Reduced Order Models of Integrated RF Spiral Inductors with Geometrical and Technological Automatic Parameterization

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Motivation

• Advancement in Wireless Communication Industry:
  – Smaller, Cheaper transceivers in Gigahertz Range.
  – On-Chip Inductive elements address both these requirements.

• Accurate tools for predicting and optimizing the characteristics of on-chip Inductive elements.
Spiral RF Inductors

- Inductance, $L$. 
- Quality of Inductor, $Q$.

\[ Q \propto \frac{E_{\text{stored}}}{E_{\text{dissipated}}} \]

- Energy Dissipation
  - Displacement Current
  - Eddy Current

A Square Spiral Inductor
Parasitic Losses

- Impressed device current
- Magnetically Induced Eddy Currents
- Electrically Induced conduction and displacement currents

Typical parasitic Losses in Spiral Inductors
Timeline

• Ruehli et al, 1974
  – Concept of PEEC.

• Pettenpaul et al, 1988
  – Model each spiral segment individually.

• Niknejad, PhD Dissertation 2000.
  – Model the Substrate & Eddy Current losses.
  – Based on full simulation of Spiral using PEEC.

• Daniel et al., 2003 proposed geometrical parameterization technique for Spiral inductors.
  – Can’t model the substrate losses.
Contribution

- In this paper: Substrate Aware Geometrical Parameterization for Spiral Inductors.
  - Model Substrate Losses
  - Parameterized
  - More accurate
  - Extendable
Parameterization

- **Design Parameters**
  - ✓ Width of Wire, \( w \).
  - ✓ Separation between turns, \( d \).
  - ✓ Frequency of operation.
  - ❌ Height of Inductor from substrate.
  - ❌ Number of turns.
  - ❌ Number of sides, or angle of turn, \( \alpha \).
Outline

Theoretical Background
  – Volume Integral Equations
  – PEEC Technique

• Substrate Modeling
• Parameterization & Model Reduction
• Results
• Conclusions & Future Work
Volume Integral Equations

- Current-Voltage relation
  \[
  \frac{J(r)}{\sigma} + j\omega \frac{\mu}{4\pi} \int_V \frac{J(r')}{|r - r'|} dr' = -\nabla \phi,
  \]

- Charge-Voltage relation
  \[
  \frac{1}{4\pi\varepsilon} \int_S \frac{\rho(r')}{|r - r'|} dr' = \phi(r'),
  \]

- Charge and current conservation
  \[
  \hat{n} \cdot J(r) = j \omega \rho(r) \quad \nabla \cdot J(r) = 0
  \]
PEEC Technique [Ruehli 74]

Discretization
- Meshed into 3D Filaments and 2D Panels.
- Filaments capture the constant current density.
- Panels capture the constant charge density on surface.

Non-uniform discretization
- Capture skin effects.
- Boundary effects.
PEEC Construction
Modeling PEEC Circuit

\[
\begin{bmatrix}
M_f L_f M_f^T & 0 \\
0 & P^{-1}
\end{bmatrix}
\begin{bmatrix}
I_m \\
\phi
\end{bmatrix}
= -\begin{bmatrix}
M_f R_f M_f^T & M_p \\
-M_p^T & 0
\end{bmatrix}
\begin{bmatrix}
I_m \\
\phi
\end{bmatrix}
+ \begin{bmatrix}
V_{m,s} \\
0
\end{bmatrix}
\]

\[sLx = -Rx + BV_{src}\]
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Substrate Modeling
Substrate Modeling

Inductor Circuit

Capacitive coupling

Substrate Network

Panels

Inductor Impedences

Substrate resistances
Substrate Modelling

Green Function based FFT technique.

\[ \phi(r) = \int_{V} \rho(r')G(r, r')d^3r' \]

\[ [Q] = [c][\phi] \]

\[ C_{ij} = -c_{ij} \]

\[ C_{ig} = c_{ii} + \left( \sum_{j=1}^{N} c_{ij} \right) \]

\[ R_{ij} = \frac{1}{C_{ij}} \]
Substrate Modeling

Inductor Circuit + \( V_{\text{src}} \) - \( I_{\text{src}} \)

Capacitive coupling

Substrate Network

Panels

Inductor Impedences

Substrate resistances
Modeling Equations

\[
\begin{bmatrix}
Z_m^f & Z_m^{fp} & Z_m^{fp} & 0 & 0 & 0 & 0 & 0 \\
Z_m^{fp^T} & Z_m^p & Z_m^{fp} & 0 & 0 & 0 & M^{ps} & 0 \\
Z_m^{fp^T} & Z_m^p & Z_m^{fp} & -Z_m^{gs} & -Z_m^{fs} & 0 & 0 & M^{cs} \\
0 & 0 & -Z_m^{fs^T} & Z_m^g & Z_m^{fs} & 0 & 0 & 0 \\
0 & 0 & -Z_m^{fs^T} & Z_m^{fs^T} & Z_m^s & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & M^p & 0 \\
0 & -pM^{ps^T} & 0 & 0 & 0 & -pM^p & sI & 0 \\
0 & 0 & -p_cM^c & 0 & 0 & 0 & 0 & sI \\
\end{bmatrix}
\begin{bmatrix}
I_m^f \\
I_m^{fp} \\
I_m^{fs} \\
I_m^{sg} \\
I_m^s \\
I_m^p \\
V_b^f \\
V_b^{fp} \\
V_b^{fs} \\
V_b^{sg} \\
V_b^s \\
V_b^p \\
V_b^c \\
\end{bmatrix} = \begin{bmatrix}
V_b^f \\
V_b^{fp} \\
V_b^{fs} \\
V_b^{sg} \\
V_b^s \\
V_b^p \\
0 \\
\end{bmatrix}
\]

