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An efficient and accurate MEMS accelerometer model with sense finger dynamics for applications in mixed-technology control loops

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Outline

- Introduction
- MEMS capacitive accelerometer
- Accurate VHDL-AMS accelerometer model with sense finger dynamics
- Simulation results
- Conclusion

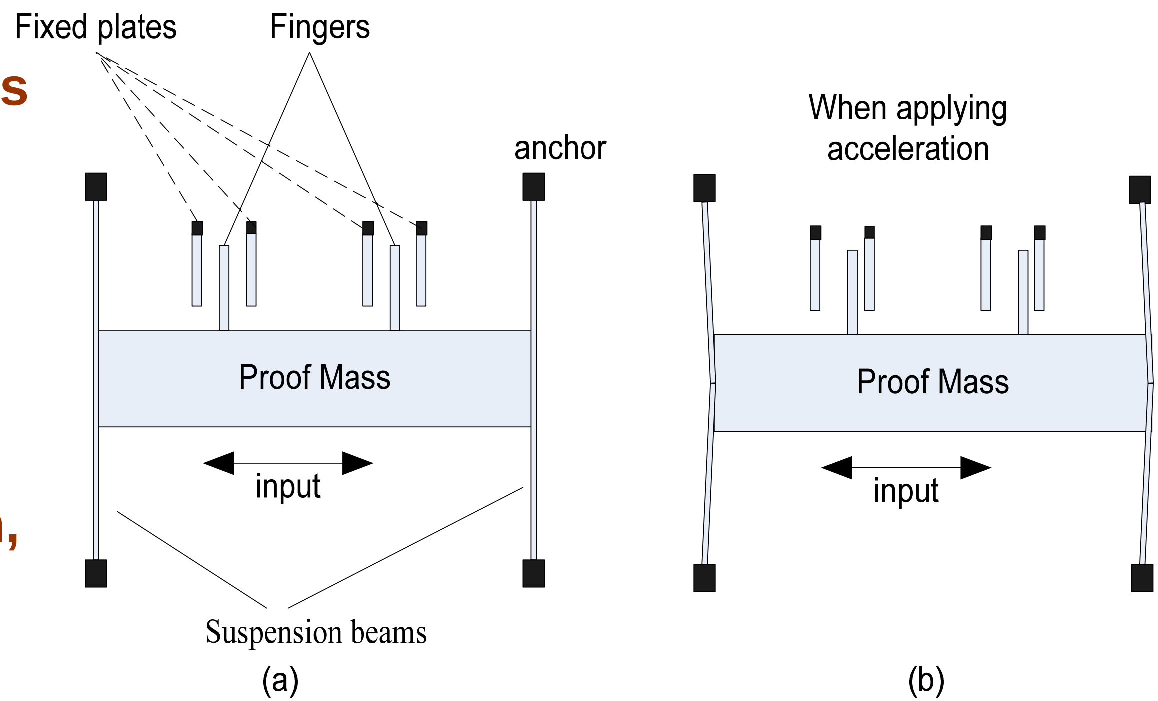
Introduction

- Effects of sense finger dynamics in MEMS capacitive accelerometer studied and modeled accurately
- Distributed mechanical sensing element model
- Finite Difference Approximation (FPA) approach
- Well known failure of Sigma-Delta accelerometers when fingers resonate is modeled correctly

