Passivity-Based Sample Selection and Adaptive Vector Fitting Algorithm for Pole-Residue Modeling of Sparse Frequency-Domain data

D. Deschrijver, T. Dhaene
University of Antwerp, Belgium

- GOAL: model the spectral response of passive electrical structures, over freq. range of interest
  - Samples are computational expensive (EM solver)
  - Minimize number of samples, and model complexity
  - Maximize accuracy
  - No prior knowledge of system's dynamics

$$S(j\omega) = \sum_{n=1}^{N} \frac{c_n}{j\omega - a_n} + d + j\omega h$$

- PROBLEM :
  - (1) Total simulation cost can be excessive
  - (2) Parameterization can be ill-conditioned
  - (3) Models are often not passive

- SOLUTION:
  - (1) Adaptive modeling techniques
    - → Adaptively select optimal sample distribution
    - → Adaptively select minimal model complexity
  - (2) Robust rational fitting techniques
    - → Vector Fitting : Robust pole-residue modeling technique
    - → Iterative least-squares approximation
  - (3) Passivity detection and enforcement
    - → Hamiltonian matrices
    - → Passivity-based sample selection
    - → First order matrix perturbations

1. Adaptive sampling techniques

GOAL: automatic build pole/zero rational model

- Maximize model accuracy, minimize samples

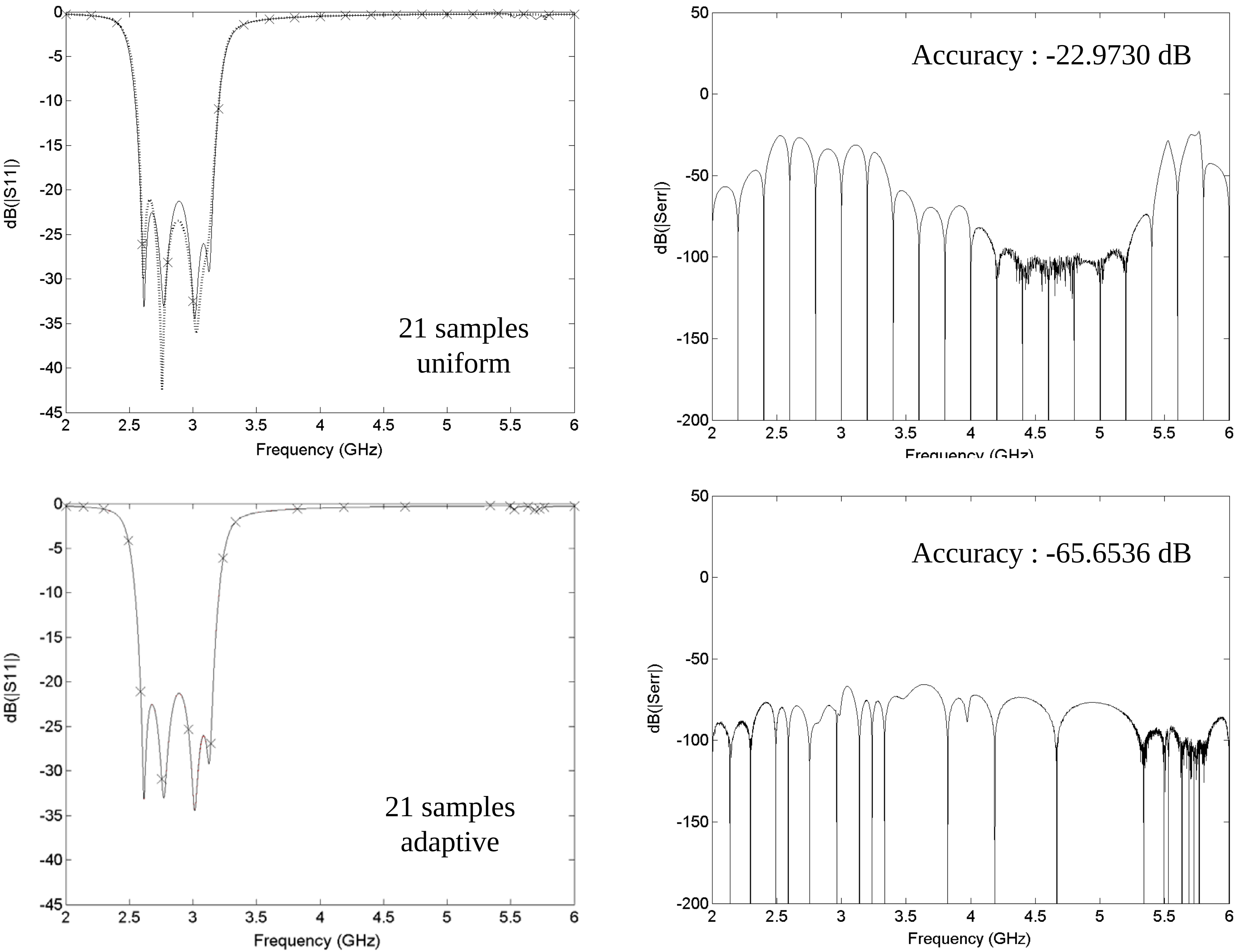
- Start
  - Simulate 4 equidistant selected samples
- Adaptive modeling loop
  - Build several rational models with different complexity [N/D]
  - Check error in all sample points
  - Increase model complexity till : error < threshold (e.g. -80dB)
  - Select *best* & *2nd best* model
  - Difference between 2 models : estimated fitting error

