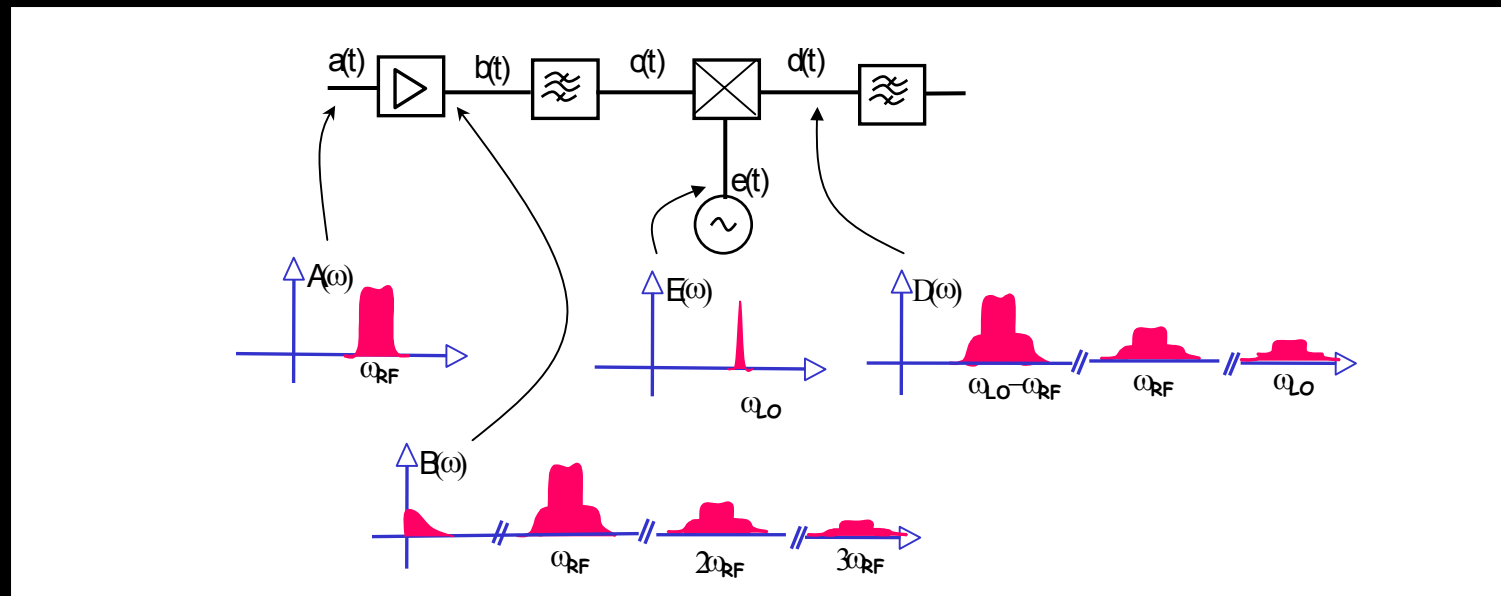
- Top-down design



# Bottom-up block modeling



## RF & microwave subsystem modeling challenge

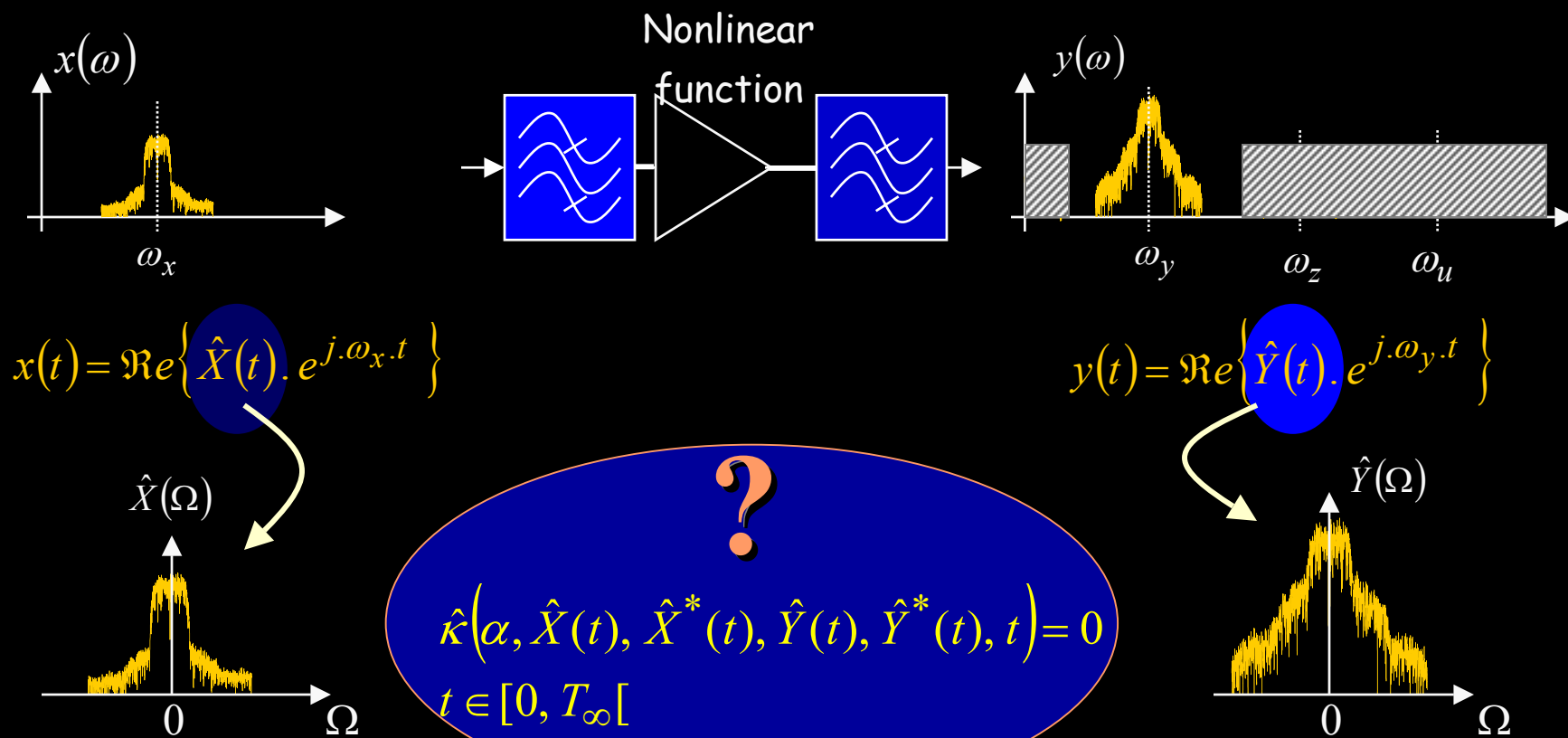


- Time scales disparity (Baseband signals, carriers, noise, spurious)
- Pass band components (High Q filters, transmission lines, ..)
- Nonlinear functions (PA, converters, mixer,..)

