Behavioral models of Frequency Pulling in Oscillators

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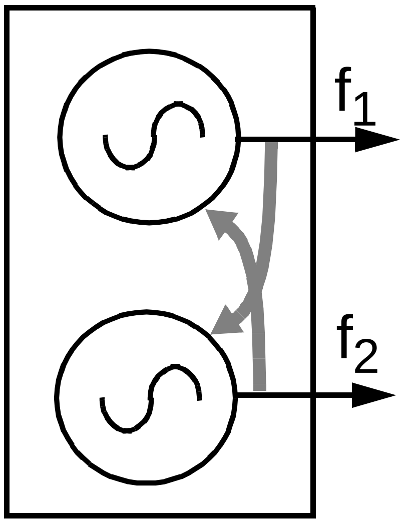
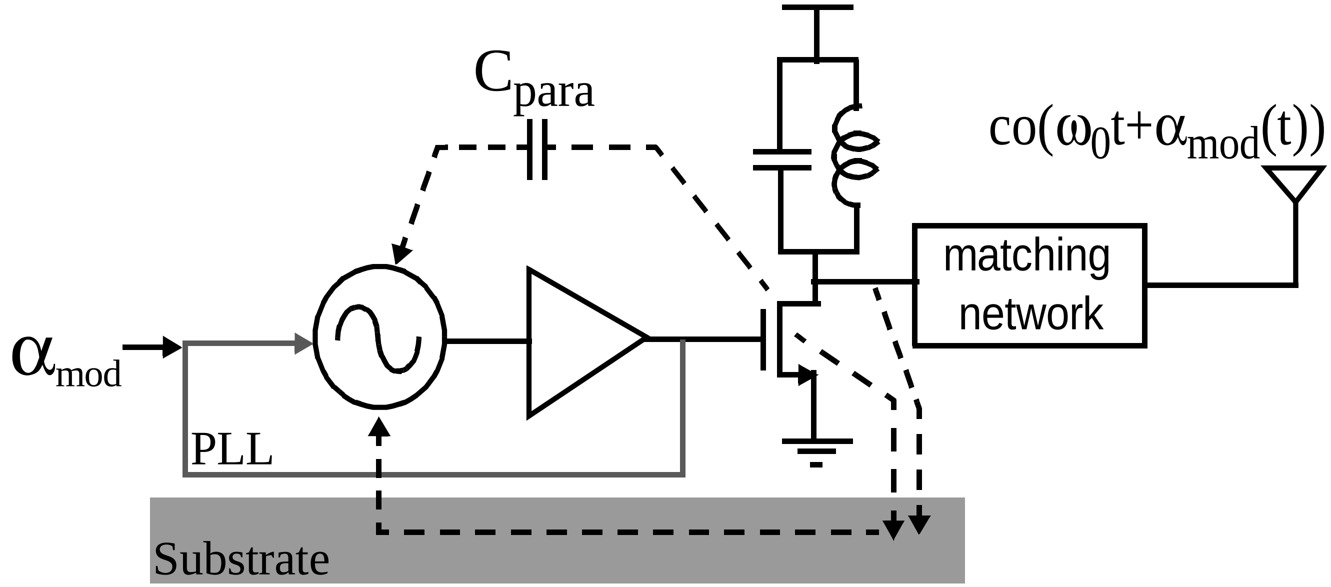
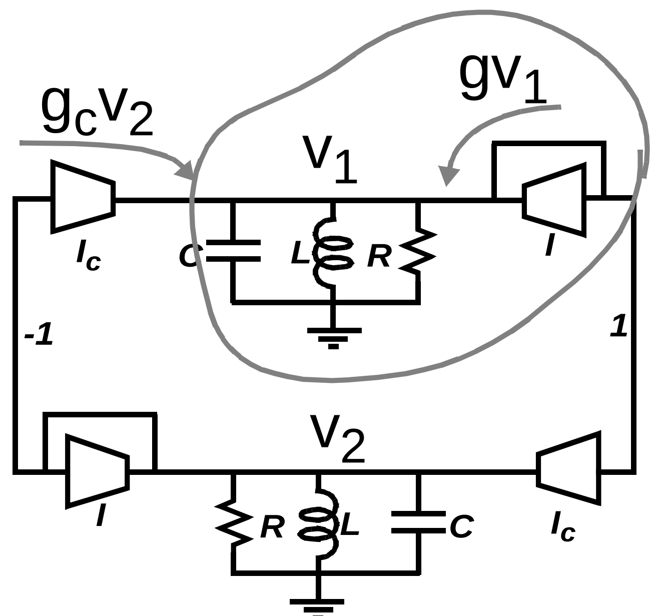
Pulling, useful and harmful effects



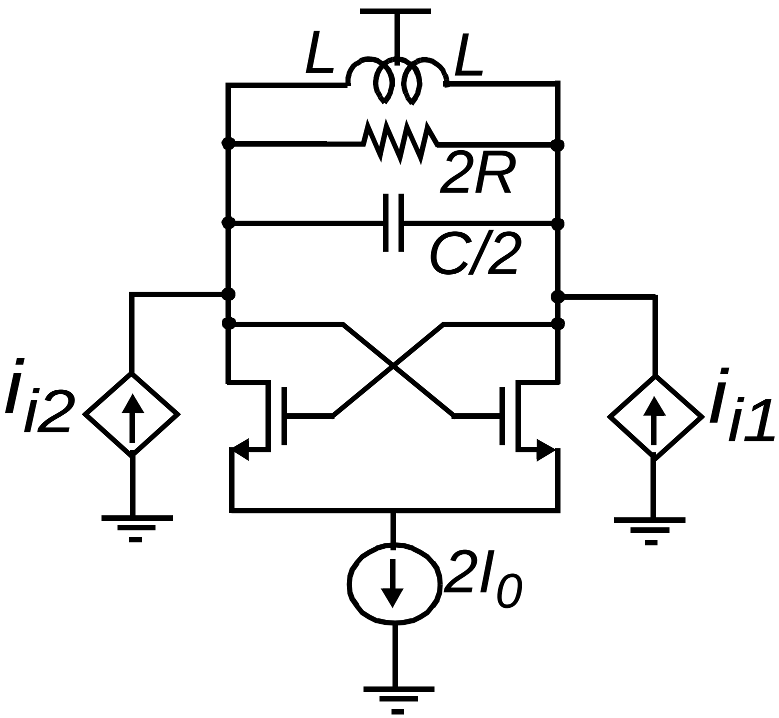


Examples of pulling:

