Modeling of Spring Constant and Pull-down Voltage of Non uniform RF MEMS Cantilever

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ABSTRACT

In this paper, we are going to present a model of spring constant and pull down voltage for non uniform RF MEMS cantilever. In order to reduce the pull down voltage, it is usual to use a beam, which is narrower close to anchor and wider at the end or electrode area for a cantilever. Compare to uniform beam, this beam will have lower spring constant which will reduce the pull down voltage. A comprehensive model for spring constant and pull down voltage of the non uniform cantilever is developed through basic force deflection mechanism of the suspended beam.

1. INTRODUCTION

Nowadays RF MEMS components are becoming popular, due to their very good performance at RF and microwave frequencies. RF MEMS switches have very low insertion loss at on state and very high isolation at off state. The operating principle of an electrostatic actuated RF MEMS switch is very simple. A beam (bridge or cantilever) is suspended from the anchor with an actuation electrode placed underneath. When a DC voltage is applied between the beam and actuation electrode the beam moves down due to an electrostatic force. The DC actuation voltage, at which the beam fully moves down, is called the pull down voltage.

Usually the actuation voltage for RF MEMS switches is higher than their solid-state counterparts. In order to reduce the pull down voltage for RF MEMS switches, techniques like folded spring, narrower beam close to the anchor than actuation electrode are used [1]. To calculate the required pull down voltage for a beam, it is necessary to have an accurate mechanical model for the spring constant of the beam. The spring constant will determine the pull down voltage. The pull down voltage (spring constant) also depends on the position and orientation of the actuation electrode. The model of the spring constant and the pull down voltage for a uniform beam is presented in the literature [1]. For non uniform beam, some work has been published recently [2, 3]. In the work presented in [2], the model assumes that the force is concentrated on the tip of the cantilever. Therefore the accuracy strongly depends on the size and position of the actuation electrode. In [3], a comparison of pull down voltage between uniform and non uniform beam is presented using numerical simulations. It shows that a non uniform beam may have a lower pull down voltage than a uniform beam. In this paper we develop an analytical model for the spring constant and pull down voltage for a non uniform cantilever taking into account that the force may be distributed along the beam. The model will be very useful for analysis of spring constant and pull down voltage of non uniform beam, using simple mathematical program. It will be much faster and simpler compared to the commercial tools using 3-D modeling. The model of the spring constant and its verification is described in section 2.The modeled pull down voltage is compared with CoventorWare simulation in section 3. The paper is concluded in section 4 followed by an appendix.

2. MODELING OF CANTILEVER

There are two basic types of RF MEMS switches, fixed-fixed bridge and cantilever. Usually the spring constant of fixed-fixed bridge is higher than cantilever, because the bridges are rigidly anchored at both sides. In circuit point of view, bridges are more useful in shunt configuration and cantilevers are more useful in series configuration. The cantilever can be used both as a DC and capacitive contact switch. For DC contact switch, a separate actuation electrode is required. For capacitive contact switch the same electrode may be used both for actuation and capacitive contact. In this section we are going to develop the model for the spring constant of a non

uniform cantilever with a wider section at the end of the beam.
The top view of a non uniform cantilever is shown in figure 1. The width of the beam close to the anchor is ‘w’ and the width of the beam above the actuation electrode is ‘wy’, where ‘y’ is a constant and it can be 1≥y≥1. For RF MEMS application y≥1 is desired, as it will reduce pull down voltage.

A side view of the non uniform cantilever is shown in figure 2. The width of the beam above the actuation area is higher than the rest of the beam. This will give higher actuation force, with lower spring constant compared to a uniform beam. The pull down voltage will be reduced.