MEMS capacitive accelerometer

- Deflection of proof mass is caused by input acceleration

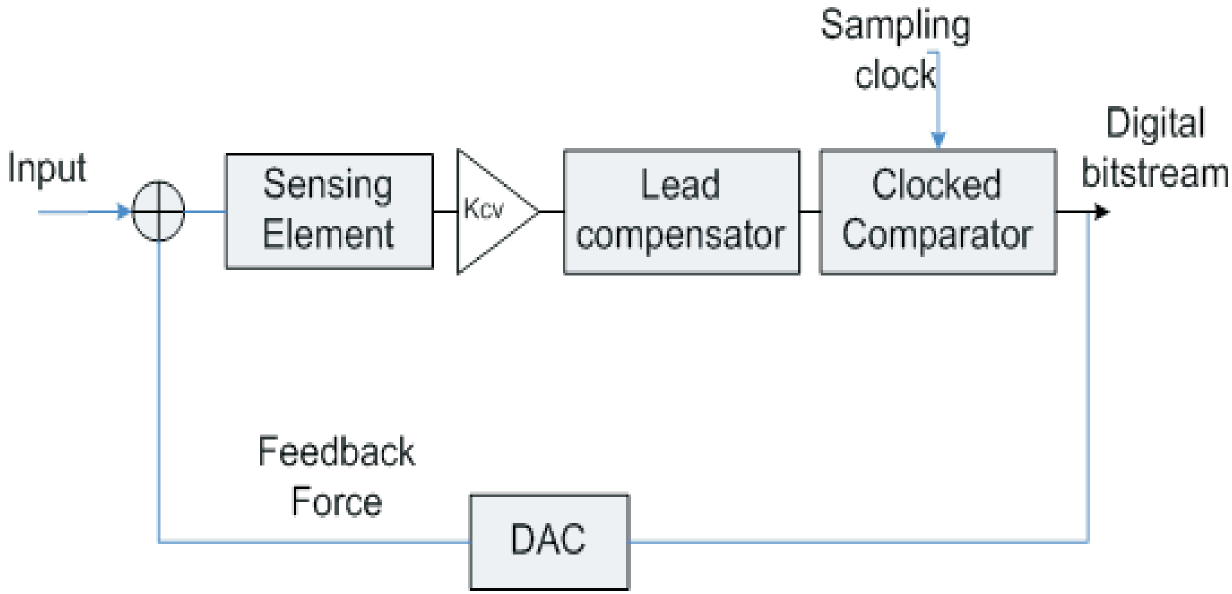
- Differential change in capacitance, which is proportional to deflection, is measured



2nd-order electromechanical Sigma-Delta modulation accelerometer

MEMS sensing element is an integral part of the system

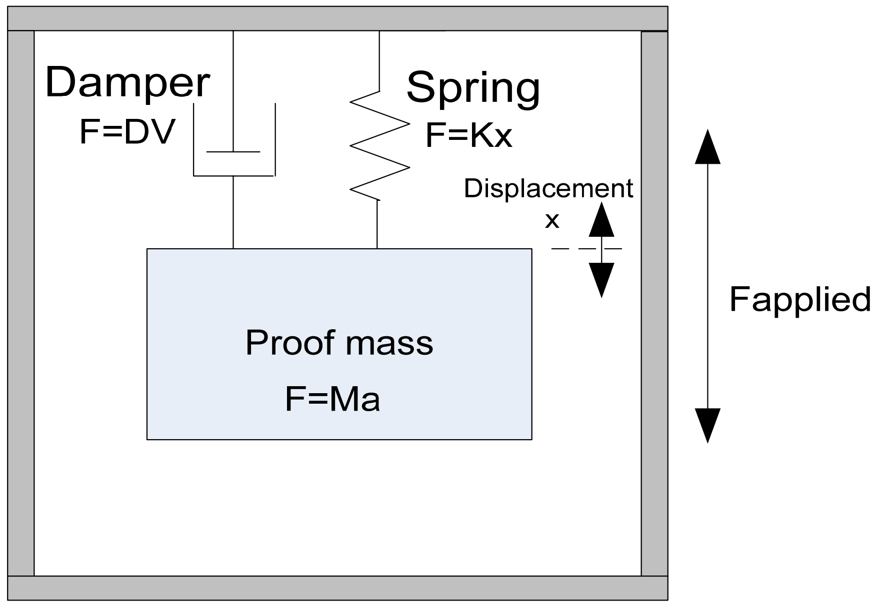
- Advantages
- Better bandwidth, dynamic range, linearity
- Direct digital output



Conventional sensing element model

- Mass-Damper-Spring system is modeled by a 2nd order differential equation:

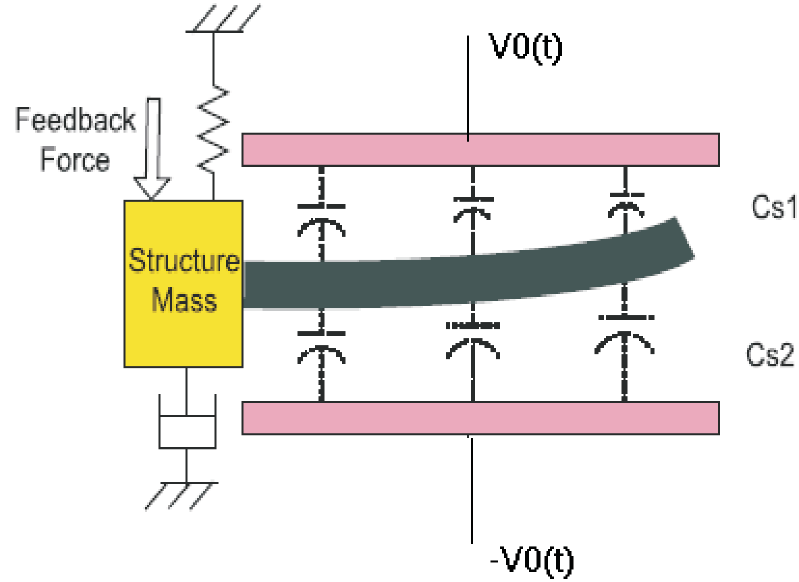
$$F(t)=M\frac{d^2x}{dt^2}+D\frac{dx}{dt}+Kx$$