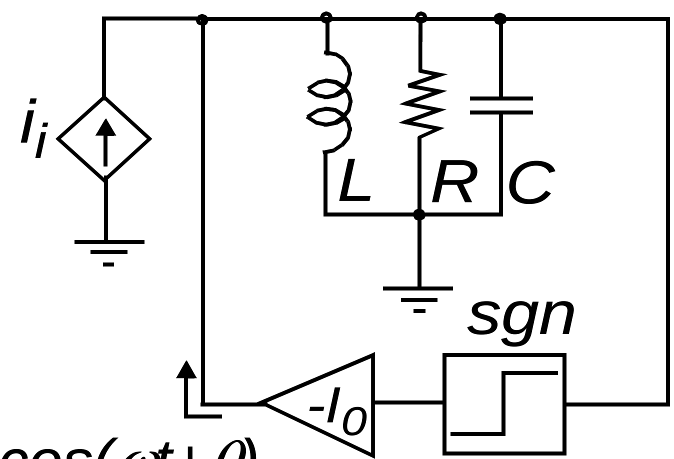
- Synthesizer
- Transmitter
- Quadrature Oscillator



Pulling in an LC oscillator, Adler's equation

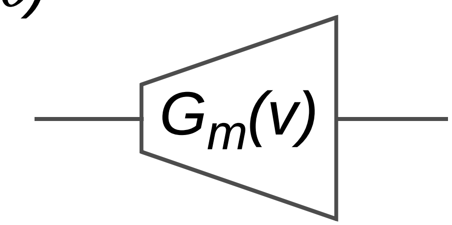


$i_i(t)=I_i\cos(\omega t)$ $\quad v(t)=v_e(t)\cos(\omega t+\theta)$



$i(t)=I\cos(\omega t+\theta)$

$I=(4/\pi)I_0$



$i_{out}=G_m(v)=G_{m0}v-\alpha x^3$

KCL at output:

$$\frac{v}{R}+\frac{1}{L}\int v\,dt+C\frac{dv}{dt} = I_i\cos(\omega t)+I\cos(\omega t+\theta)$$

Pulling in an LC oscillator, Adler's equation

$i_i(t) = I_i(t)e^{j\omega t}$ $\qquad v(t) = v_e(t)e^{j\omega t}e^{\theta}$ $\qquad i(t) = I(t)e^{j\omega t}e^{\theta}$

$$\frac{1}{R}\frac{dv_e}{dt} + j\frac{1}{R}\left(\omega + \frac{d\theta}{dt}\right)v_e + \frac{1}{L}v_e + C\frac{d^2v_e}{dt^2} + j2C\left(\omega + \frac{d\theta}{dt}\right)\frac{dv_e}{dt} + jC\frac{d^2\theta}{dt^2}$$

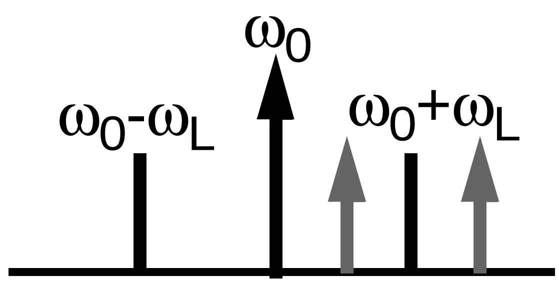
$$-C\left(\omega + \frac{d\theta}{dt}\right)^2 v_e = j\omega I_i e^{-j\theta} + jI\left(\omega + \frac{d\theta}{dt}\right)$$

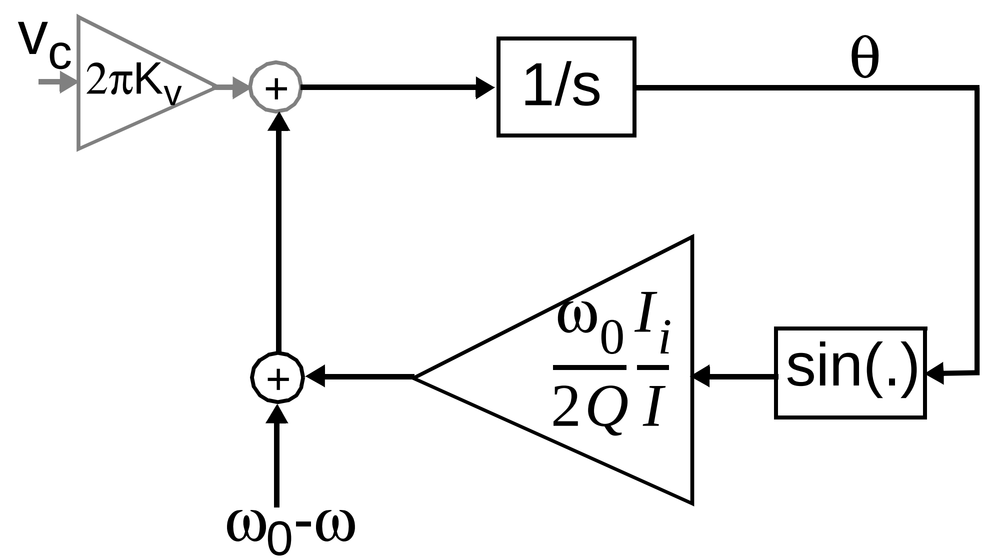
$$\frac{1}{R}\frac{dv_e}{dt} + \frac{1}{L}v_e + C\frac{d^2v_e}{dt^2} - C\left(\omega + \frac{d\theta}{dt}\right)^2 v_e = I_i\omega\sin(\theta)$$

$v_e \approx RI = QL\omega I \qquad I_i \ll QI$

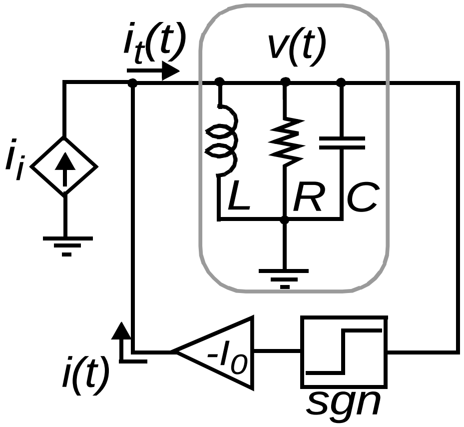
$$\boxed{\frac{d\theta}{dt} = \omega_0 - \omega - \frac{\omega_0}{2Q}\frac{I_i}{I}\sin(\theta)}$$

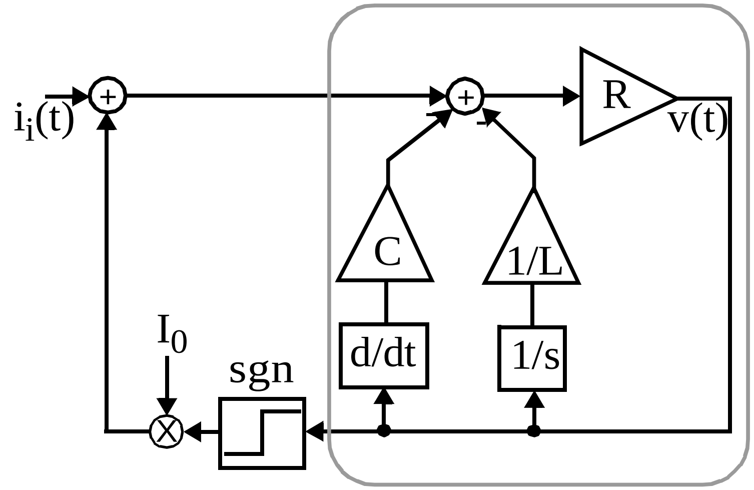
$$\omega_L = \frac{\omega_0}{2Q}\frac{I_i}{I}$$