➡ **Pass band – Enveloppe concept**

# Pass band subsystem

amplifier  
mixer  
multiplier  
VCO



# Functional modeling



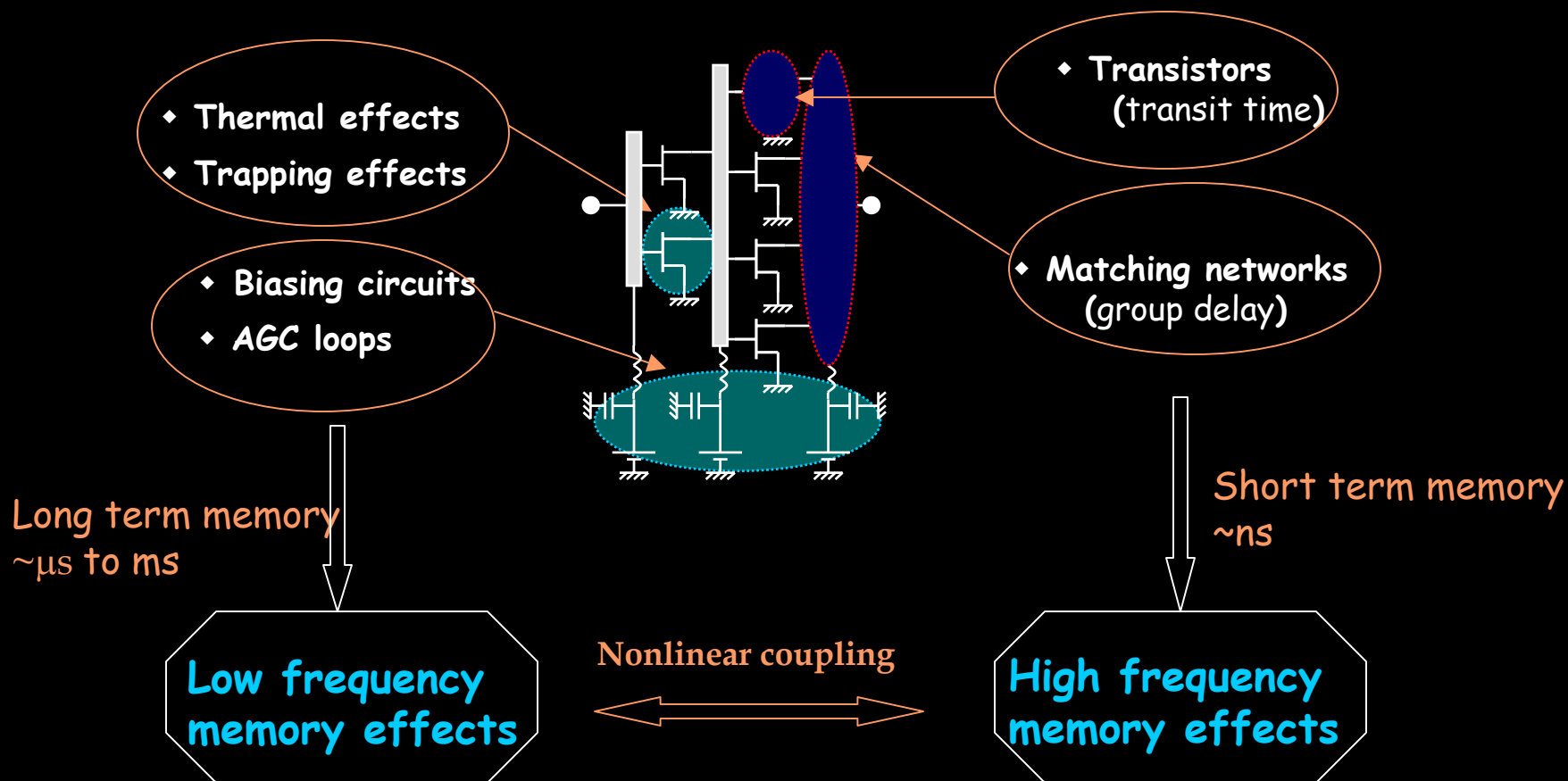
$$\hat{k}(\alpha, \hat{X}(t), \hat{X}^*(t), \hat{Y}(t), \hat{Y}^*(t), t) = 0$$

$$t \in [0, T_\infty[$$

## Three major problems

- Nonlinearity
- Interstage mismatch
- Memory effects
  - High speed applications
  - High capacity

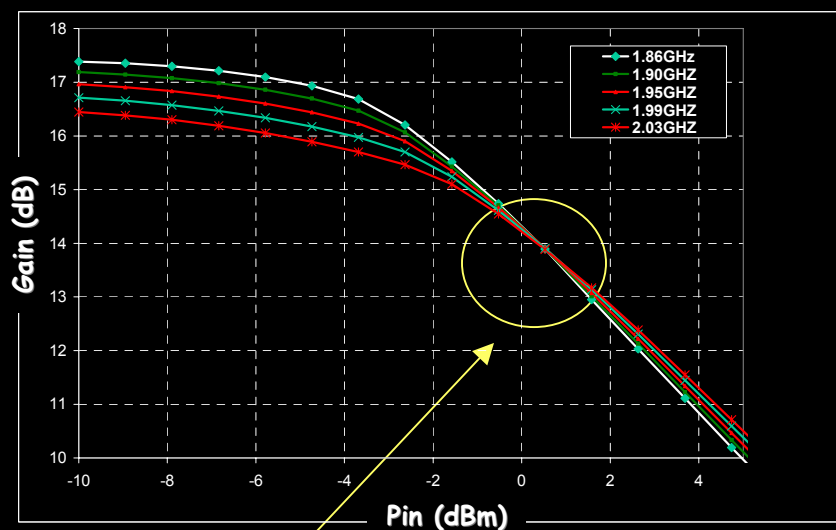
# Severe memory effects in solid state devices





# Memory effects highlights : Typical narrow band BiCMOS LNA

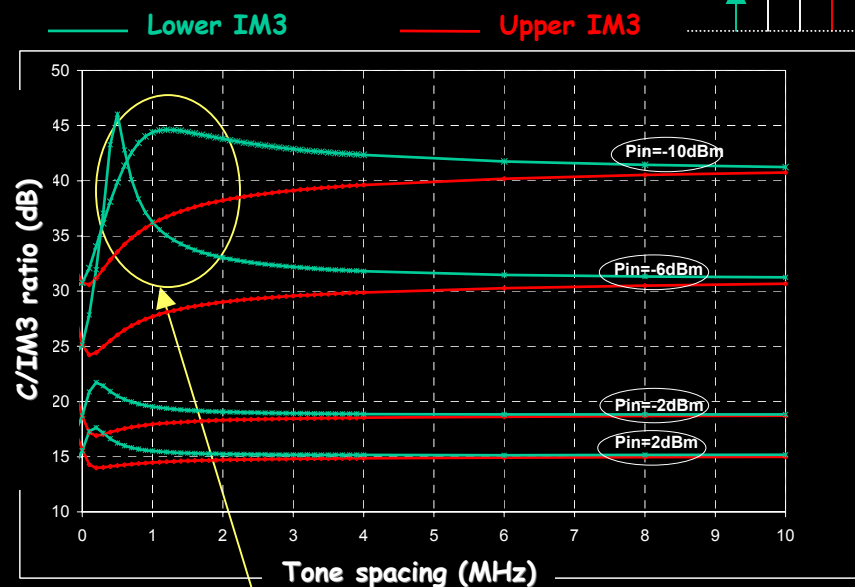
Short term memory : Single tone measurements