- Too simple and inaccurate
- Does not reflect finger dynamics

Mechanical Sensing Element

- Higher resonant modes of sense finger cannot be reflected in conventional model
- Distributed model for sense fingers is needed
- Non-collocated system



ACCURATE MODEL WITH SENSE FINGER DYNAMICS

- **Motion of fingers could be modeled by following partial differential equation (PDE):**

$$\rho S \frac{\partial^2 y(x,t)}{\partial t^2} + C_D I \frac{\partial^5 y(x,t)}{\partial x^4 \partial t} + EI \frac{\partial^4 y(x,t)}{\partial x^4} = F_e(x,t)$$

- **E, I, C_D, ρ , S are all physical properties of the beam.**
- **Fe(x, t) - distributed electrostatic force along the beam:**

$$F_e(x,t) = \frac{1}{2}\varepsilon A\left[\frac{V_0^{\ 2}}{(d_0 - y(x,t))^2} - \frac{V_0^{\ 2}}{(d_0 + y(x,t))^2}\right]$$

Finite Difference Approximation (FDA) approach

- VHDL-AMS cannot support PDEs
- Convert PDE to a series of ODEs
- Finger is divided into N segments (5 segments in this design)
- Partial derivatives wrt position can be replaced with:

$$\frac{\partial y_n(t)}{\partial x} = \frac{y_n(t) - y_{n-1}(t)}{\Delta x} \qquad n = 0,1,2\ldots N$$

Distributed model for sense finger

- Partial differential equation (PDE) for the motion of the sense fingers:

$$\rho S \frac{\partial^2 y(x,t)}{\partial t^2} + C_D I \frac{\partial^5 y(x,t)}{\partial x^4 \partial t} + EI \frac{\partial^4 y(x,t)}{\partial x^4} = F_e(x,t)$$

- Series of ODE (Ordinary Differential Equations) converted from the PDE by FDA:

$$\rho S \frac{d^2 y_n}{dt^2} + \frac{C_D I}{(\Delta x)^4}\left(\frac{dy_{n+2}}{dt} - 4\frac{dy_{n+1}}{dt} + 6\frac{dy_n}{dt} - 4\frac{dy_{n-1}}{dt} + \frac{dy_{n-2}}{dt}\right) + \frac{EI}{(\Delta x)^4}(y_{n+2} - 4y_{n+1} + 6y_n - 4y_{n-1} + y_{n-2}) = \frac{f_{E_n}(t)}{\Delta x}$$

$$n = 0, 1, 2 \ldots N$$

Boundary conditions

- Equations for the border segments are a little different

  - At root (x=0)

$$y(0,t) = z(t)$$

$$\frac{\partial y(0,t)}{\partial x} = 0$$

  - At free end (x=l)