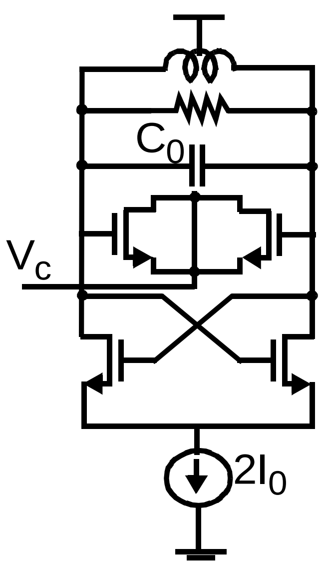


Singal flow graph of an LC oscillator

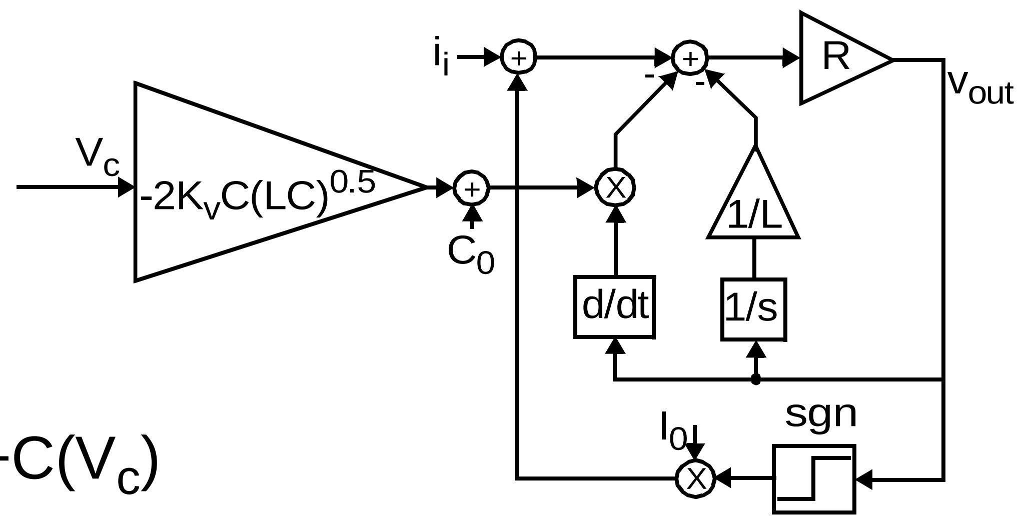




$$\frac{1}{L}\int_{-\infty}^{t} v(t)(dt + C)\frac{dv(t)}{dt} + \frac{v(t)}{R} = i_t(t) \qquad v(t) = R\left[i_t(t) - \frac{1}{L}\int_{-\infty}^{t} v(t)dt - C\frac{dv(t)}{dt}\right]$$



$C_{tot}=C_0+C(V_c)$



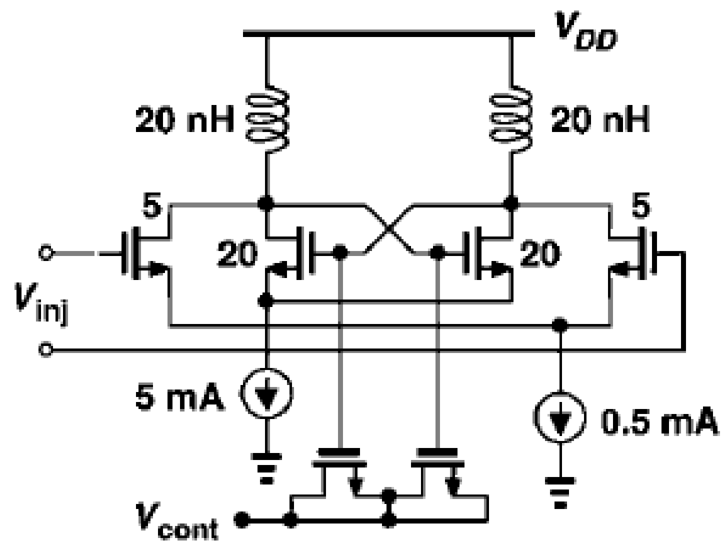
Experimental validation

B. Razavi, “A study of injection locking and pulling in oscillators“

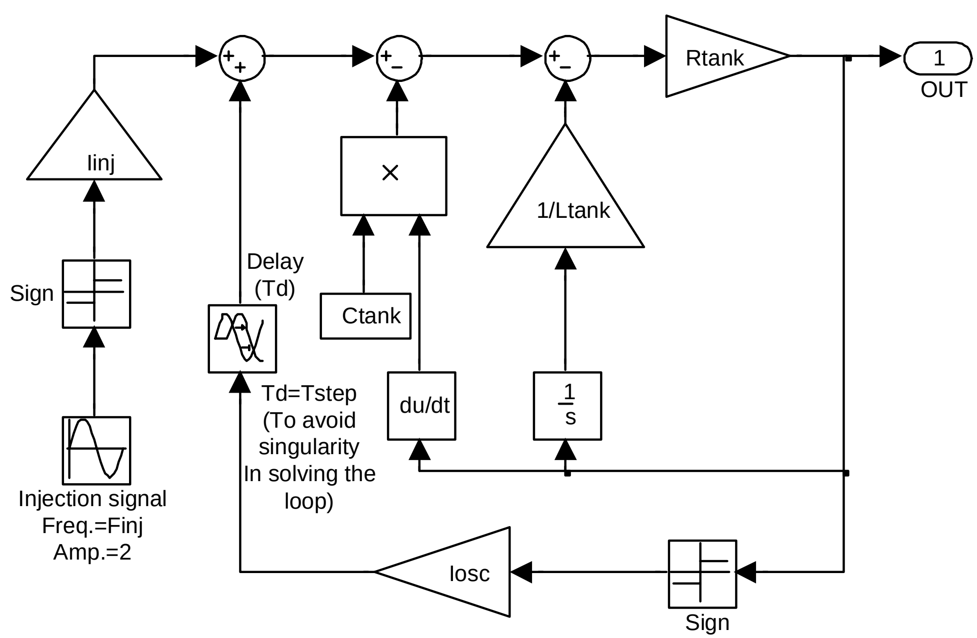
$$\frac{d\theta}{dt} = \omega_0 - \omega - \frac{\omega_0}{2Q}\frac{I_i}{I}\sin(\theta)$$