wrapping



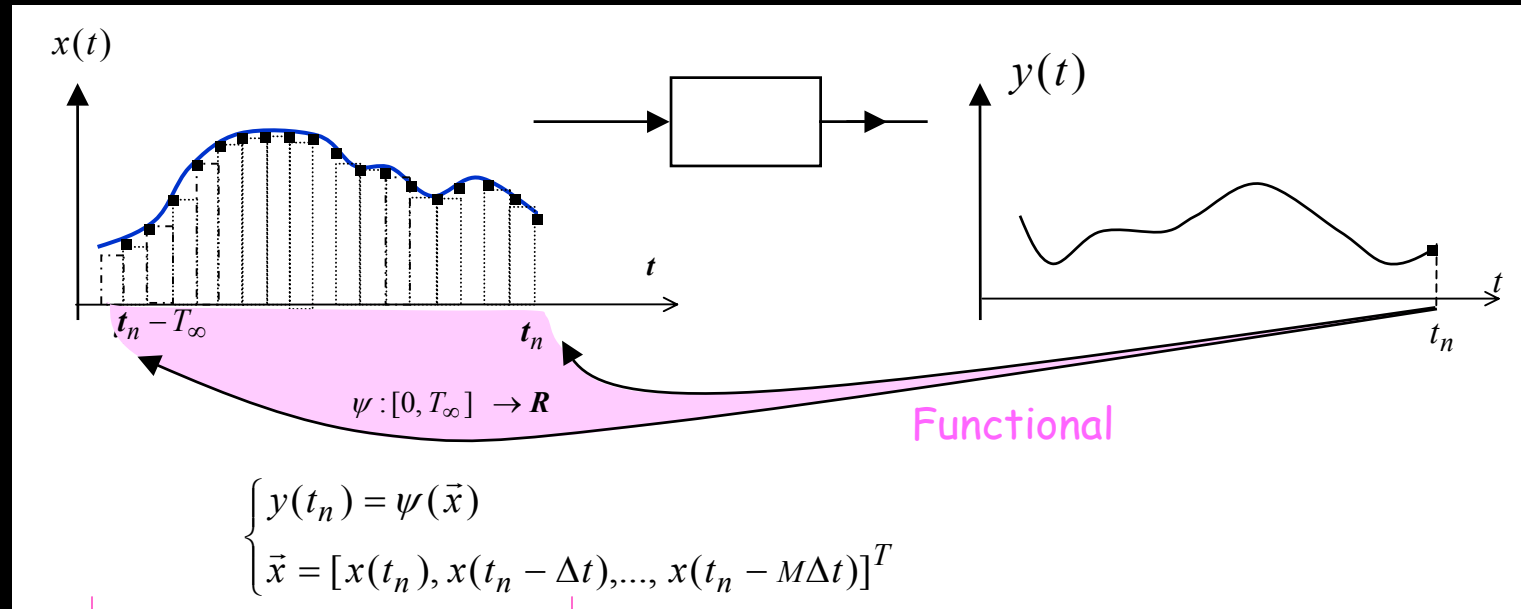
Long term memory : Two-tone measurements



Resonance & asymmetry

Need rigorous but yet simple modeling approach (affordable with either simulation and measurement tools)

# Analytic approach: Volterra series expansion



$T_\infty$  Memory duration

## Power series expansion of the functional

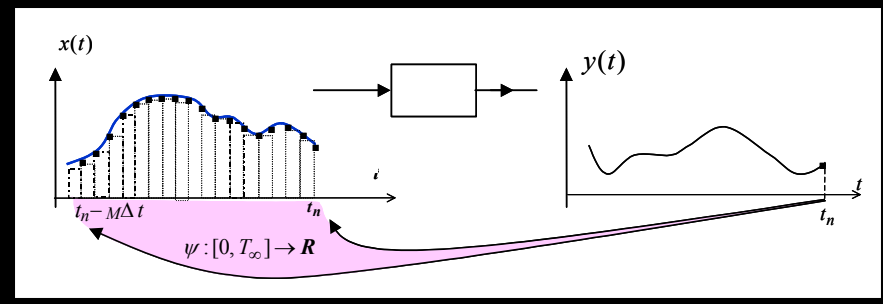
$$y(t_n) = \psi(\vec{x}_0) + \Delta \psi(\vec{x}_0)^T (\vec{x} - \vec{x}_0) + \frac{1}{2} (\vec{x} - \vec{x}_0)^T [\Delta^2 \psi(\vec{x}_0)] (\vec{x} - \vec{x}_0) + \dots$$

↳  $\vec{x}_0$  is an arbitrary input record

# Volterra series approach

$$\vec{x}_0 = \vec{0}, \quad \Delta t \rightarrow 0,$$

$$y(t) = \sum_{n=1}^{\infty} y_n(t)$$



$$y_1(t) = \int_0^{T_\infty} h_1(\tau_1) x(t - \tau_1) d\tau_1 \quad \longrightarrow \quad \text{linear response}$$

$$y_2(t) = \int_0^{T_\infty} \int_0^{T_\infty} h_2(\tau_1, \tau_2) x(t - \tau_1) x(t - \tau_2) d\tau_1 d\tau_2 \quad \longrightarrow \quad \text{quadratic}$$

...

$$y_n(t) = \int_0^{T_\infty} \dots \int_0^{T_\infty} h_n(\tau_1, \dots, \tau_n) x(t - \tau_1) \dots x(t - \tau_n) d\tau_1 \dots d\tau_n \quad \longrightarrow \quad \text{order } n$$

Volterra kernel of order n

• Nonlinear circuit  $\implies$  consider  $n > 1$   $\implies$  practically inefficient



# Modified Volterra series

$$\vec{x}_0 = \vec{f}(t_n) \neq \vec{0},$$

$$y(t) = \sum_{n=0}^{\infty} y_n(t)$$

$$y_0(t) = \psi(x(t))$$

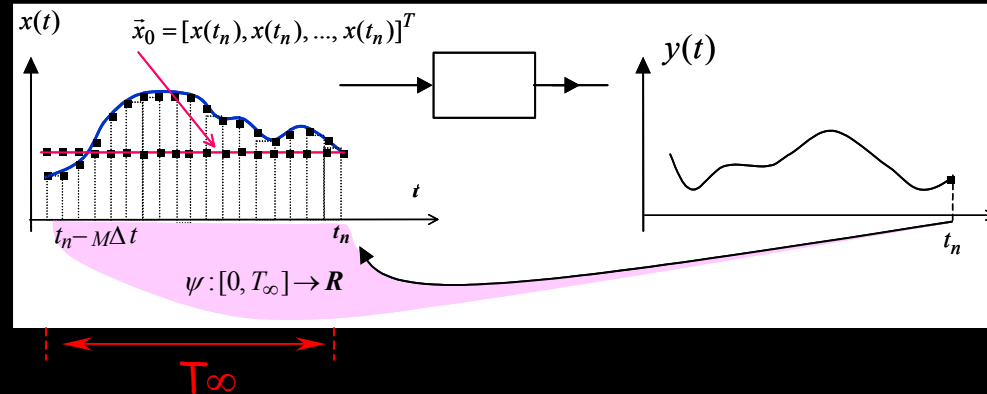
$$y_1(t) = \int_0^{T_\infty} h_1(x(t), \tau_1) [x(t - \tau_1) - x(t)] d\tau_1 \quad \text{Parametric linear}$$

