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Modeling of Spring Constant and Pull-down Voltage of Non uniform RF MEMS Cantilever

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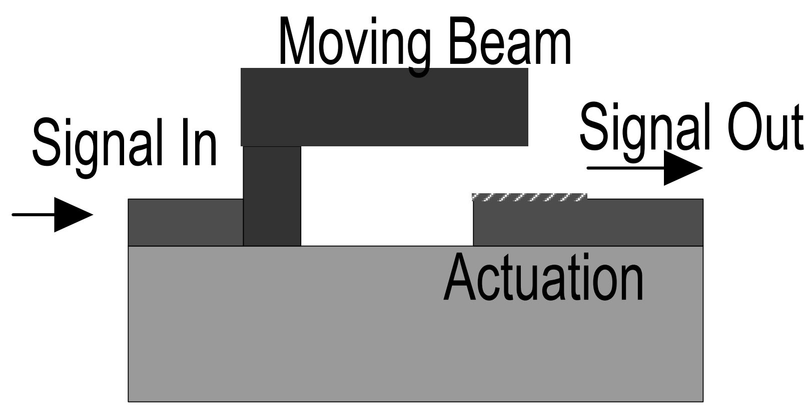
Outline

1. A brief introduction of RF MEMS
2. Motivation
3. Proposed model
4. Comparison of the model with standard text book
5. Variation of spring constant with width and length
6. Pull down voltage and comparison of the model with CoventorWare results
7. Conclusion

Introduction to RF MEMS

- Radio Frequency Micro Electromechanical Systems
- Basic mechanical movement and switching
- Very low insertion loss during on state at high frequency
- Very high isolation during off state
- Electrostatic actuation mechanism is most commonly used due to its very low power consumption and easy operation



Introduction to RF MEMS (applications)

- High frequency switching application
- High Q varactor for filter and oscillator
- The switches can be used in:
  1) Phase shifters
  2) Matching network
  3) Antenna Systems etc.

Introduction to RF MEMS (configuration)

- Resistive contact
- Capacitive contact

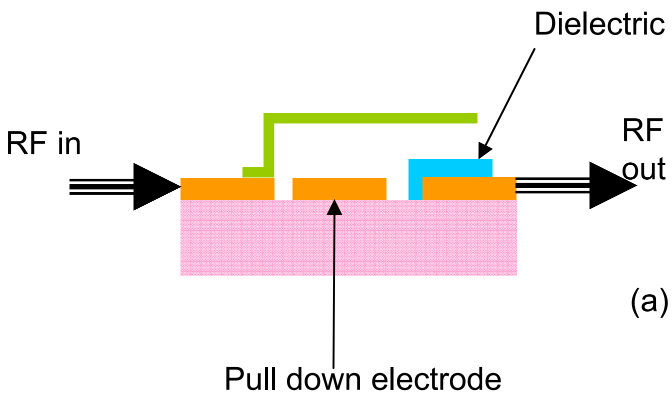
- Shunt
- Series

- Microstrip
- CPW

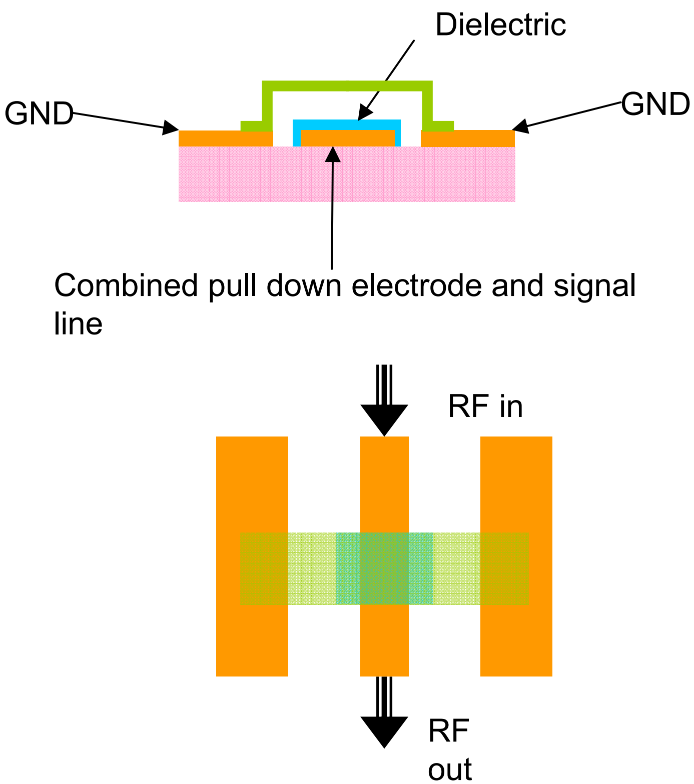
- Broadside
- Inline

- Bridge
- Cantilever

Switch configuration



Capacitive series inline switch, Cantilever



Capacitive shunt broadside switch, Bridge

Introduction to RF MEMS (limitation)