1. **Adaptive sampling techniques**

Adaptive sampling loop:

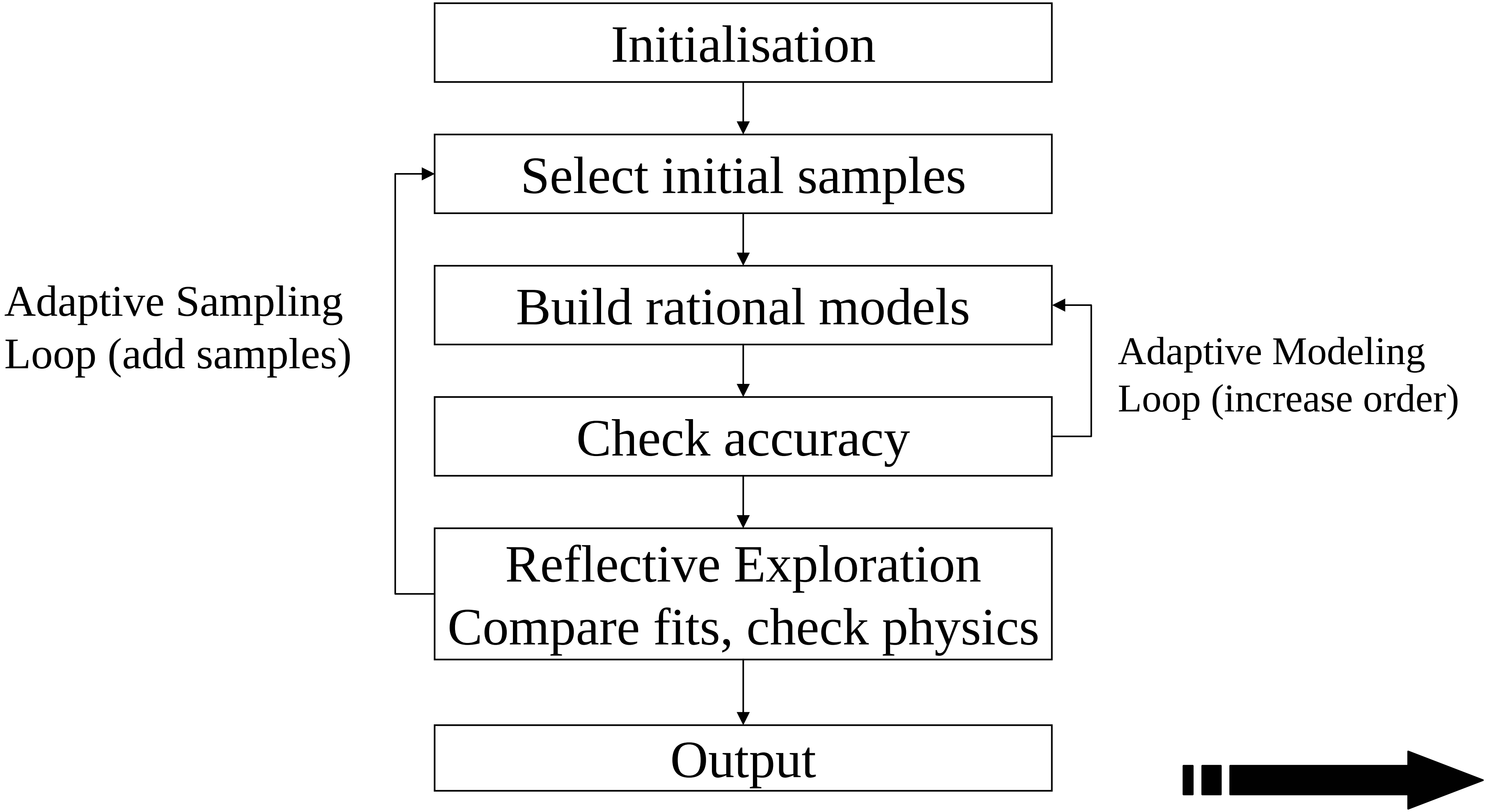
- Add sample where:
  - Mag(estimated fitting error) > magnitude-threshold (e.g. -60dB)
  - Phase(estimated fitting error) > phase-threshold (e.g. 5 deg)
  - unphysical behavior (e.g. |S| > 1)
- Extra heuristics
  - Avoid oversampling: cluster data, to avoid ringing
  - Avoid undersampling: check phase variation between samples