...

$$y_n(t) = \int_0^{T_\infty} \dots \int_0^{T_\infty} h_n(x(t), \tau_1, \dots, \tau_n) [x(t - \tau_1) - x(t)] \dots [x(t - \tau_n) - x(t)] d\tau_1 \dots d\tau_n$$

Modified Volterra kernel of order  $n$

Signal speed



truncate to  $n = 1$

• Short memory duration:  
 $T_\infty \ll$  signal period

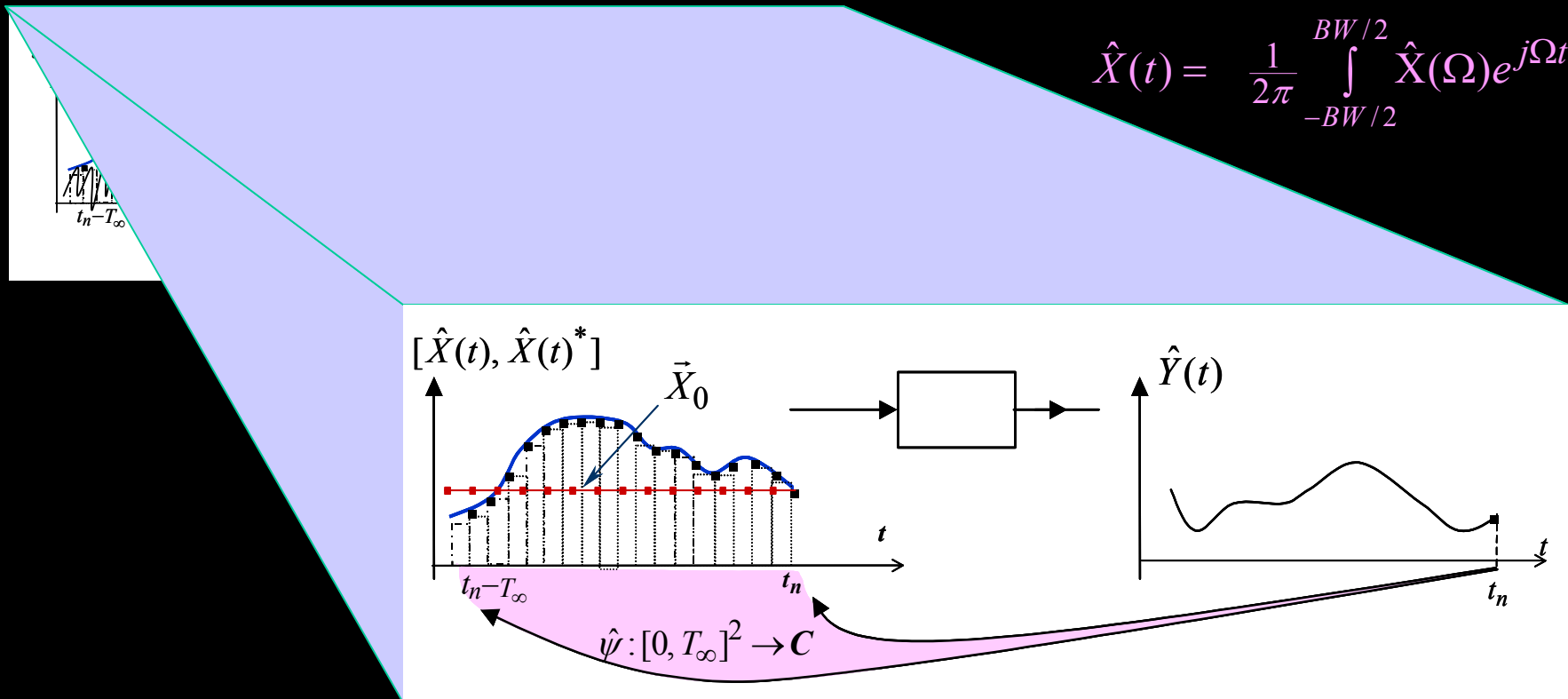
achieve some practical efficiency

# Envelope domain modeling enforces short memory duration conditions

$$x(t) = \Re[\hat{X}(t)e^{j\omega_x t}]$$

$$y(t) = \Re[\hat{Y}(t)e^{j\omega_y t}]$$

$$\hat{X}(t) = \frac{1}{2\pi} \int_{-BW/2}^{BW/2} \hat{X}(\Omega)e^{j\Omega t} d\Omega$$



Short memory duration =  $T_\infty \ll \frac{1}{BW} = \text{more realistic assumption}$

# Envelope domain modified Volterra model

$$\hat{Y}(t) = \sum_{n=0}^{\infty} \hat{Y}_n(t)$$

$$\hat{Y}_0(t) = \hat{\psi}(\hat{X}(t), \hat{X}(t)^*)$$

Order 0: AM/AM, AM/PM model

$$\hat{Y}_1(t) = \int_0^{T_{\infty}} h_I(\hat{X}(t), \hat{X}(t)^*, \tau) \cdot (X(t-\tau) - X(t)) d\tau + \int_0^{T_{\infty}} h_Q(\hat{X}(t), \hat{X}(t)^*, \tau) \cdot (X(t-\tau)^* - X(t)^*) d\tau$$

Parametric linear (I-Q)

...

$$\hat{Y}_n(t) = \int_0^{T_{\infty}} \dots \int_0^{T_{\infty}} \sum_{i_1=1}^2 \dots \sum_{i_n=1}^2 \hat{h}^{i_1 \dots i_n}(\hat{X}(t), \hat{X}(t)^*, \tau_1, \dots, \tau_n) \cdot (X^{i_1}(t-\tau_1) - X^{i_1}(t)) \cdot \dots \cdot (X^{i_n}(t-\tau_n) - X^{i_n}(t)) d\tau_1 \dots d\tau_n$$

Parametric order n

$$X^1(t) \stackrel{\Delta}{=} \hat{X}(t), \quad X^2(t) \stackrel{\Delta}{=} \hat{X}(t)^*$$