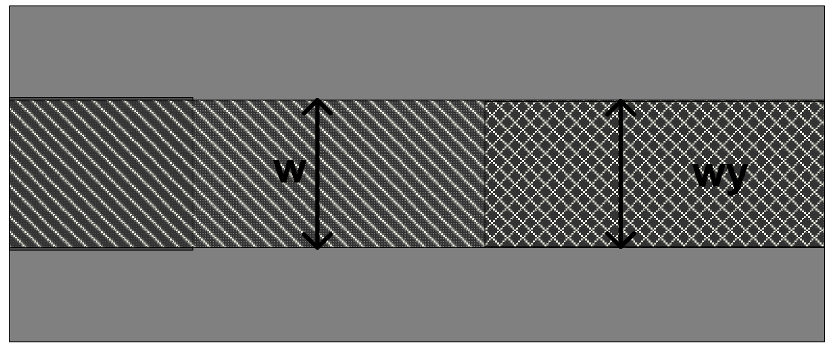
- High actuation voltage (in the range of few volts to several tens of volts)
- Low switching speed (usually in the range of μs)
- Need hermetic packaging

Motivation

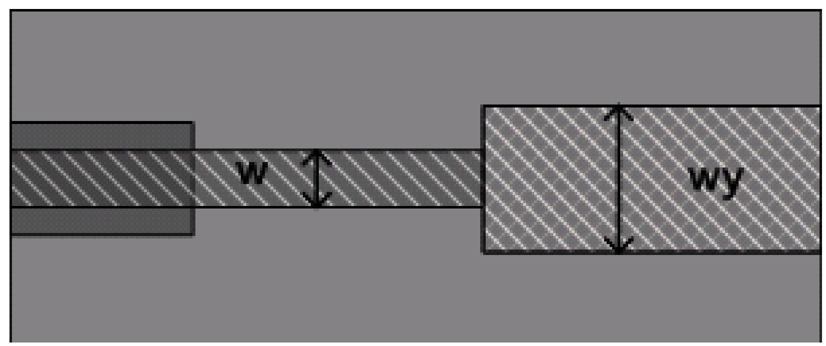
- To reduce the actuation voltage, different techniques are used
  - Beam narrower at the anchor and wider at the actuation area (non uniform beam)
  - Folded spring type
- The proposed design will have lower spring constant and lower pull down voltage
- Requires proper modeling of the spring constant and pull down voltage before fabrication

Motivation

- The model for uniform beam is available in standard text book
- For non uniform beam, a new model is recently published [1]
- The actuation force is assumed concentrated, thus the accuracy varies a lot
- We have developed a more accurate and elaborate model



Uniform Cantilever

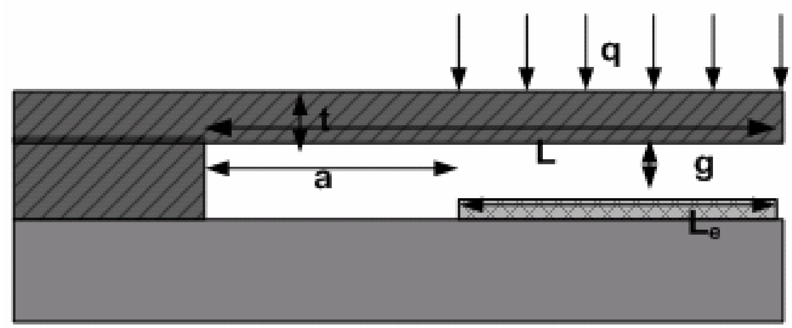


Non-uniform Cantilever

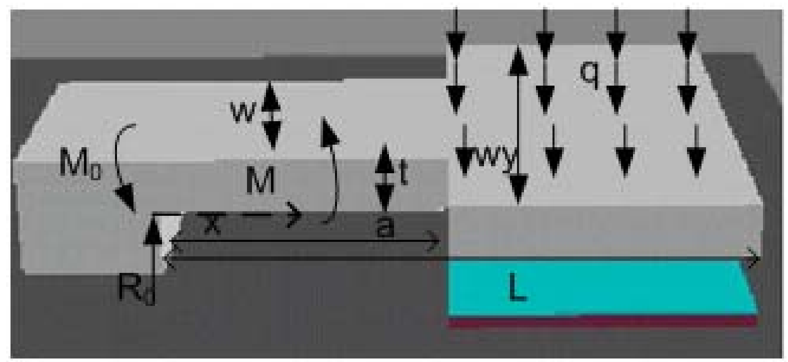
[1] S. Afrang et al, *Design and Simulation of Simple and Varying Section Cantilever and Fixed-Fixed End Types MEMS Switches*, Proceeding of ICSE 2004. pp. 593-596.