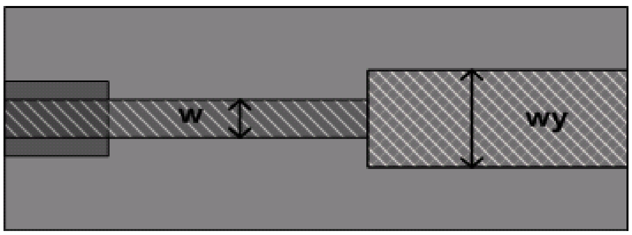

Figure 1: Top view of a non uniform cantilever

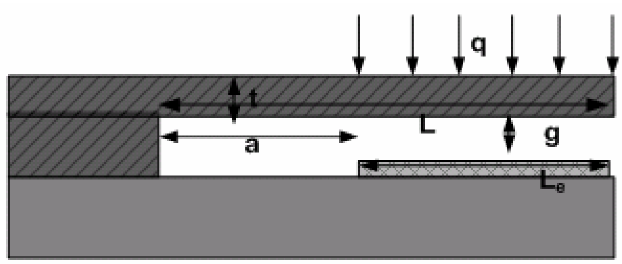

Figure 2: Side view of a non uniform cantilever

In order to develop the model, we need to use the beam diagram with actuation force, moment and reaction force acting on the beam at different positions. Figure A1 in the appendix shows such a model.

The derivation of the spring constant of the non uniform beam is presented in the appendix using the Euler-Bernoulli theory [4]. The equation of the deflection at any position of beam is given by,

$$v = \frac{q}{24EIy}(4Lx^3 - 6L^2x^2 - x^4) + C_3x + C_4 \quad (1)$$

With C_3 and C_4 given by,

$$C_3 = \frac{qaL(a-L)}{2EI}\left(1 - \frac{1}{y}\right) + \frac{qa^3}{6EIy} \quad (2)$$

$$C_4 = \frac{qa^2L}{12EI}(3L-4a)\left(1 - \frac{1}{y}\right) + \frac{qa^4}{12EI}\left(1 + \frac{1}{2y}\right) - \frac{qa^4}{6EIy} \quad (3)$$

For a cantilever, the maximum deflection occurs at the end of the beam or at x=L. The deflection at the end is given by,

$$v_L = -\frac{qL^4}{8EIy} + C_3L + C_4 \quad (4)$$

After simplification the maximum deflection of the beam becomes,

$$v_L = \frac{-q(L-a)^2(L+a)^2}{8EIy} - \frac{q(L-a)a(a+3L)}{12EI} - \frac{qa^2L}{2EI}(L-a)^2(1-\frac{1}{y}) \quad (5)$$

The spring constant of the cantilever is given by,

$$k = -\frac{P}{v_L} = -\frac{q(L-a)}{v_L} \quad (6)$$

If we insert equation (5) into equation (6), the spring constant becomes,

$$k = \frac{24EIy}{3(L-a)(L+a)^2 + 2ya^2(a+3L) + 12aL(L-a)(y-1)} \quad (7)$$

When the beam is uniform, i.e. y=1, the expression for C_3 and C_4 becomes,

$$C_{3u} = \frac{qa^3}{6EI} \quad (8)$$

$$C_{4u} = -\frac{qa^4}{24EI} \quad (9)$$

The deflection of the cantilever at any point is given by,

$$v = -\frac{q}{24EI}\left[x^4 - 4Lx^3 + 6L^2x^2 - 4a^3x + a^4\right] \quad (10)$$

This matches with the expression of deflection in [4].

To test the more validity of this expression we can take some well known limits in equation (7). If we take the limits, a=L, a=0 and y=1 we obtain the following spring constants,

$$k \xrightarrow{a=0} \frac{8EIy}{L^3} = \frac{2Ewy}{3}\left(\frac{t}{L}\right)^3 \quad (11)$$

$$k \xrightarrow{a=L} \frac{3EI}{L^3} = \frac{Ew}{4}\left(\frac{t}{L}\right)^3 \quad (12)$$

$$k \xrightarrow{y=1} 2Ewt^3 \frac{L-a}{3L^4 - 4La^3 + a^4} \quad (13)$$

The expressions for spring constant (11)-(13) match with the expressions given in [1, Ch2].

Using equation (7) we have calculated the spring constant in MathCAD, for two different beam lengths of the cantilever versus width of the electrode. The two beam lengths are 150 μm and 200 μm respectively and the thickness of the beam is 2 μm. The width of the beam at the anchor is 100 μm. The length of the electrode is L_e=100 μm and the width of the electrode is varied from 100 μm to 300 μm ($1 \leq y \leq 3$). The material of the beam is chosen Aluminum. The variation of the spring constant versus the electrode width is shown in figure 3. From figure 3 it can be seen that, the spring constant increases to some extent with electrode width, from uniform beam i.e. w=100 μm, and it is more prominent for shorter beam. For a long beam the spring constant is much lower than for the short beam and its spring constant varies very little with electrode width. But as the actuation area increases, it will reduce pull down voltage significantly.