Short term memory conditions



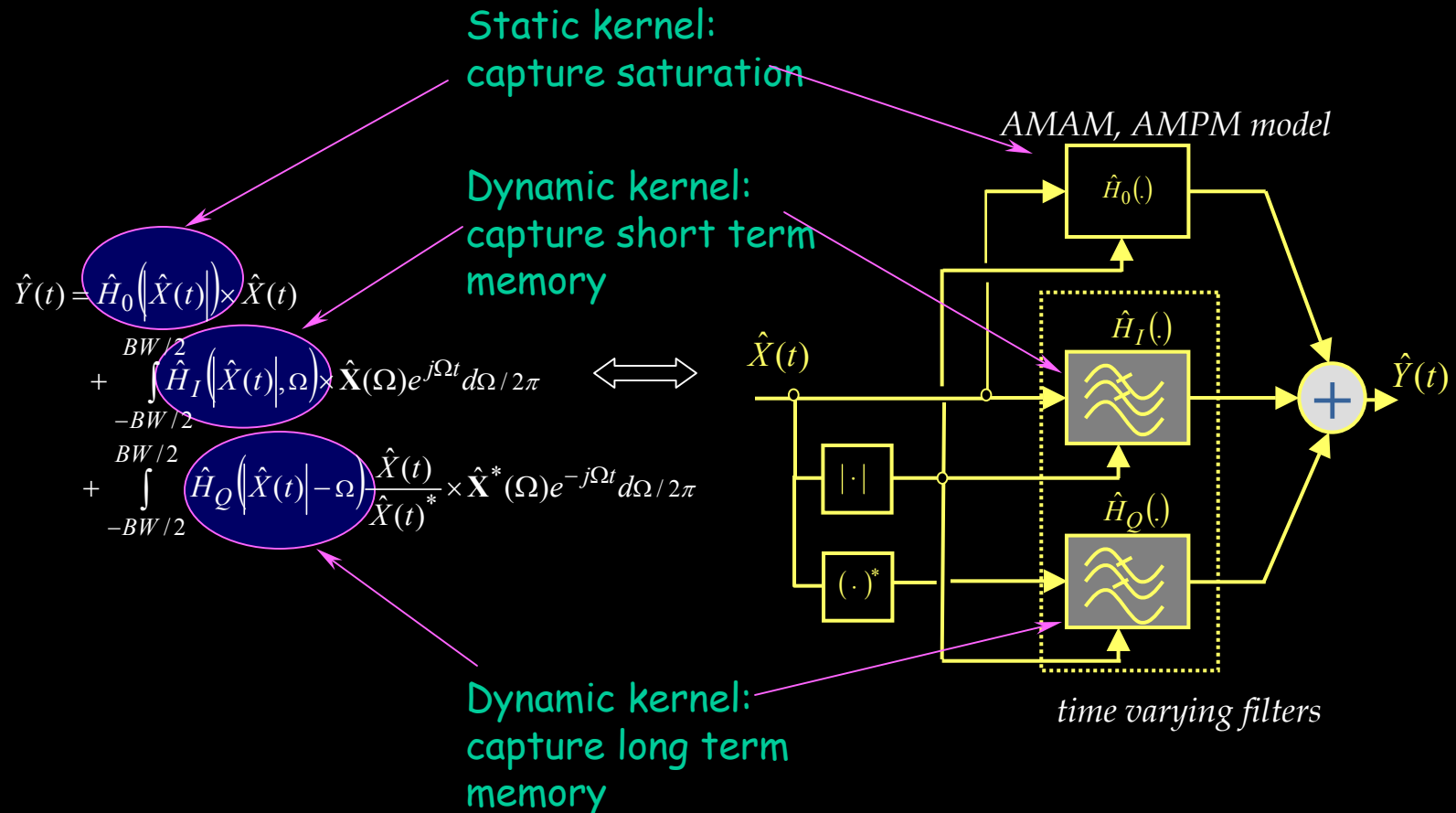
$n = 1$

$$T_{\infty} \ll \frac{1}{BW}$$

$$\hat{Y}(t) = \hat{\psi}(\hat{X}(t), \hat{X}^*(t)) + \int h_I(\hat{X}(t), \hat{X}^*(t), \tau) \times (\hat{X}(t-\tau) - \hat{X}(t)) d\tau + \int h_Q(\hat{X}(t), \hat{X}^*(t), \tau) \times (\hat{X}^*(t-\tau) - \hat{X}^*(t)) d\tau$$

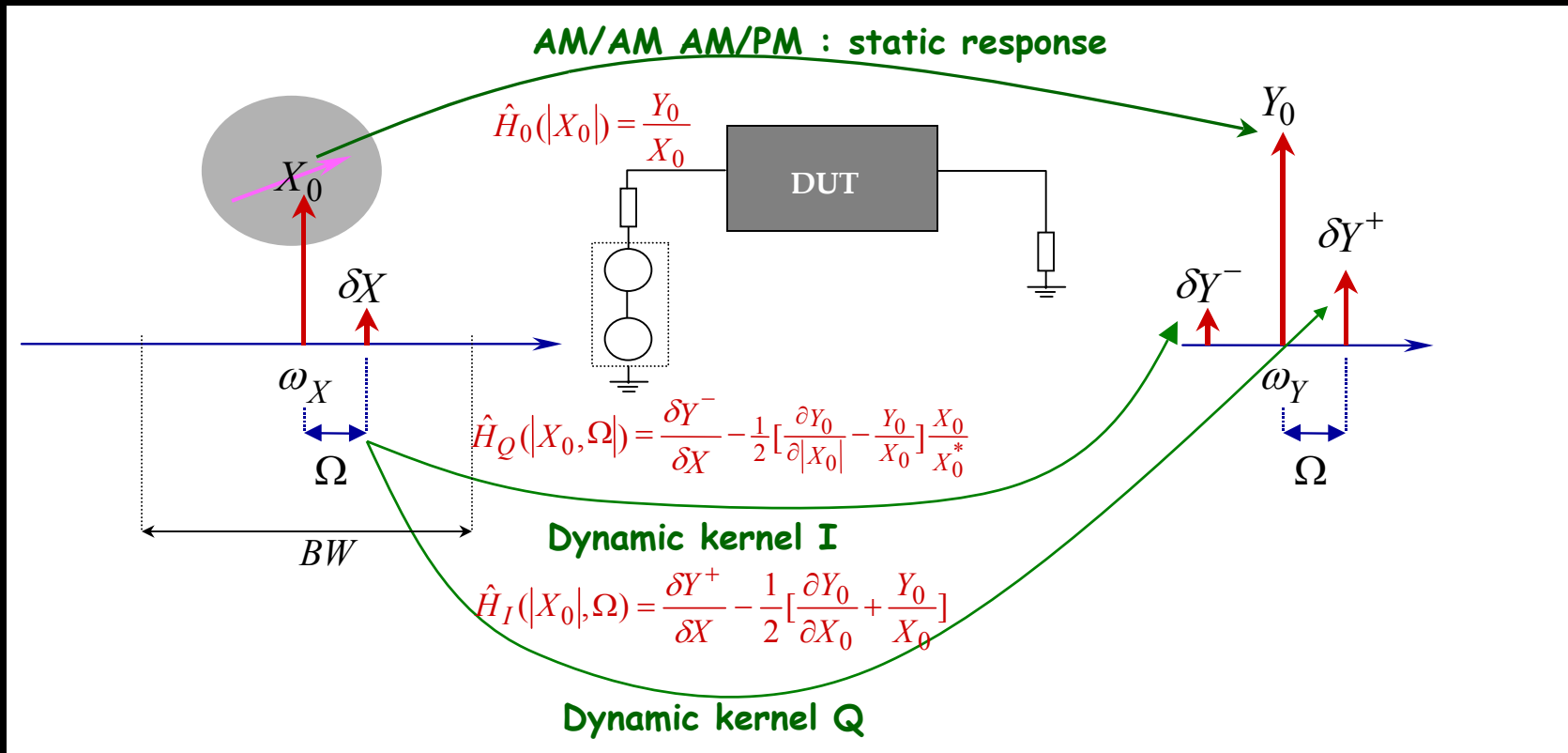
Amplifier  
Mixer  
VCO

# 1st order modified Volterra model : summary



# 1st order modified Volterra model : Kernel extraction

- Two-tone measurements



⇒ Harmonic Balance, shooting simulation or Experimental characterization with VNA or digital scope