$$\frac{\partial^2 y(l,t)}{\partial x^2} = 0$$

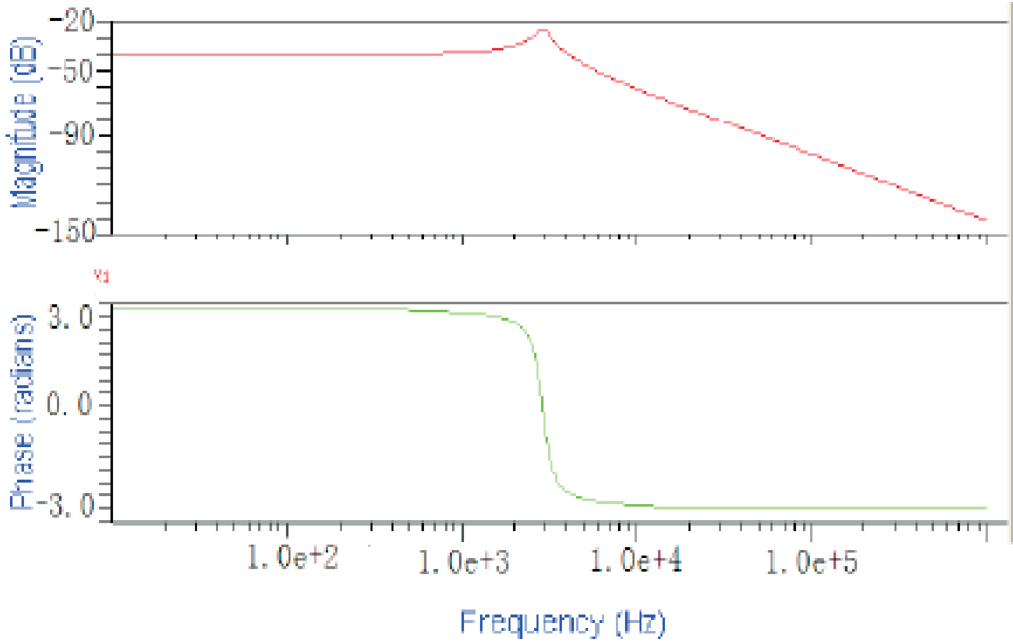
$$\frac{\partial^3 y(l,t)}{\partial x^3} = 0$$

Simulation results of conventional model

- **Frequency response of conventional model**

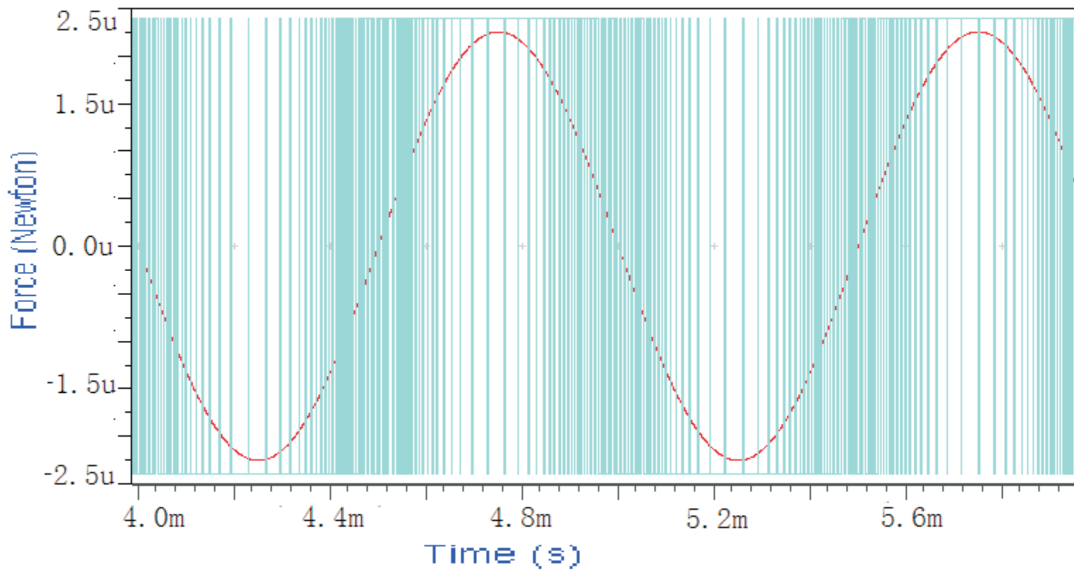
- **Lowest resonant mode caused by the dynamics of the structure mass**