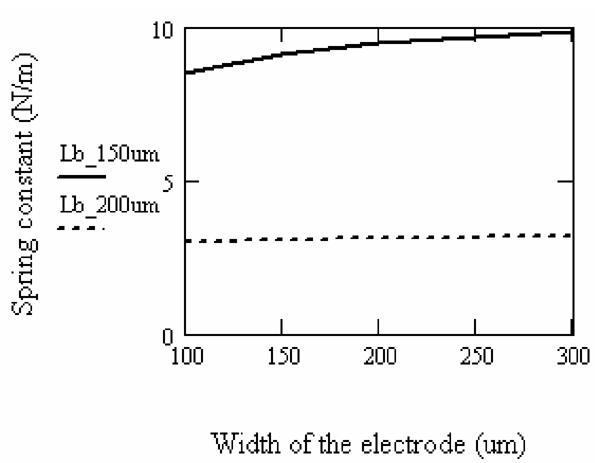

Figure 3: Variation of spring constant of cantilevers with electrode width

We have also compared the spring constant of beam with different lengths and same electrode width. For uniform beam, the electrode width and length are 100 μm and 100 μm respectively. For non uniform beam, the beam width is 100 μm. The electrode length is 100 μm and width is 200 μm. The thickness of the beam is 2 μm. The variation of spring constant and comparison between non uniform beam (NU) and uniform beam width (U) with beam length is shown in figure 4.

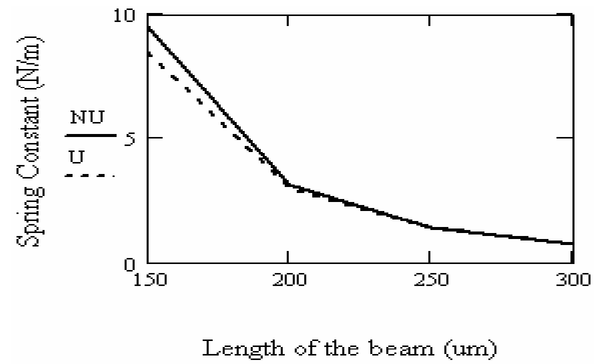

Figure 4: The variation of spring constant with beam length

3. COMPARISON OF MODEL WITH SIMULATION RESULT

The pull down voltage of a cantilever beam for electrostatic actuation is given by [1, 5].

$$V_p = \sqrt{\frac{8kg_0^3}{27\varepsilon_0 L_e wy}} \quad (14)$$

Here 'k' is the spring constant, 'g_0' is the initial gap height and 'wyL_e' is the actuation electrode area of the cantilever. We have implemented the analytical expressions in MathCAD to calculate the spring constant and pull down voltage for a cantilever. We also simulated the pull down voltage of the beam using CoventorWare, to compare the results. The simulation in CoventorWare is done, assuming no stress gradient and residual stress in the beam material. The dimensions of the cantilever are as follows. The length of the cantilever is 150 μm, length of the electrode is 100 μm, initial gap is 2 μm and thickness of the cantilever is 2 μm. The beam material is aluminum with young's modulus E=77 GPa. The results are shown in table 1. The simulation results of pull down voltage in CoventorWare are shown in figure 5. For convenience we have stopped the simulation, just before the beam collapses, to save simulation time. However we can extract the pull down voltage, at which the beam moves close to one third of the initial gap.

Table 1: Pull down voltage comparison of different electrode widths for a 150 μm long and 100 μm wide beam at anchor

Electrode width, wy (μm)	Spring constant (N/m)	Pull down voltage (V)	Pull down voltage in Coventor-Ware (V)	Error (%)
100	8.50	15.20	18.4	17
150	9.10	12.90	15.5	17
200	9.50	11.40	13.6	16
300	9.90	9.50	11.3	16

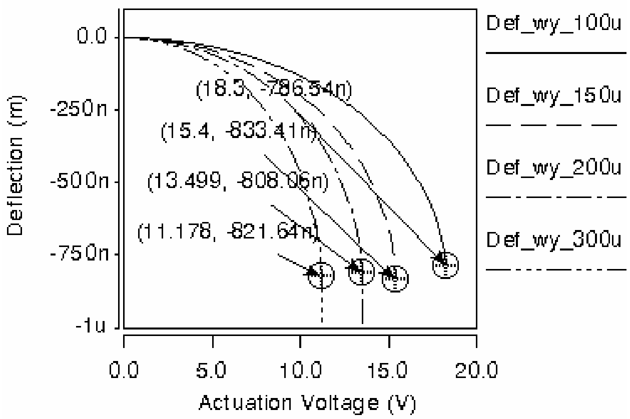

Figure 5: The pull down voltage simulation in CoventorWare for 150 μm long and 100 μm wide beam.