## Constant envelope systems (gmsk, radar, ..)

⇒ Long term memory is not excited

$$\hat{Y}(t) = \hat{H}_0\left(\left|\hat{X}(t)\right|\right) \times \hat{X}(t) + \int_{-BW/2}^{BW/2} \hat{H}_I\left(\left|\hat{X}(t)\right|, \Omega\right) \times \hat{X}(\Omega) e^{j\Omega t} d\Omega / 2\pi + \int_{-BW/2}^{BW/2} \hat{H}_Q\left(\left|\hat{X}(t)\right|, -\Omega\right) \frac{\hat{X}(t)}{\hat{X}(t)^*} \times \hat{X}^*(\Omega) e^{-j\Omega t} d\Omega / 2\pi$$

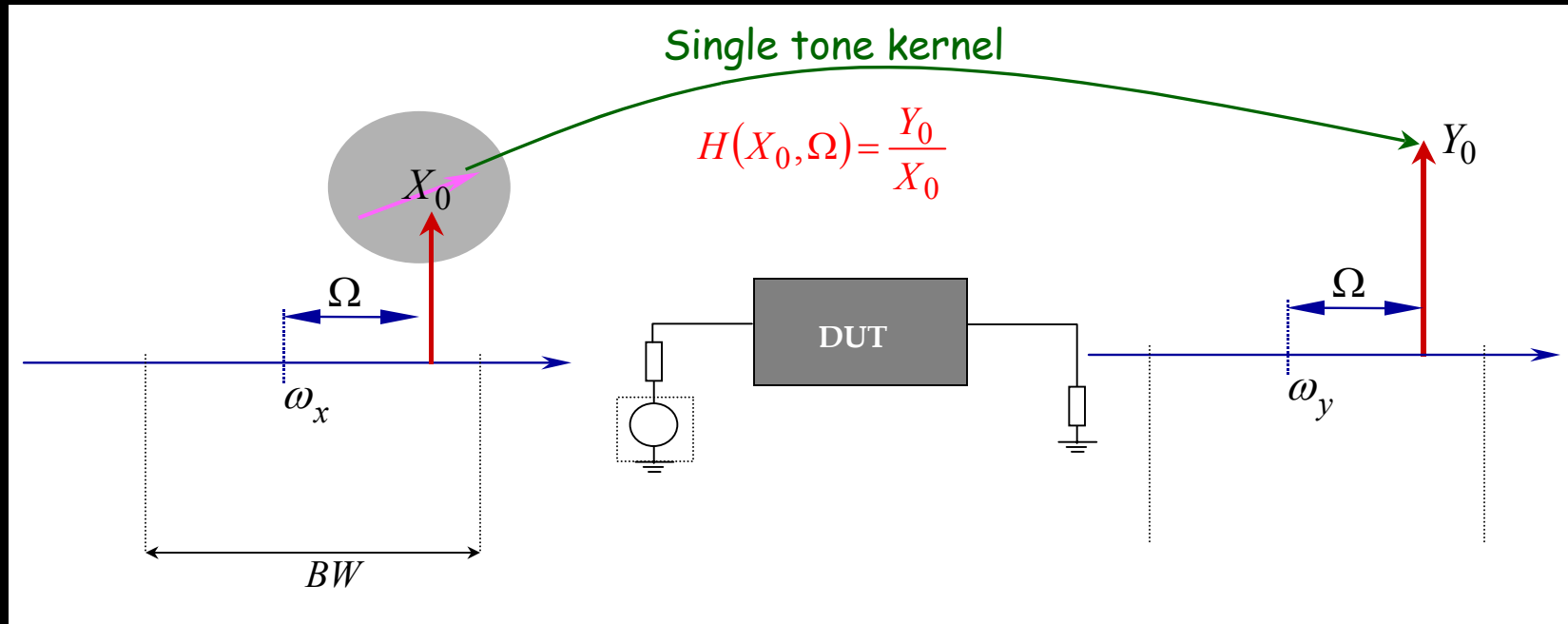
⇒ Very effective model for constant envelope systems

$$\hat{Y}(t) = \int_{-BW/2}^{BW/2} \hat{H}\left(\left|\hat{X}(t)\right|, \Omega\right) \times \hat{X}(\Omega) e^{j\Omega t} \frac{d\Omega}{2\pi} = \int_0^{T_\infty} \hat{h}\left(\left|\hat{X}(t)\right|, \tau\right) \times \hat{X}(t-\tau) d\tau$$

Kernel = Single tone complex gain vs power and frequency

# Quasi-constant envelope systems (gmsk, radar, ..)

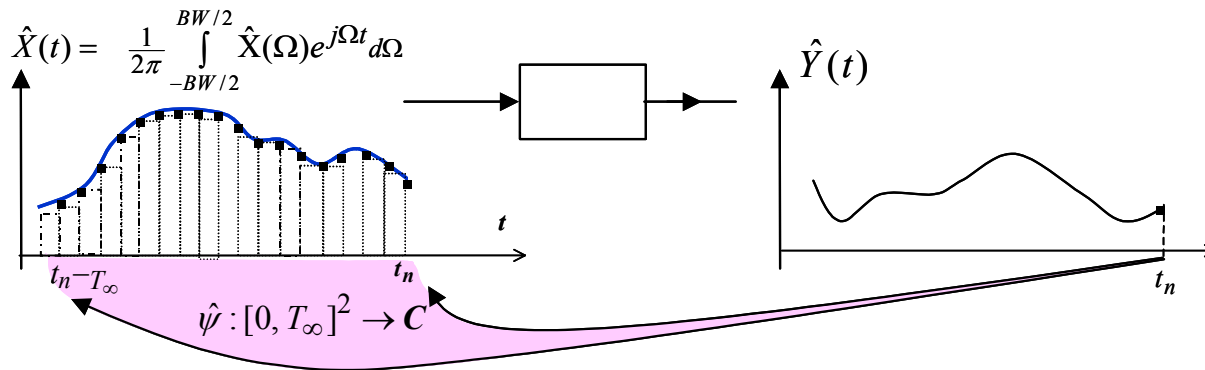
⇒ Very simple kernel extraction



## Certain circuits do not satisfy short memory duration conditions

Long term memory effects:  $T_\infty > \frac{1}{BW}$

- Poorly designed biasing circuits - AGC loops - Thermal effects



$$\begin{cases} \hat{Y}(t_n) = \hat{\psi}(\vec{X}) \\ \vec{X} = [\hat{X}(t_n), \hat{X}(t_n)^*, \hat{X}(t_n - \Delta t), \hat{X}(t_n - \Delta t)^*, \dots, \hat{X}(t_n - M\Delta t), \hat{X}(t_n - M\Delta t)^*]^T \end{cases}$$