$$\omega_L = \frac{\omega_0}{2Q}\frac{I_i}{I} = 2\pi \times 1.5MHz$$

$$\omega_{osc} = 2\pi \times 900MHz \qquad \frac{I_i}{I} = -38dB$$

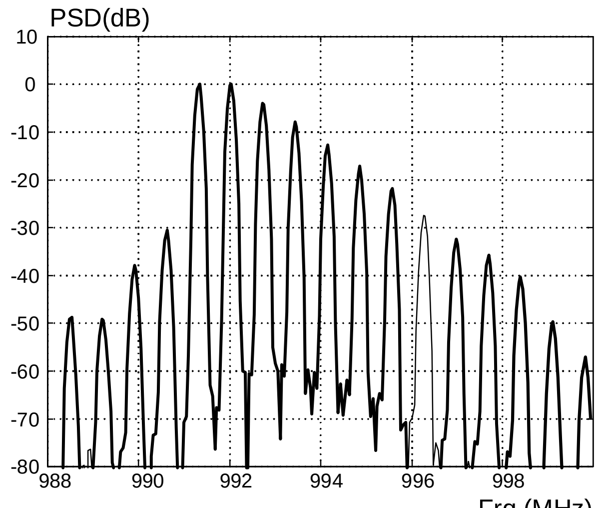



```
Iosc=2.5e-3;        % tail current
F0=1e9;             % nominal oscillation frequency
Fosc=992936860; % actual oscillation frequency
                    % measured under "no-pulling" condition
Flock=1.5e6;        % lock range
Iinj=Iosc/80;       % Iinj(dB) = Iosc(dB)- 38

Qtank= 5.8;         % Quality factor of inductor
Ltank = 20e-9;      % inductor
Rtank=Qtank*Ltank*(2*pi*F0);  % parallel resistor  of tank
Ctank=1/(Ltank*(2*pi*F0)^2);     % capacitor

Tstep=(1/F0)/200;      % time step of simulation
Tmax=(2^21)*Tstep;  % simulation time
Finj=Fosc-Flock-710e3; % frequency of injected signal

sim('simulink_model_of_osc',Tmax)
```

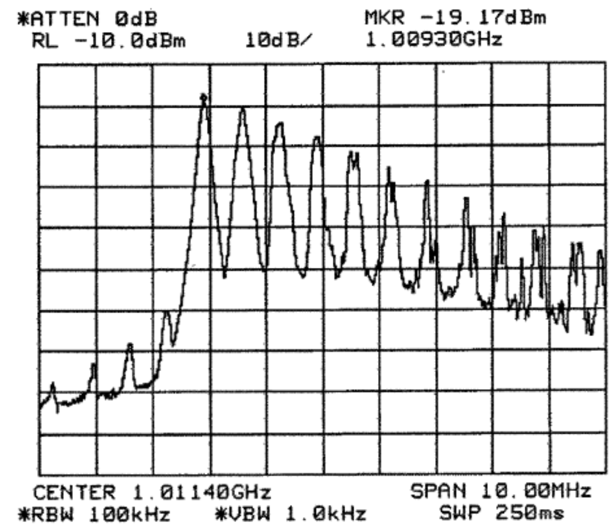


Simulation vs. Measurement

f_{inj}=110kHz out of lock range

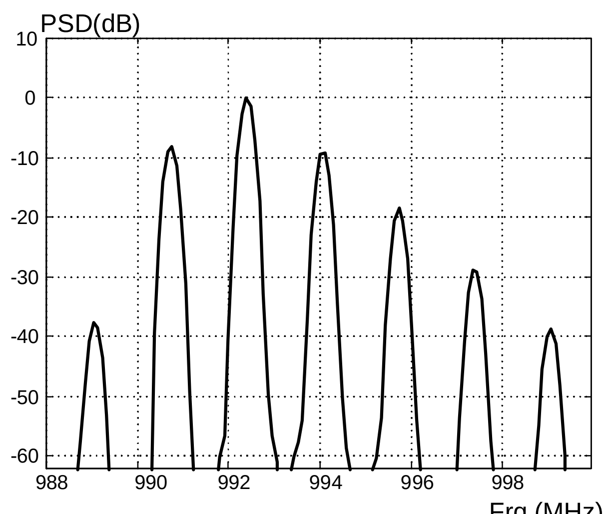


(a)

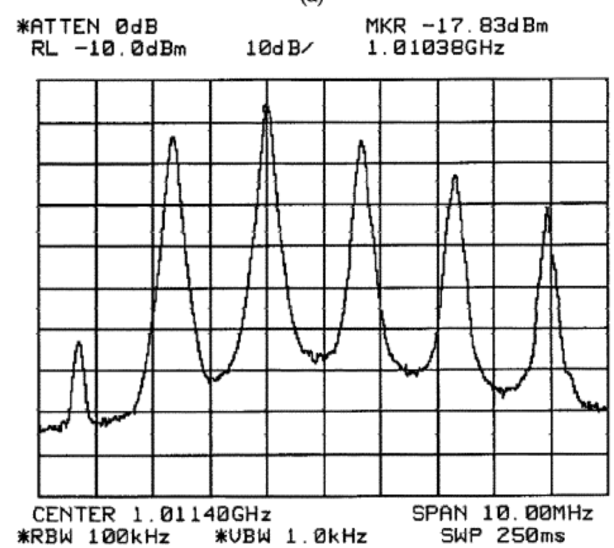


(a)

f_{inj}=710kHz out of lock range

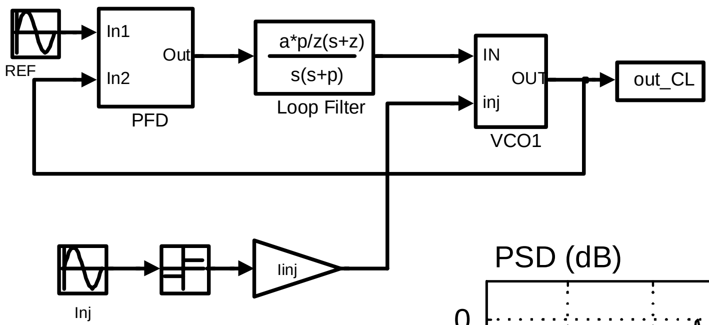


(b)