Proposed model (I)

- A side view with actuation force is shown on the right (top)
- A 3-D view with force moment diagram is shown the right (bottom)
- The force-moment and deflection equations are seperated into two regions



A side view of the cantilever



A 3D view of the cantilever with force moment diagram

Proposed model (I I)

- In region 1 (0≤x≤a), the deflection is given by:

$$v = \frac{qx^2}{12EI}\left[2(L-a)x - 3(L^2 - a^2)\right]$$

- In region 2 (a≤x≤L), the deflection is given by:

$$v = \frac{q}{24EIy}(4Lx^3 - 6L^2x^2 - x^4) + C_3x + C_4$$

with C_3 and C_4 is given by:

$$C_3 = \frac{qaL(a-L)}{2EI}\left(1 - \frac{1}{y}\right) + \frac{qa^3}{6EIy}$$

$$C_4 = \frac{qa^2L}{12EI}(3L-4a)\left(1 - \frac{1}{y}\right) + \frac{qa^4}{12EI}\left(1 + \frac{1}{2y}\right) - \frac{qa^4}{6EIy}$$

Proposed model (I I I)

- The spring constant of the cantilever depends on the actuation force and the deflection at the end.
- The deflection of the cantilever at the end is given by:

$$v_L = \frac{-q(L-a)^2(L+a)^2}{8EIy} - \frac{q(L-a)a(a+3L)}{12EI}$$

$$-\frac{qa^2L}{2EI}(L-a)^2(1-\frac{1}{y})$$

- The spring constant is given by:

$$k = -\frac{P}{v_L} = -\frac{q(L-a)}{v_L}$$ After simplification $$k = \frac{24EIy}{3(L-a)(L+a)^2 + 2ya^2(a+3L) + 12aL(L-a)(y-1)}$$

Comparison of Modeling

- When the force is distributed and y=1, the deflection of the beam is given by:

$$v = -\frac{q}{24EI}\left[x^4 - 4Lx^3 + 6L^2x^2 - 4a^3x + a^4\right]$$

This matches with the standard text book formulas.

For some well known cases

- When ‘a=0’ the spring constant is given by:

$$k \xrightarrow{a=0} \frac{8EIy}{L^3} = \frac{2Ewy}{3}\left(\frac{t}{L}\right)^3$$

Comparison of Modeling

- For 'a=L' the spring constant is given by:

$$k \xrightarrow{a=L} \frac{3EI}{L^3} = \frac{Ew}{4}\left(\frac{t}{L}\right)^3$$

- For 'y=1' the spring constant is given by:

$$k \xrightarrow{y=1} 2Ewt^3 \frac{L-a}{3L^4 - 4La^3 + a^4}$$

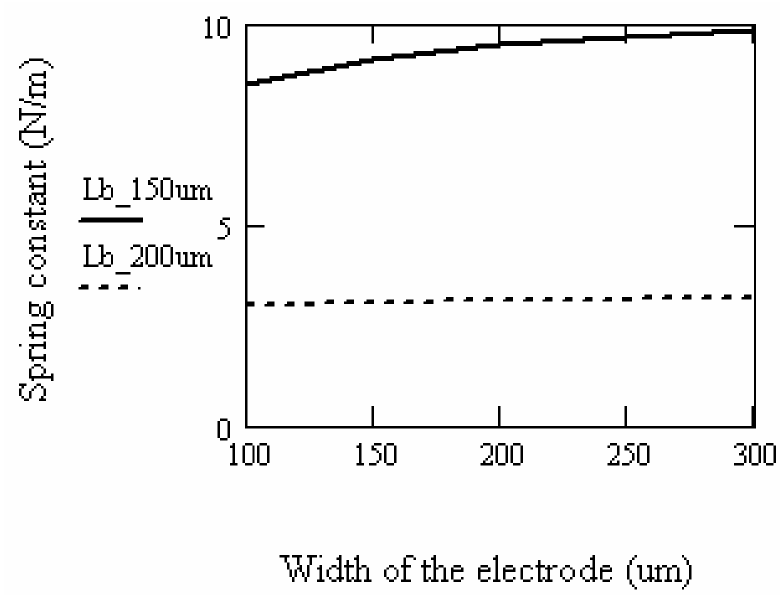
- They all match with the expressions mentioned in [2]

[2] G.M. Rebeiz, *RF MEMS Theory, Design and Application,* New Jersey, *John* Wiley and Sons 2003.