⇒ **Reconsider the functional decomposition :**  
**Use more effective basis functions instead of power expansions**

• **Previously modified Volterra series**

Basis function order n :  $B_n(X) = X^n$

$$\hat{Y}(t) = \psi(\hat{X}(t), \hat{X}(t)^*) + \sum_{k=0}^{\infty} [h_I^1(\hat{X}(t), \hat{X}(t)^*, k\Delta t) \quad h_Q^1(\hat{X}(t), \hat{X}(t)^*, t_k)] \cdot \begin{bmatrix} \hat{X}(t-k\Delta t) - \hat{X}(t) \\ \hat{X}(t-k\Delta t)^* - \hat{X}(t)^* \end{bmatrix} + \sum_{k_1 k_2}^{\infty} [\hat{X}(t-k_1\Delta t) - \hat{X}(t) \quad \hat{X}(t-k_1\Delta t)^* - \hat{X}(t)^*] \cdot \begin{bmatrix} \hat{h}_{II}^2(\tau_1, \tau_2) & \hat{h}_{IQ}^2(\tau_1, \tau_2) \\ \hat{h}_{QI}^2(\tau_1, \tau_2) & \hat{h}_{QQ}^2(\tau_1, \tau_2) \end{bmatrix} \cdot \begin{bmatrix} \hat{X}(t-k_2\Delta t) - \hat{X}(t) \\ \hat{X}(t-k_2\Delta t)^* - \hat{X}(t)^* \end{bmatrix} + \dots$$

**Change basis functions**

First order term is forced to be linear as to signal speed

• **Extension**

Basis function order n :  $B_n(X) = \sum_{k=0}^{Kn} \alpha_{n,k} X^n$

$$\hat{Y}(t) = \psi(\hat{X}(t), \hat{X}(t)^*) + \sum_{k=0}^{\infty} f^1(\hat{X}(t), \hat{X}(t)^*, \hat{X}(t-k\Delta t), \hat{X}(t-k\Delta t)^*, k\Delta t) + \sum_{k_1 k_2}^{\infty} f^2(\hat{X}(t), \hat{X}(t)^*, \hat{X}(t-k_1\Delta t), \hat{X}(t-k_1\Delta t)^*, \hat{X}(t-k_2\Delta t), \hat{X}(t-k_2\Delta t)^*, k_1\Delta t, k_2\Delta t) + \dots$$

First order term is allowed to be nonlinear as to signal speed



# 1st order truncation => Nonlinear impulse response



$$\hat{X}(t) = |\hat{X}(t)| e^{j \int_0^t \Omega_x(\tau) d\tau}$$

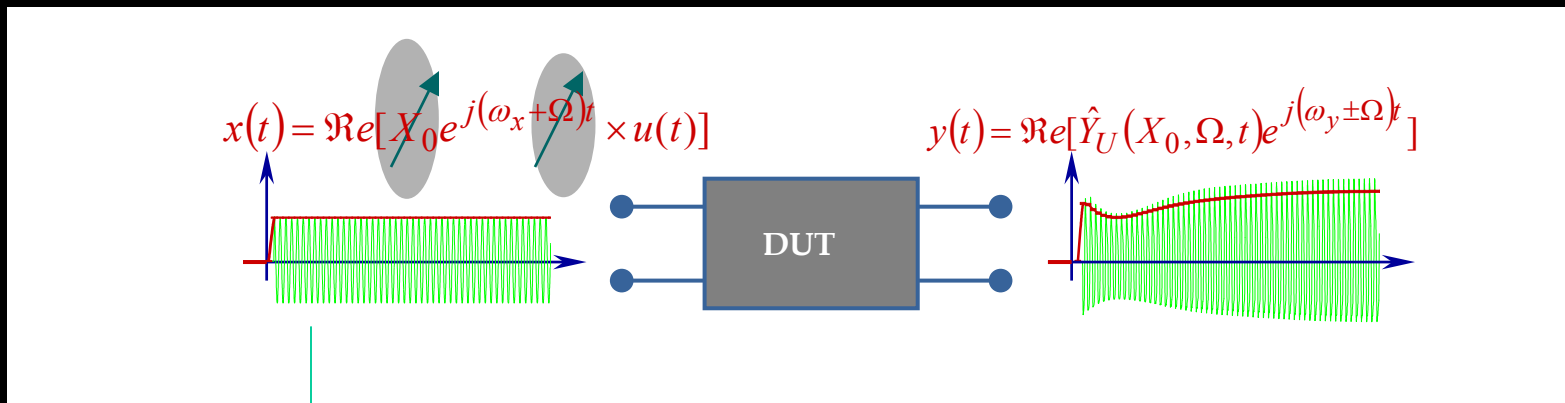
Time varying amplitude

Time varying frequency

$$\hat{Y}(t) = \int_0^{T_\infty} \hat{h}\left(|\hat{X}(t-\tau)|, \Omega_x(t-\tau), \tau\right) \times \hat{X}(t-\tau) d\tau$$

# Nonlinear impulse response extraction

- Unit step envelope drive:

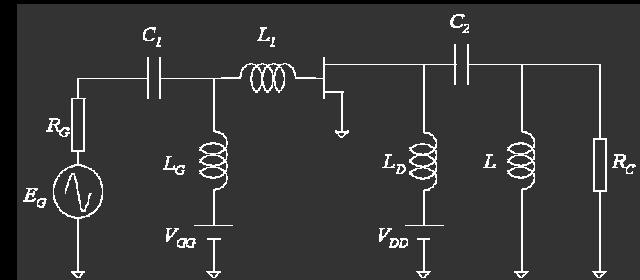


$$\hat{Y}(t) = \int_0^{T_\infty} \hat{h}(\hat{X}(t-\tau), \Omega(t-\tau), \tau) \times \hat{X}(t-\tau) d\tau$$