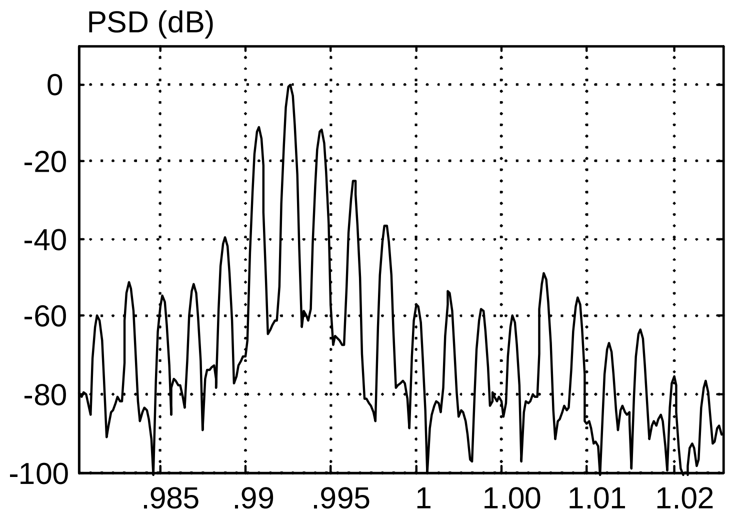
(b)

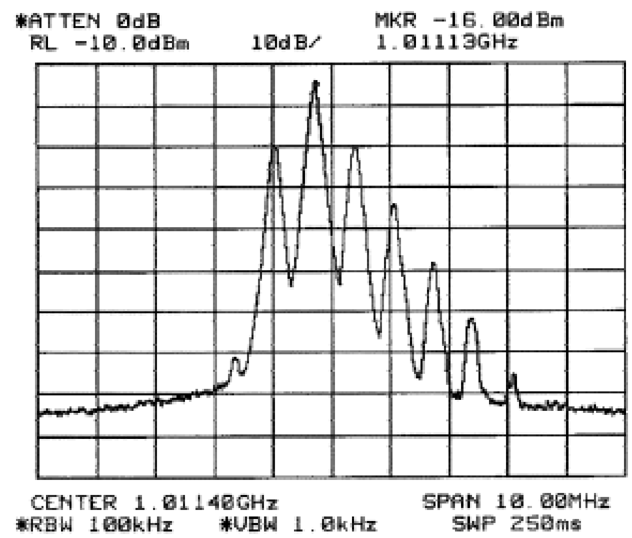
Simulation vs. Measurement



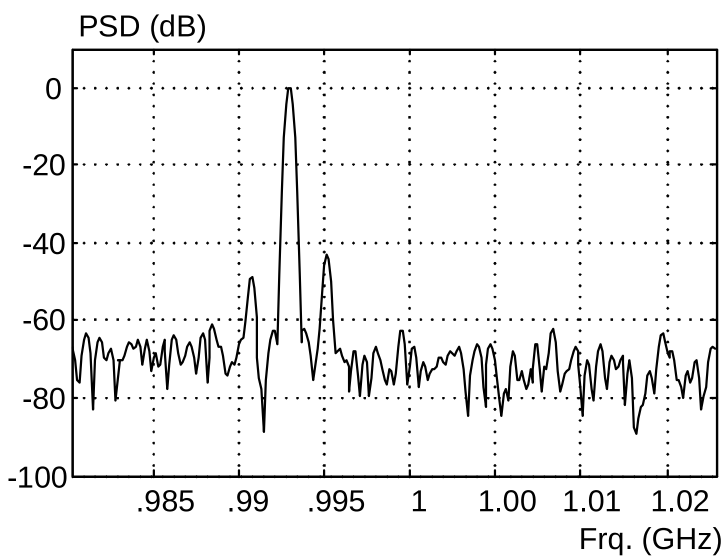
$F_{ref}=F_{osc}$
Fosc=0.929GHz
I_{inj}/I_{osc}=-40dB