$$\omega_0 = \sqrt{K / M}$$



Simulation results of conventional model

- Output bitstream
- Pulse density is inversely proportional to the input signal

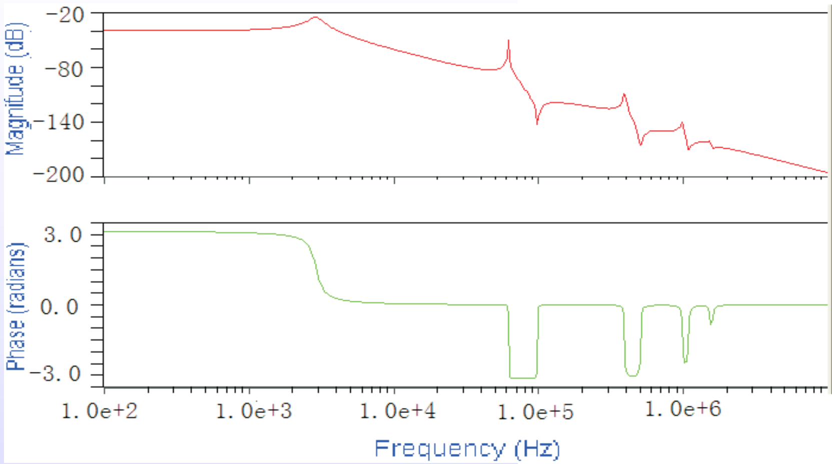


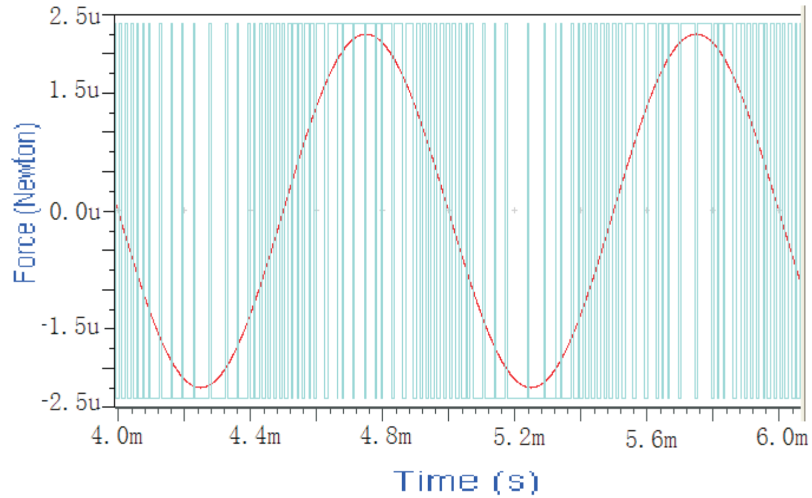
Higher resonant mode

- The higher resonant mode is related to the sense finger resonance
- Fingers bend significantly while the lumped mass has a small deflection.
- Resonant frequency

$$\omega_i = \alpha_i^2 \frac{W}{L}\sqrt{\frac{E}{12\rho}} \qquad \alpha_1 = 1.875, \alpha_2 = 4.694$$

Simulation results of the distributed model





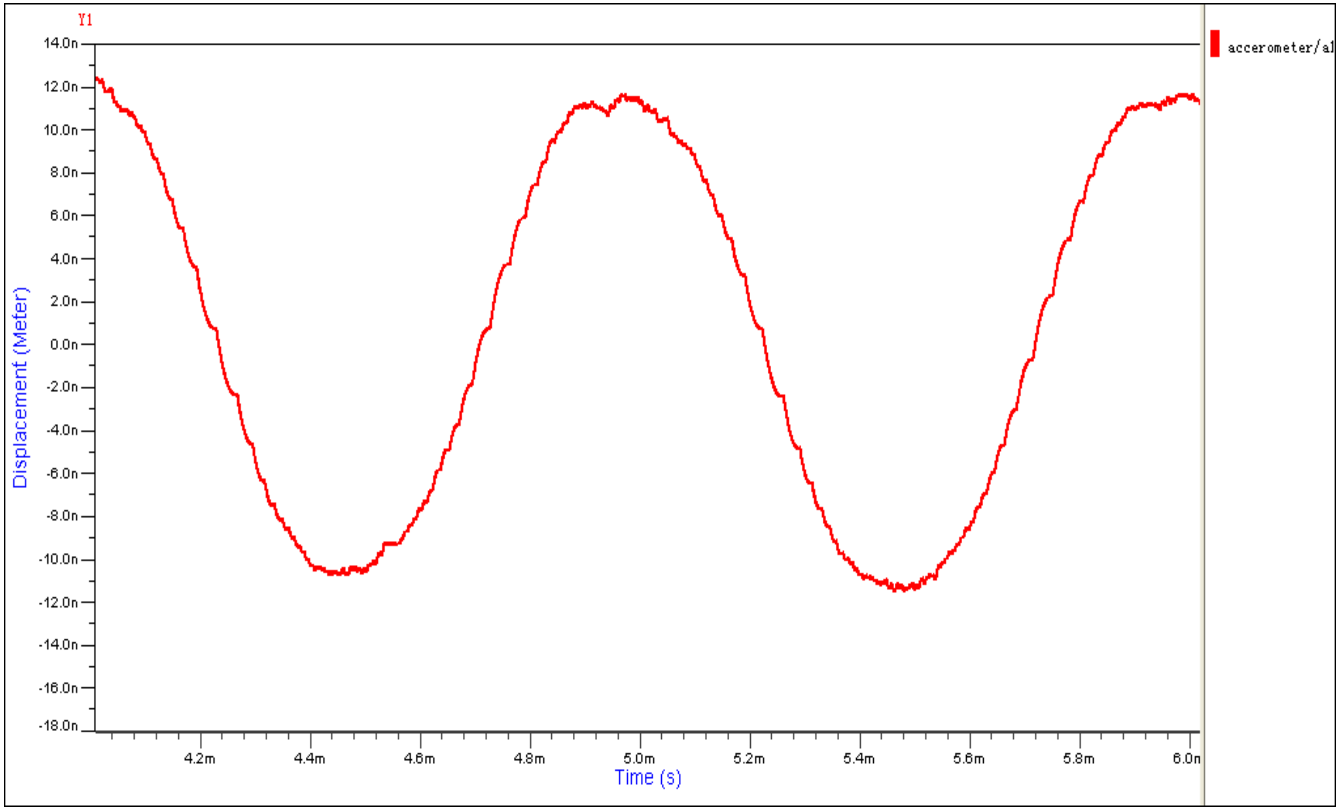
- **The finger dimensions in this design are:**