Spring constant variation

- The dimensions are as follows

• The beam lengths, L_1=150 μm and L_2= 200 μm
• The thickness of the beam, t= 2 μm.
• The width of the beam at the anchor, W= 100 μm.
• The length of the electrode, L_e=100 μm
• Width of the electrode is varied from 100 μm to 300 μm (1≤y≤3)

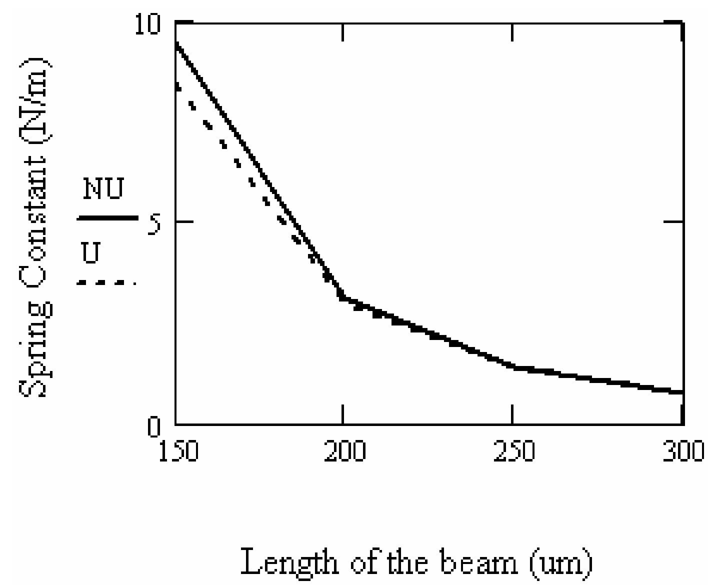


Variation of spring constant of cantilevers with electrode width

Spring constant variation

- The dimensions of the cantilever are as follows

- For uniform beam, the electrode width Wy= 100 µm and length L_e=100 µm.
- For non uniform beam, the beam width w=100 µm, the electrode length L_e=100 µm and Wy= 200 µm.
- The thickness of the beam is 2 µm.



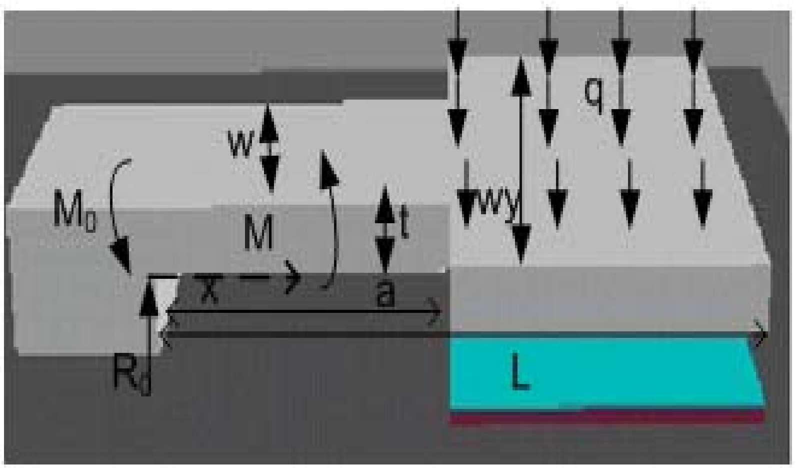
The variation of spring constant with beam length

Pull down voltage

- The pull down voltage of a beam is given by:

$$V_p = \sqrt{\frac{8kg_0^3}{27\varepsilon_0 L_e wy}}$$

- This calculation is done using the assumption that the beam collapses when it moves one third of its initial gap.