We have also made a comparison for a 200 μm long beam with 100 μm long electrode. The initial gap is 2 μm and thickness of the cantilever is 2 μm. The comparison is shown in table 2. The simulation results for pull down voltage in CoventorWare are also shown in figure 6.

Table 2: Pull down voltage comparison of different electrode widths for a 200 μm long and 100 μm wide beam

Electrode width, wy (μm)	Spring constant (N/m)	Pull down voltage (V)	Pull down voltage in CoventorWare (V)	Error (%)
100	3.01	9.05	10.5	14
150	3.08	7.48	8.7	14
200	3.12	6.52	7.5	13
300	3.16	5.40	6.2	13

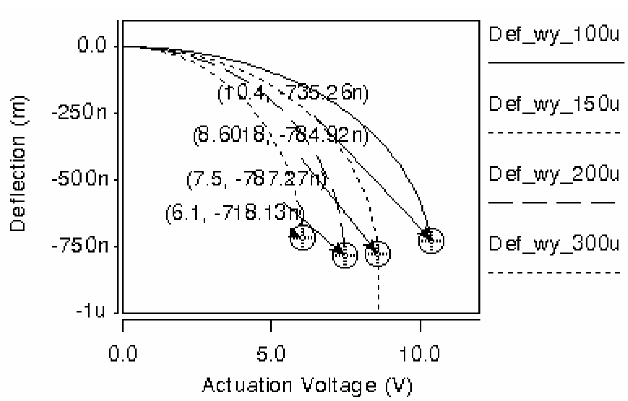

Figure 6: The pull down voltage simulation in CoventorWare for 200 μm long and 100 μm wide beam at anchor.

In the table 1 and table 2, we have seen that, the model has much lower error compare to the presented model in [2]. The error is almost constant for different electrode widths compare to a large variation of error for different electrode size and location presented in [2]. The model also matches for uniform beam formula presented in standard text book [1]. However, the pull down voltage is underestimated. In our analytical model, it is assumed that, the beam will collapse when the cantilever end moves 1/3rd of its initial gap. In CoventorWare the beam end needs to go beyond the 1/3rd of the initial gaps before it collapses, which may require higher pull down voltage. Another reason may be the non-linearity, which may further increase the pull down voltage in CoventorWare.

4. CONCLUSION

We have made a comprehensive analytical model for non uniform cantilever. The model matches quite closely with the CoventorWare simulation result. The ratio between the model and CoventorWare result is almost constant for different electrode widths. When the cantilever is longer (200 μm) the spring constant is very low and the electrode width has very low effect on spring constant. This will reduce the pull down voltage as the actuation force is higher. This model will be very useful for predicting pull down voltage of non uniform cantilever. This model can be implemented by any simple mathematical tools and calculation can be done within very short time.

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APPENDIX

A force moment diagram of cantilever is shown in figure A1.

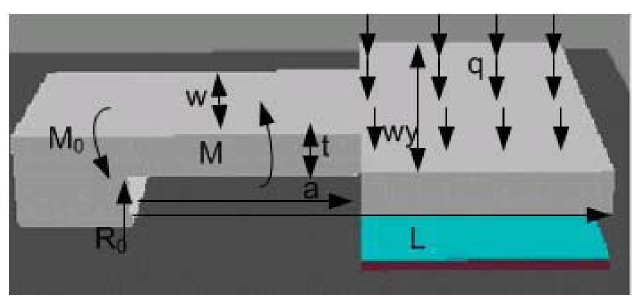

Figure A1: A force moment diagram of non uniform cantilever

In figure A1, 'L' is the length of the cantilever, 't' is the thickness of the cantilever, 'w' is the width of the cantilever close to the anchor and 'wy' is the width of the cantilever above the actuation electrode region. The distributed force acting on the cantilever electrode is 'q' per unit length. The distance from anchor to start of the actuation electrode is 'a'. The anchor is rigid so a moment of inertia will be applied to keep the cantilever fixed at that position (no vertical movement and rotation). M_0 is the moment working at the anchor positions. R_0 is the reaction forces acting opposite to the actuation force to balance the vertical force [4].