- Samples are selected until all criteria are satisfied


dB(|S11|)
Frequency (GHz)
21 samples
uniform
dB(|Serr|)
Frequency (GHz)
Accuracy : -22.9730 dB
dB(|S11|)
Frequency (GHz)
21 samples
adaptive
dB(|Serr|)
Frequency (GHz)
Accuracy : -65.6536 dB

1. **Adaptive sampling techniques**

Flowchart :



2. Robust rational fitting technique (Vector Fitting)

- Spectral response : Rational pole-residue model

$$S(j\omega) = \sum_{n=1}^{N} \frac{c_n}{j\omega - a_n} + d + j\omega h$$

- Vector Fitting identifies unknown system variables
- Sanathanan-Koerner type of iteration (Iterative least squares)
- Unstable poles flipped into right half plane
- Poles and residues real or complex conjugate pairs

→ A set of initial poles are used, and relocated to optimal location

→ Residues are calculated to minimize the fitting error

3. Passivity considerations

Definition 1 :

System with scattering matrix S(jω) is **passive** if transfer function is bounded real

$$I - S(j\omega^{*})S(j\omega) \geq 0 \qquad \forall \omega$$

or $$\max(\sigma(S(j\omega)) \leq 1 \qquad \forall \omega$$

Definition 2 :

System with scattering matrix S(jω) is **asymptotically passive** if it is passive for $\omega \to \infty$

3. Passivity considerations

Theorem 1 :

A system with scattering matrix S(jω) is **passive** if

⇔ H has no imaginary eigenvalues

$$H = \begin{pmatrix} A - BR^{-1}D^T C & -BR^{-1}B^T \\ C^T Q^{-1} C & -A^T + C^T D R^{-1} B^T \end{pmatrix}$$

$Q = DD^T - I$

$R = D^T D - I$

Theorem 2 : → **Algebraic passivity tests**

$1 \in \sigma(S(j\omega_i))$ ⇔ $j\omega_i$ is an eigenvalue of H

3. Passivity considerations

→ Calculate slopes of singular value curves at frequencies

→ Eigenvalue sweep provides exact boundaries of passivity violations

Select samples within regions of passivity violation $[\omega_k,\omega_{k+1}]$ where