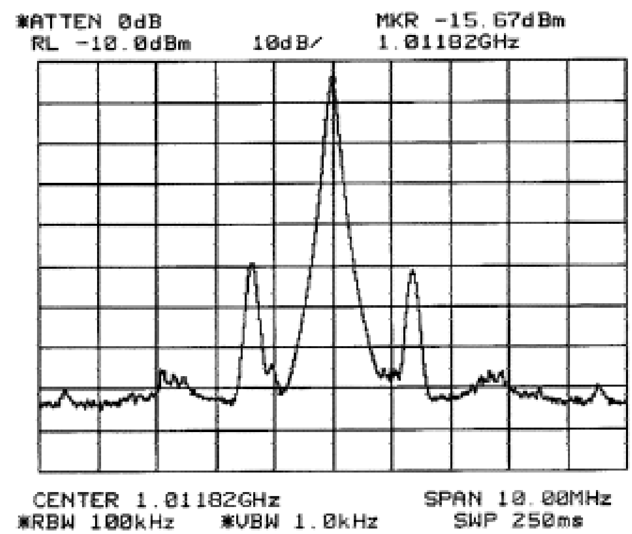
open loop



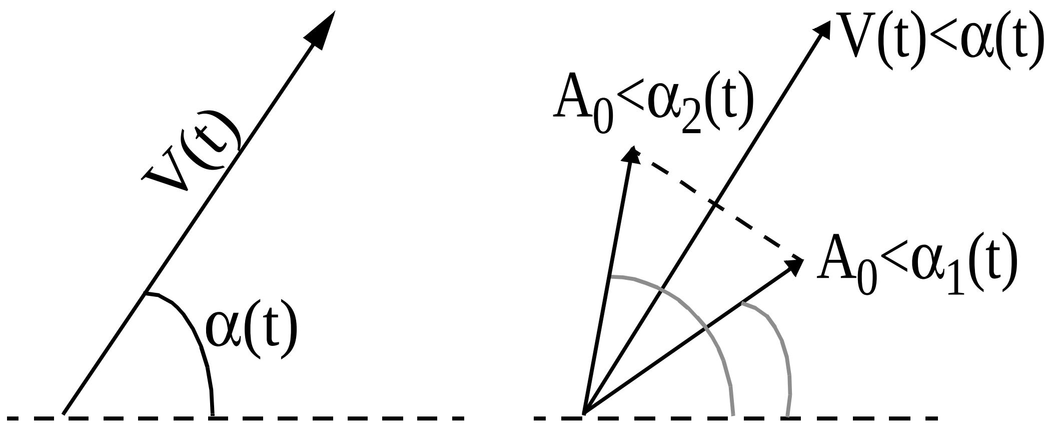


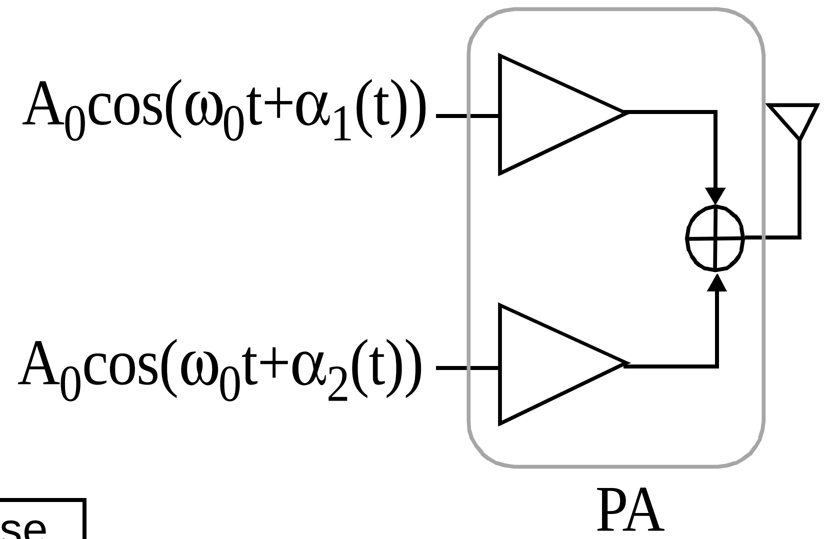
closed loop

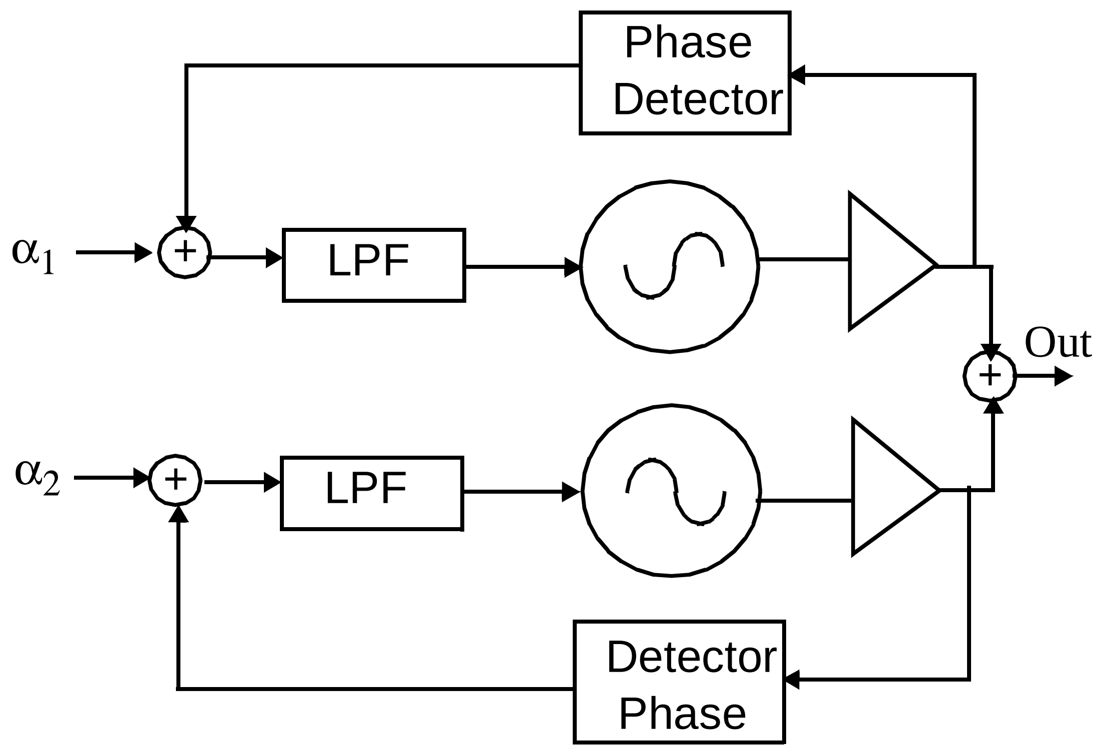




An Example: Outphasing transmitter