Comparison of pull down voltage with CoventorWare

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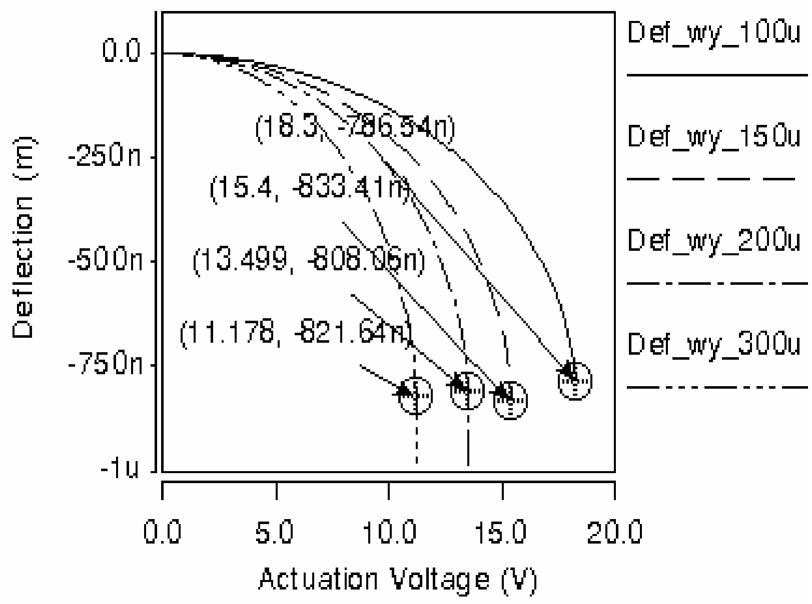
Electrode width, wy (µm)	Spring constant (N/m)	Pull down voltage (V)	Pull down voltage in Coventor-Ware (V)	Error (%)
100	8.50	15.20	18.4	17
150	9.10	12.90	15.5	17
200	9.50	11.40	13.6	16
300	9.90	9.50	11.3	16



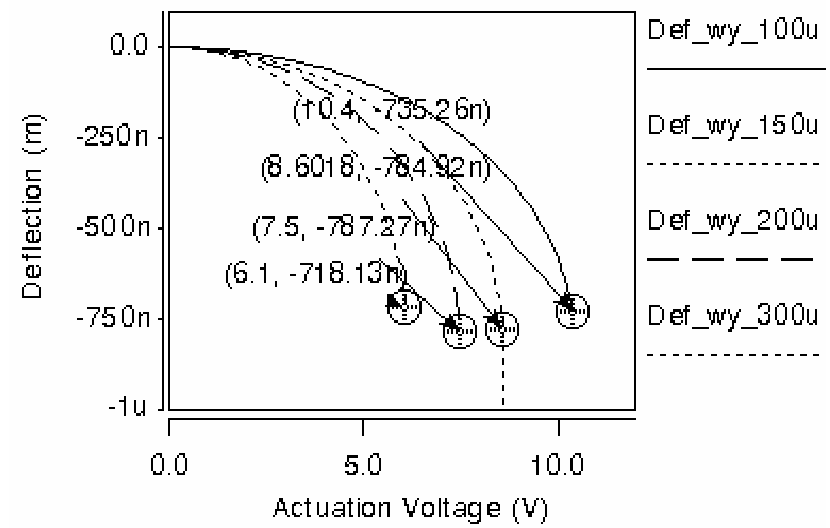
The pull down voltage simulation in CoventorWare

Here L=150 µm, L_e= 100 µm, g_0= 2 µm and t= 2 µm. The beam material is aluminum with young's modulus E=77 GPa.

Comparison of pull down voltage with CoventorWare

BMAS 2006

Electrode width, wy (µm)	Spring constant (N/m)	Pull down voltage (V)	Pull down voltage in Coventor-Ware (V)	Error (%)
100	3.01	9.05	10.5	14
150	3.08	7.48	8.7	14
200	3.12	6.52	7.5	13
300	3.16	5.40	6.2	13



The pull down voltage simulation in CoventorWare

Here L=200 µm, L_e= 100 µm, g_0= 2 µm and t= 2 µm. The beam material is aluminum with young's modulus E=77 GPa.

Recent publication of non uniform beam [1]

For uniform beam, L=120 μm, w=30 μm, t=1.5 μm and g_0=1.5 μm

Electrostatic area $(\mu m)^2$	20*30	30*30	30*45
V_{th} Calculation (a)	14.5	11.85	9.67
V_{th} Simulation (a)	17.1	14.8	13.35
V_{th} Simulation (b)	22.4	20	18.2
Error (a)	18%	25%	38%
Error (b)	55%	68%	88%

For non-uniform beam, L=120 μm, w_1=15 μm W_2=30 μm t=1.5 μm and g_0=1.5 μm

Electrostatic area $(\mu m)^2$	30*20	30*30	30*45
V_{th} Calculation results	10.26	8.47	7
V_{th} Simulation results	12.4	10.6	9.5
Error	20%	25%	35%

- The accuracy varies a lot for both uniform and non uniform beam

Conclusions

- The proposed model is very simple
  - Gives faster calculation of the pull-down voltage compared to the standard method using 3-D model
  - The model can be implemented with simple mathematical tools
- The model accuracy is very close to state of the art commercial tools
- It is much more accurate than the earlier published model [1]
- It can also be applied directly to a uniform beam i.e. y=1

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Thanks for your attention