$$\max(\sigma(j\omega)) \qquad \forall\omega\in[\omega_k,\omega_{k+1}]$$

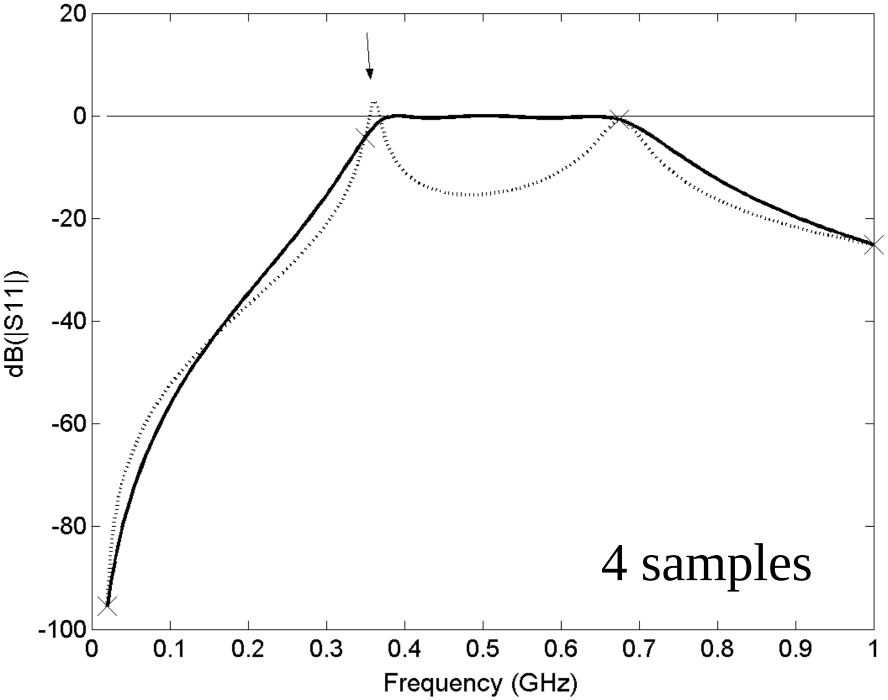
is maximal, until

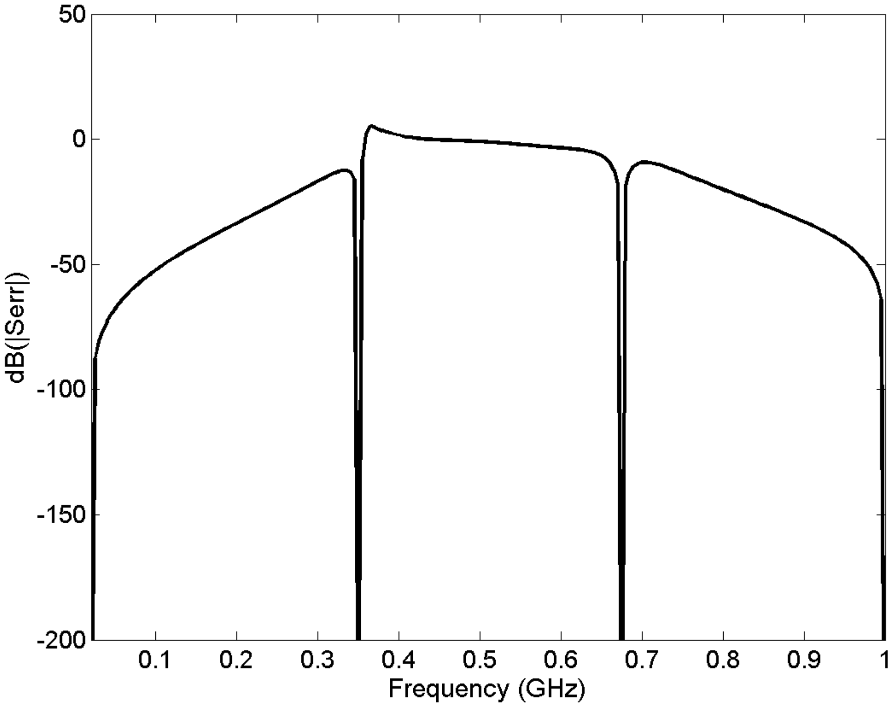
$$\max(\sigma(j\omega))<\varepsilon \qquad \forall\omega\in[\omega_0,\omega_K]$$