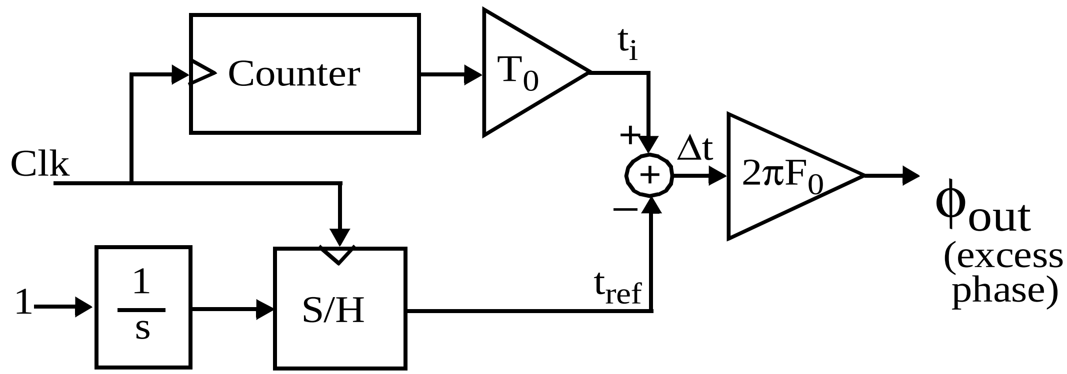


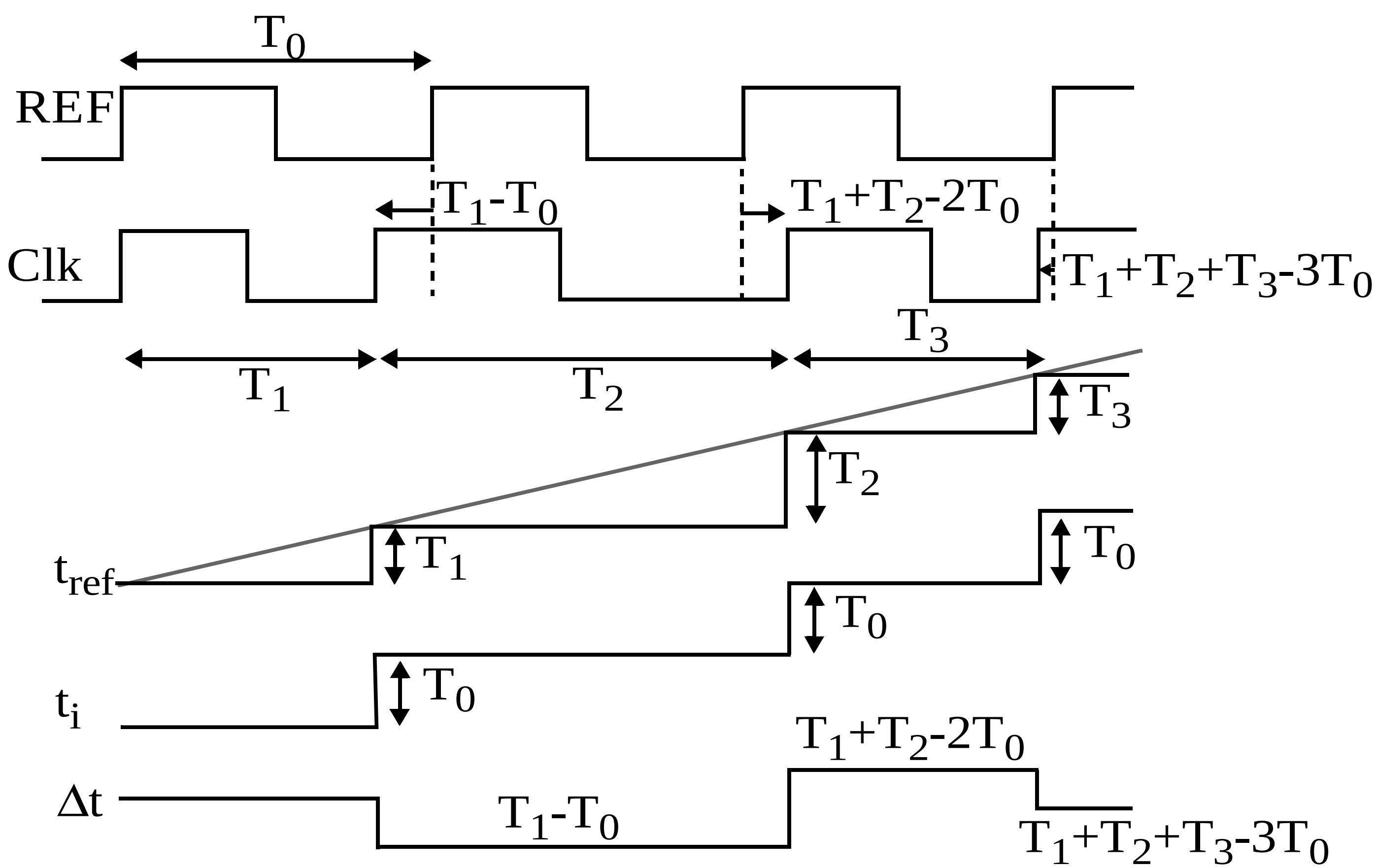
Behavioral model for Phase Detector

REF is reference of time

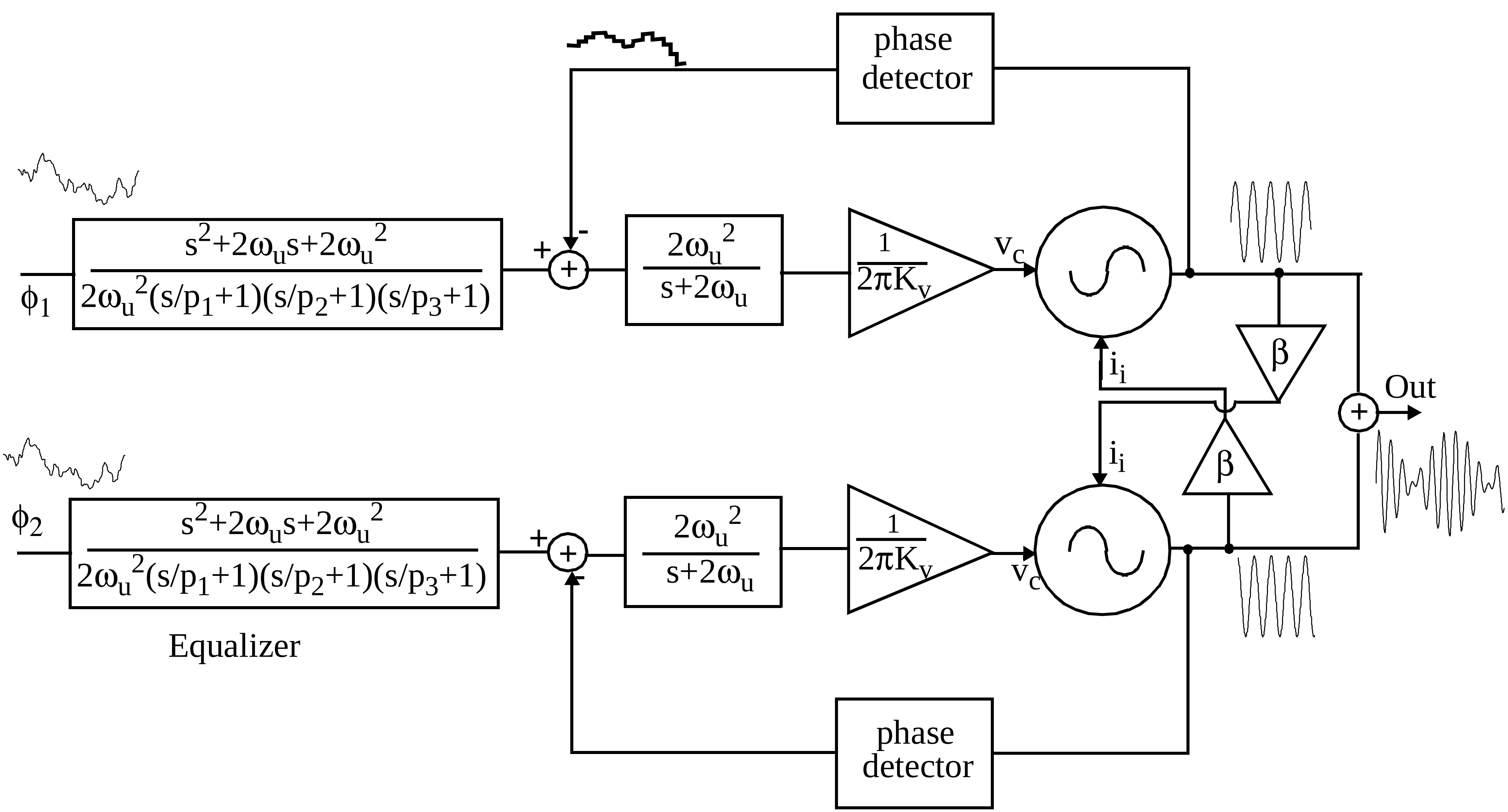
This reference shows itself as T_0 in the figure.