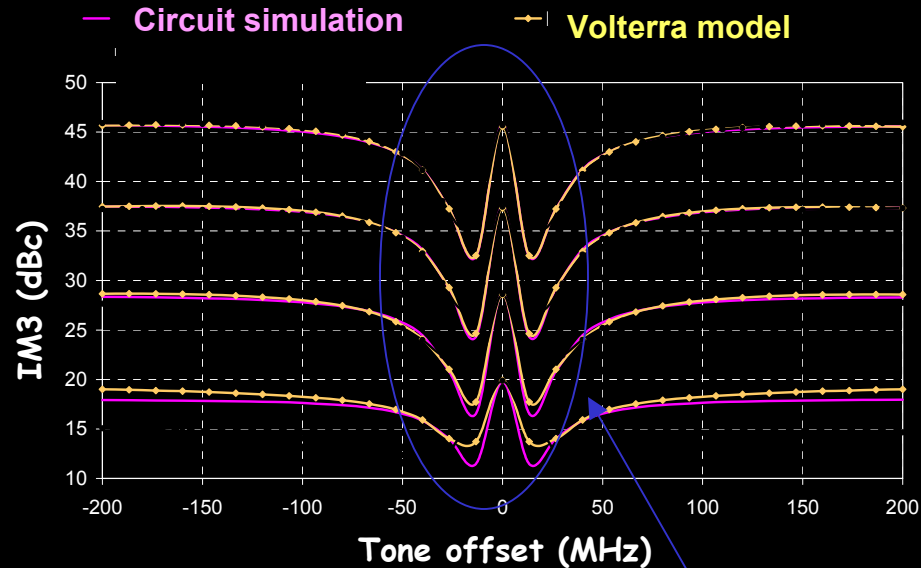
$$\hat{h}(X_0, \Omega, t) = \frac{1}{X_0} \frac{\partial \hat{Y}_U(X_0, \Omega, t)}{\partial t} e^{\pm j\Omega t}$$

⇒ Envelope transient simulation or Experimental envelope characterization with digital scope

# Elementary amplifier modeling

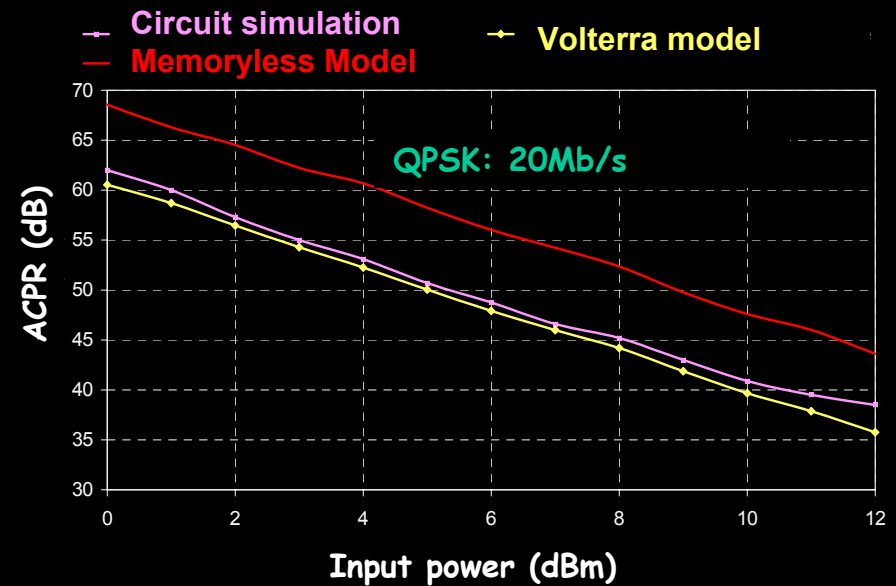


## • Two-tone test



Excellent prediction of  
long term memory

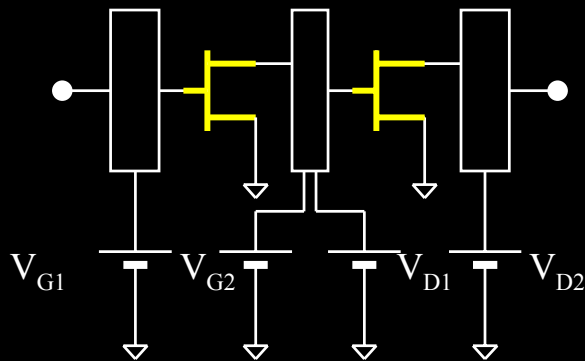
## • Multitone test



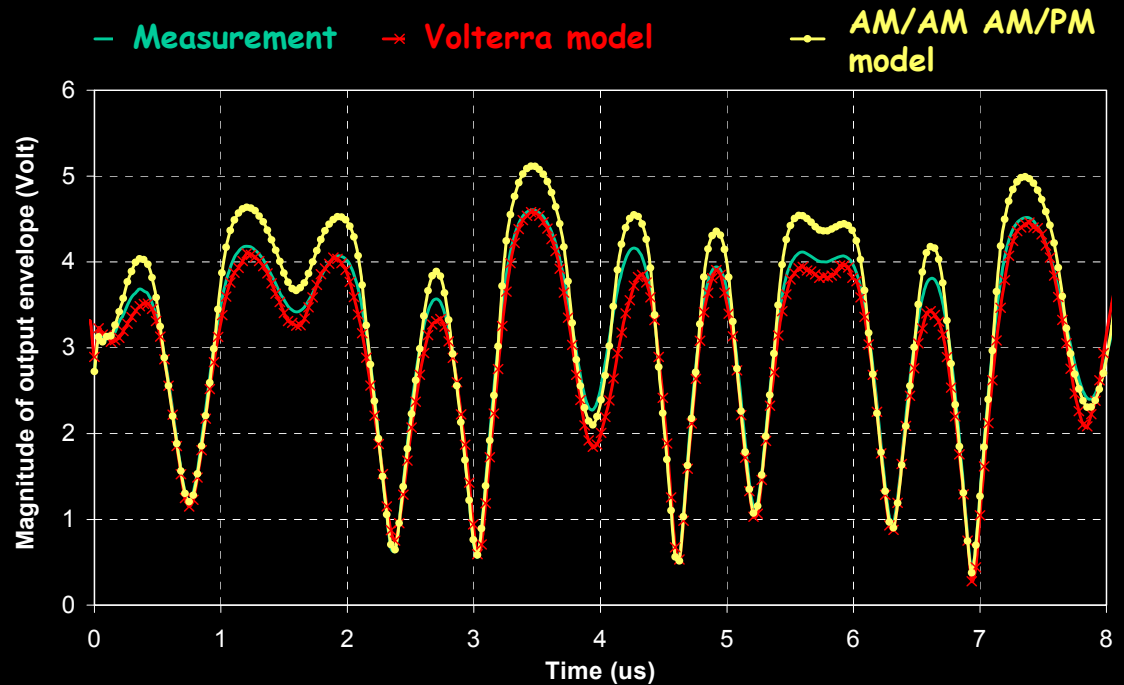
# Experimental modeling

Experimentally extracted model from time domain envelope measurement setup  
(Tektronix AWG2021 and TDS 754D)

space qualified  
2stage 1.2mm HFET amplifier  
 $f_0 = 1.6$  GHz  
 $P_{out} = 350$  mW  
 $BW = 80$  MHz



QPSK signal @ 1 MB/s rate,  
-16 dBm input





## Summary

- Envelope domain modeling + Modified Volterra series concept give an effective solution for RF & microwave circuit modeling
- Models can be extracted from either circuit simulation tools and physical measurement equipments
- Model equations can be implemented in analog HDL



# Model justification

$$\hat{Y}(t) = \int_0^{T_\infty} \hat{h}\left(\hat{X}(t-\tau), \Omega(t-\tau), \tau\right) \times \hat{X}(t-\tau) d\tau$$