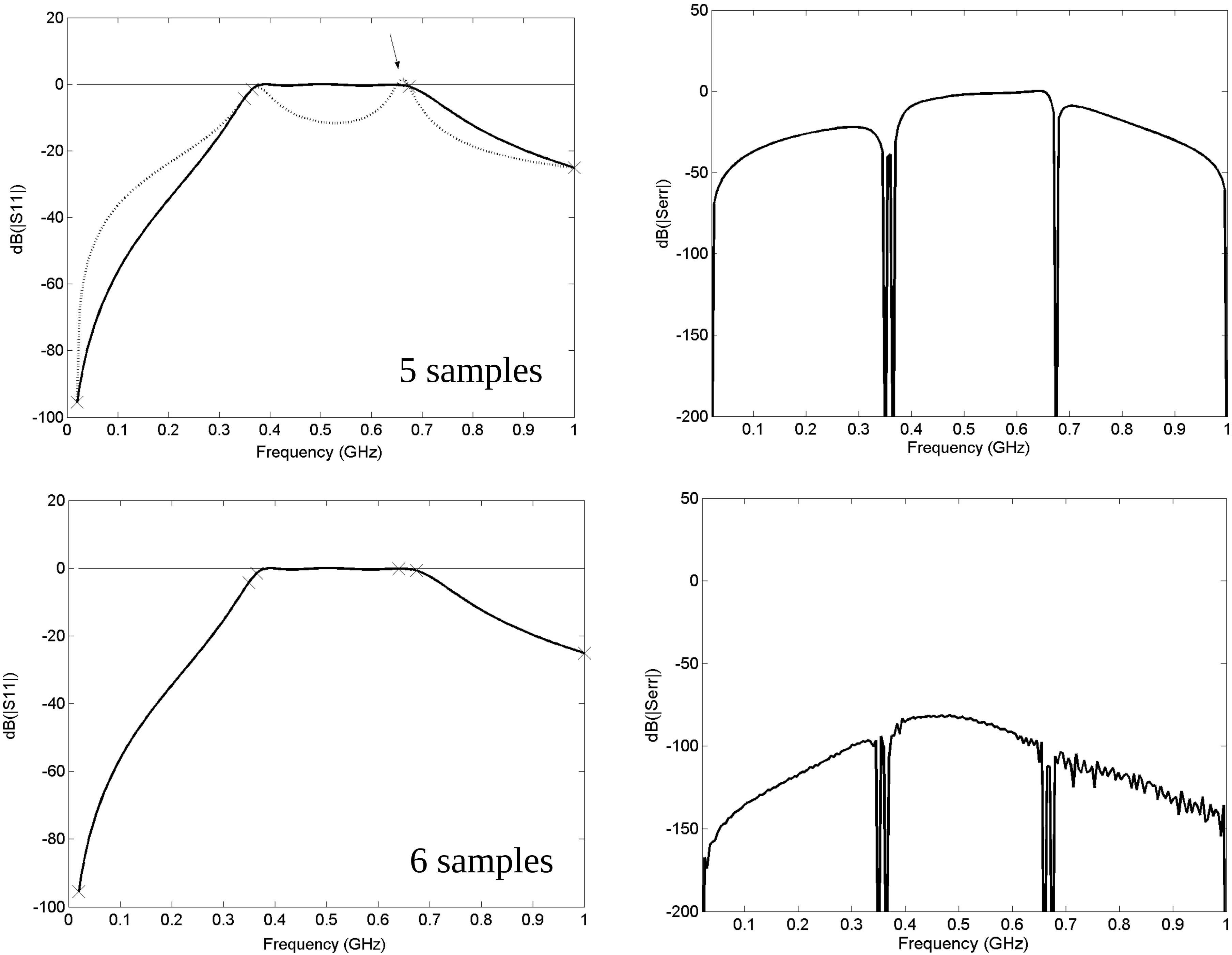
4. Example

One-port Bandpass filter modeled over [0.02 GHz-1 GHz]

Desired accuracy : $dB(|S_{ref} - S_{fit}|) < -60$







5 samples
6 samples
dB(|S11|)
dB(|Serr|)
Frequency (GHz)

Eigenvalues Hamiltonian (4 samples) :

```
+0.0000000000000 - 0.35556635157541i
-0.0000000000000 - 0.36797493144273i
-0.0000000000000 + 0.35556635157541i <--
+0.0000000000000 + 0.36797493144273i <--
+0.0092067761414 7 - 0.67657524224380i
+0.0092067761414 7 + 0.67657524224380i
-0.0092067761414 7 - 0.67657524224380i
-0.0092067761414 7 + 0.67657524224380i
```

Passivity violation :

```
[0.35556635157541i, 0.36797493144273i]
```

Eigenvalues Hamiltonian (5 samples) :

```
+0.0000000000000 - 0.6515679182412 3i
```

Eigenvalues Hamiltonian (5 samples) :

```
+0.0000000000000 - 0.65156791824123i
-0.0000000000000 - 0.67230698556846i
-0.0000000000000 + 0.65156791824123i <--
+0.0000000000000 + 0.67230698556846i <--
+0.19217333633158 + 0.0000000000000i
-0.19217333633158 + 0.0000000000000i
+0.01014688998501 - 0.37471499581693i
-0.01014688998501 - 0.37471499581693i
+0.01014688998501 + 0.37471499581693i
-0.01014688998501 + 0.37471499581693i
```

Passivity violation :

```
[0.65156791824123i , 0.67230698556846i]
```

Eigenvalues Hamiltonian (6 samples) :

```
+0.00539549345271 - 0.38963154992961i
-0.00539549345272 + 0.38963154992961i
-0.00539549345272 - 0.38963154992962i
+0.00539549345272 + 0.38963154992961i
-0.00677392820199 - 0.50292408510574i
-0.00677392820201 + 0.50292408510575i
+0.00677392820199 - 0.50292408510573i
+0.00677392820201 + 0.50292408510572i
+0.01921270798832 - 0.64595773831741i
+0.01921270798832 + 0.64595773831741i
-0.01921270798832 - 0.64595773831741i
-0.01921270798832 + 0.64595773831742i
```

No Passivity violation

What about unpassive behaviour due to ringing effects or outside frequency range of interest ?

→ First order perturbation eigenvalues of Hamiltonian

[Grivet-Talocia, 2003]

→ Compensation of residue vector

[Saraswat, Achar, Nakhla, 2003]

→ etc …

= Post-processing techniques for SMALL passivity violations

QUESTIONS ?