The moment M_0 working at the anchor of the cantilever is given by [4],

$$M_0 = q(L-a)\left[a + \frac{L-a}{2}\right] \quad \text{(A1)}$$

$$= \frac{q}{2}(L^2 - a^2)$$

The vertical force working on the anchor is given by,

$$R_0 = q(L-a) \quad \text{(A2)}$$

The equation of moment working on the beam shown in figure A1, at the region 0≤x≤a, is given by

$$M = q(L-a)x - \frac{q}{2}(L^2 - a^2)$$
$$EIv'' = q(L-a)x - \frac{q}{2}(L^2 - a^2) \quad \text{(A3)}$$

Here 'I' is the moment of inertia of the beam in this region and given by, $I=wt^3/12$.
By integrating equation (A3) in terms of x, and simplifying we get,

$$v' = \frac{q}{2EI}\left[(L-a)x^2 - (L^2-a^2)x\right] + C_1 \quad \text{(A4)}$$

As the anchor is rigid no rotation can take place at the anchor. So at x=0, v'=0, which gives $C_1=0$ from equation (A4). So equation (A4) becomes,

$$v' = \frac{q}{2EI}\left[(L-a)x^2 - (L^2-a^2)x\right] \quad \text{(A5)}$$

By integrating equation (A5) in terms of 'x' again, we get

$$v = \frac{q}{2EI}\left[(L-a)\frac{x^3}{3} - (L^2-a^2)\frac{x^2}{2}\right] + C_2$$
$$= \frac{qx^2}{12EI}\left[2(L-a)x - 3(L^2-a^2)\right] + C_2 \quad \text{(A6)}$$

As the anchor is rigid no vertical movement can occur at that region. At x=0, v=0, which gives $C_2=0$, so the equation of deflection in this region becomes,

$$v = \frac{qx^2}{12EI}\left[2(L-a)x - 3(L^2-a^2)\right] \quad \text{(A7)}$$

For the region a≤x≤L, the equation of moment can be written as,

$$M = q(L-a)x - \frac{q}{2}(L^2-a^2) - \frac{q}{2}(x-a)^2$$
$$\Rightarrow EIyv'' = qLx - \frac{qL^2}{2} - \frac{q}{2}x^2 \quad \text{(A8)}$$

Here 'Iy' is the moment of inertia of the beam above the electrode region, with the width 'wy' and thickness 't'. By integrating equation (A8) we get,

$$EIyv' = qL\frac{x^2}{2} - \frac{qL^2x}{2} - \frac{q}{6}x^3 + C_3'$$
$$v' = \frac{q}{6EIy}(3Lx^2 - 3L^2x - x^3) + C_3 \quad \text{(A9)}$$

Using the continuity of the slope of deflection at x=a, the constant C_3 is given by.

$$C_3 = \frac{qaL(a-L)}{2EI}\left(1 - \frac{1}{y}\right) + \frac{qa^3}{6EIy} \quad \text{(A10)}$$

By integrating the equation (A9) in term of x again we can get the equation for deflection,

$$v = \frac{q}{24EIy}(4Lx^3 - 6L^2x^2 - x^4) + C_3x + C_4 \quad \text{(A11)}$$

By using continuity of deflection at x=a we find

$$C_4 = \frac{qa^2L}{12EI}(3L-4a)\left(1-\frac{1}{y}\right) + \frac{qa^4}{12EI}\left(1+\frac{1}{2y}\right) - \frac{qa^4}{6EIy} \quad \text{(A12)}$$

The maximum deflection of the cantilever occurs at x=L or at the end of beam. The deflection at x=L from the equation (A11) is given by (without replacing the value of C_3 and C_4 for simplification),

$$v_L = -\frac{qL^4}{8EIy} + C_3L + C_4 \quad \text{(A13)}$$

The spring constant of the cantilever is given by,

$$k = -\frac{P}{v_L} = -\frac{q(L-a)}{v_L} \quad \text{(A14)}$$

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