Output is phase of the ***Clk*** with respect to the reference signal and sampled at rising edges of the ***Clk***

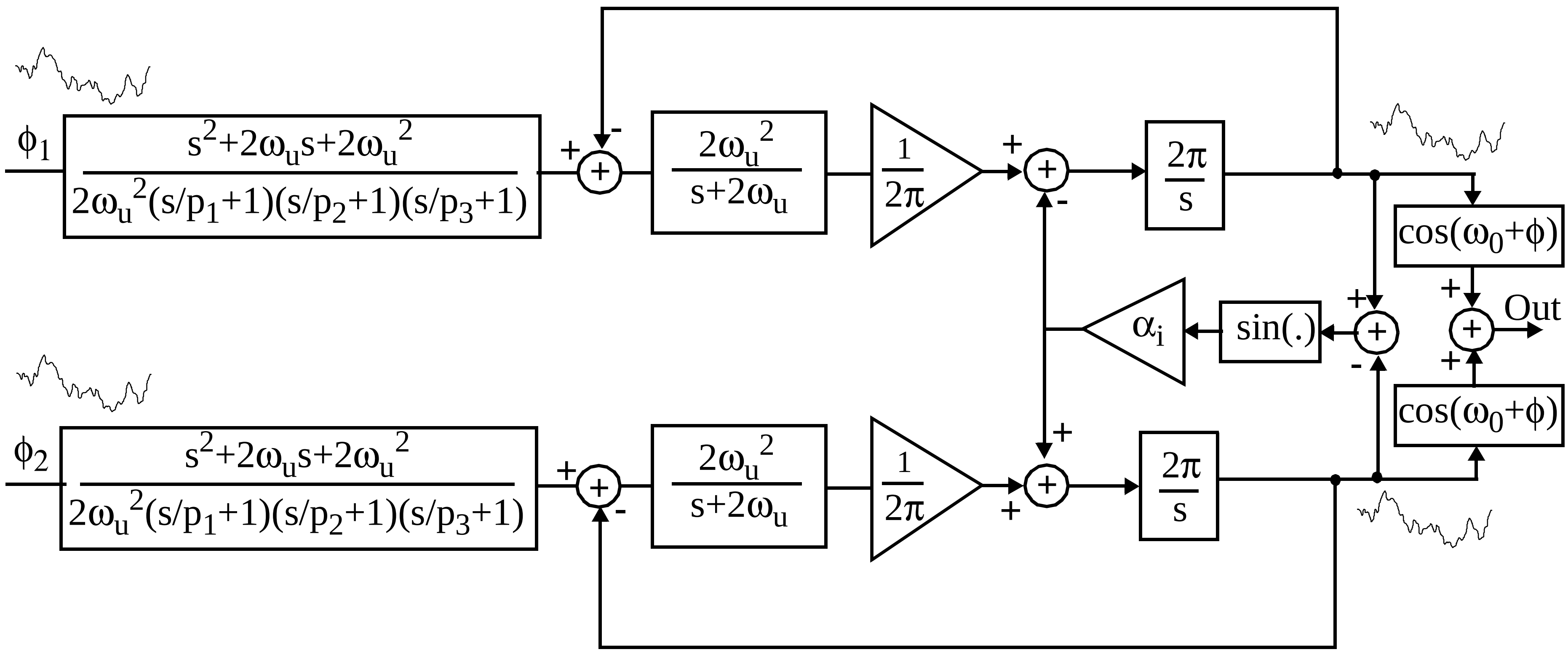




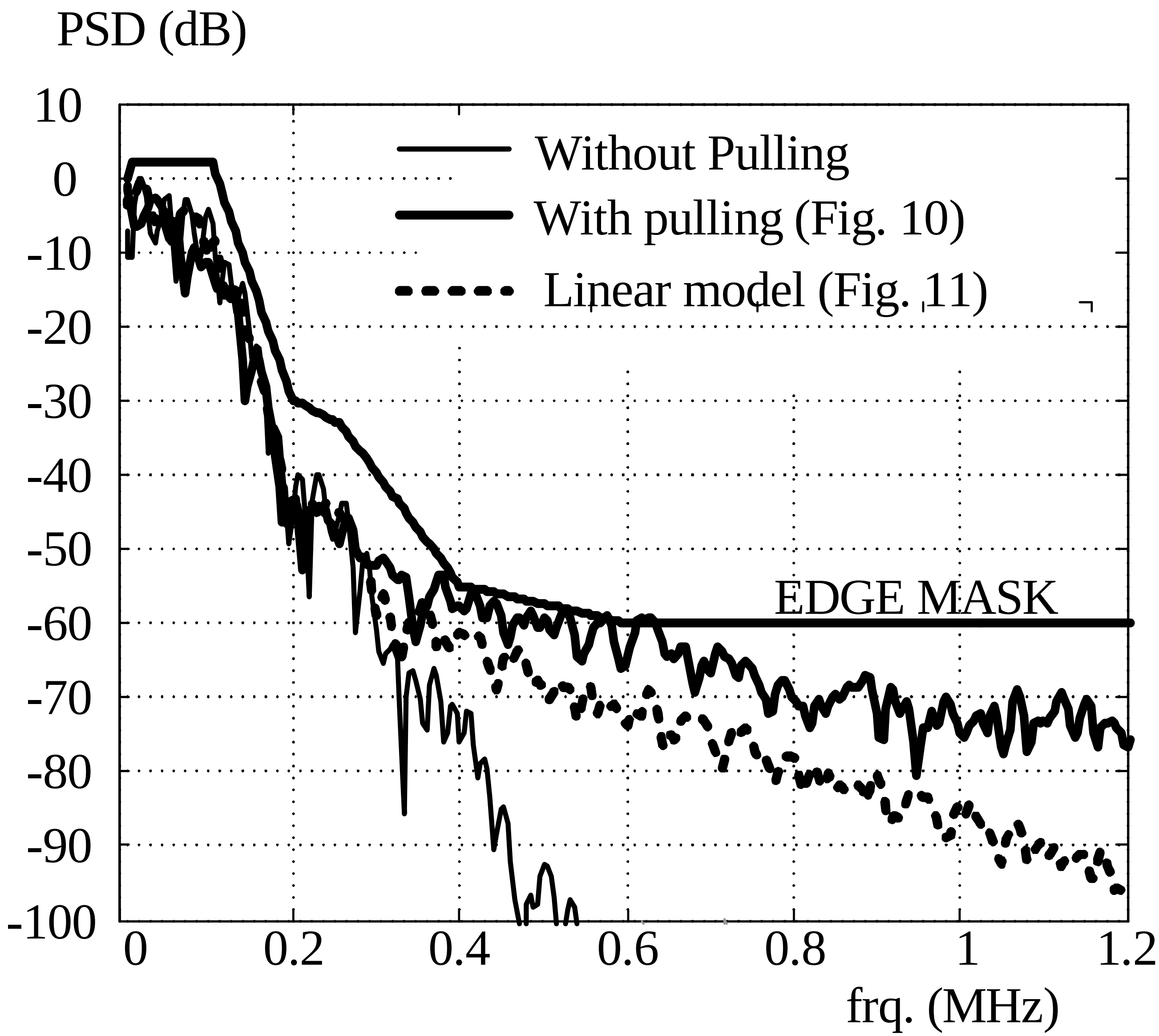
Behavioral model for an outphasing Tx



Linear model



Simulation Results



Conclusion

1. Pulling effect in differential LC oscillator is studied and Adler's equation is derived.
2. A model is introduced to capture the pulling effect in oscillator. This model is extremely simple and can be easily implemented in MATLAB, VERILOG or any high level programming tool.
3. In spite of simpleness, the model gives accurate results which are comparable with measurement results