\[sLx = -Rx + BV_{src}\]
PEEC CIRCUIT

\[ sLx = -Rx + BV_{src} \]

• Let,

\[ E = sL + R \]

\[ \Rightarrow Ex = Bu \]

• Size of E can be prohibitively large.

• E is constructed from scratch in each iteration.

• Equation need to be solved every time configuration is changed.
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  Parameterization & Model Reduction
  • Results
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Parameterization

\[ s_i = \{1, d, W, dW, d^2W, W^2\} \]
Model Reduction

\[ \begin{align*}
V^T \begin{bmatrix} \tilde{E}_0 & \tilde{E}_1 & \cdots & \tilde{E}_n \end{bmatrix} & = \begin{bmatrix} s_0 \tilde{E}_0 & s_1 \tilde{E}_1 & \cdots & s_n \tilde{E}_n \end{bmatrix} \\
\tilde{E} \begin{bmatrix} \tilde{x} \\ \tilde{b} \end{bmatrix} & = \begin{bmatrix} \tilde{b}^T \tilde{x} \end{bmatrix} \\
y & = \tilde{b}^T \tilde{x}
\end{align*} \]
Control Flow

- Compute $\tilde{E}_i$ from a set of sample points $(w_i, d_i)$.

- New configuration: $W_1 \rightarrow W_2$, $d_1 \rightarrow d_2$
  - $s_i \in \{1, d_2, W_2, d_2 W_2, d_2^2, W_2^2, s, s d_2, s W_2, s d_2 W_2, s d_2^2, s W_2^2\}$.
  - Compute new $\tilde{E}$:

  $\tilde{E} = s_0\tilde{E}_0 + s_1\tilde{E}_1 + \ldots + s_n\tilde{E}_n$.

- Solve for $y$.

  $\tilde{E} \tilde{x} = \tilde{b} u$

  $y = \tilde{b}^T \tilde{x}$
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Results

• Conclusions & Future Work
Results: Effects of Substrate on Inductance

![Graph showing Inductance Vs. Frequency with different substrate separations]
Results: Effects of Substrate on Quality Factor

![Graph showing Quality Factor Vs. Frequency]

- 2um sep w/o sub
- 4um sep w/o sub
- 2um sep with sub
- 4um sep with sub

- w/o substrate
- with sub
Effects of Separation B/W Turns

\[ R_{\text{sub}_1} \ll R_{\text{sub}_2} \]
Results: Number of Turns

![Inductance Vs. Frequency Graph](image)

- 1.5 turns
- 2 turns
- 2.5 turns
Results: Number of Turns

Quality Factor Vs. Frequency

\[ Q = \frac{\omega L}{R} \]

- 1.5 turns
- 2 turns
- 2.5 turns
Results: Simulated Vs Measured Inductance

Inductance Vs. Frequency

Simulated Inductance

Measured Inductance

Ref. Gil et al., IEEE Trans. On Microwave Theory, Sept’03.
Results: Simulated Vs Measured Quality Factor

Quality Factor Vs. Frequency

Simulated Q Values

- 2 um sep
- 4 um sep
- 6 um sep

Frequency (GHz)

Square Spiral Inductor
R=60 μm, W=145 μm, S=2 μm

Measured Q Values

Frequency (GHz)
CPU Time

- 2 GHz Intel Pentium 4 processor
- 1 GB RAM

<table>
<thead>
<tr>
<th>Configuration</th>
<th>w/o param</th>
<th>with param</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 turn</td>
<td>15 min/config</td>
<td>15 min + 2 sec/config</td>
</tr>
<tr>
<td>(10 config)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 turns</td>
<td>35 min/config</td>
<td>35 min + 2 sec/config</td>
</tr>
<tr>
<td>(10 config)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.5 turns</td>
<td>50 min/config</td>
<td>60 min + 2 sec/config</td>
</tr>
<tr>
<td>(10 config)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Modeling Octagonal Shapes

Octagonal RF Inductor
Outline

• Theoretical Background
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Conclusions & Future Work
Conclusion

• A substrate aware parameterization technique has been proposed.
• The substrate has a significant impact on quality of RF inductors.
• The simulator accurately models the inductance.
• The small discrepancy in Q is due to inability to model via resistances.
Future Work

• Scheme to model “via” resistance.
• Include Eddy Current effects in model.
• Make the model efficient
  – Using standard sparse matrix packages.
  – Optimizing on matrix operations.
Thank You