  L* = 140*µm*, *W* =2*µm*, *T* = 1*µm

- **Higher order resonant modes are exhibited in the accurate distributed model**

Simulation results ---- movement of the sense fingers

- **Deflection of the sense fingers in the conventional model**

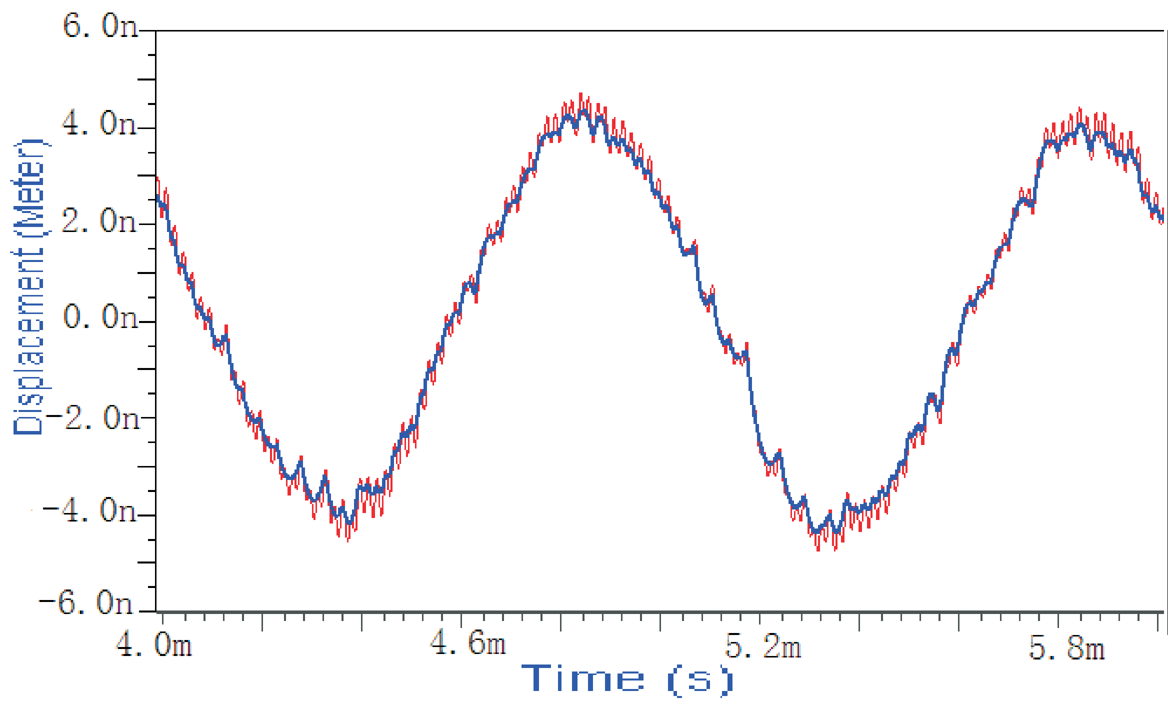


Simulation results ---- movement of the sense fingers

- **The finger dimensions in this design are:**

  L* = 140*μm*, *W* =2*μm*, *T* = 1*μm

- **Displacement of the lumped-mass (Blue) and average position of the sense finger (Red)**

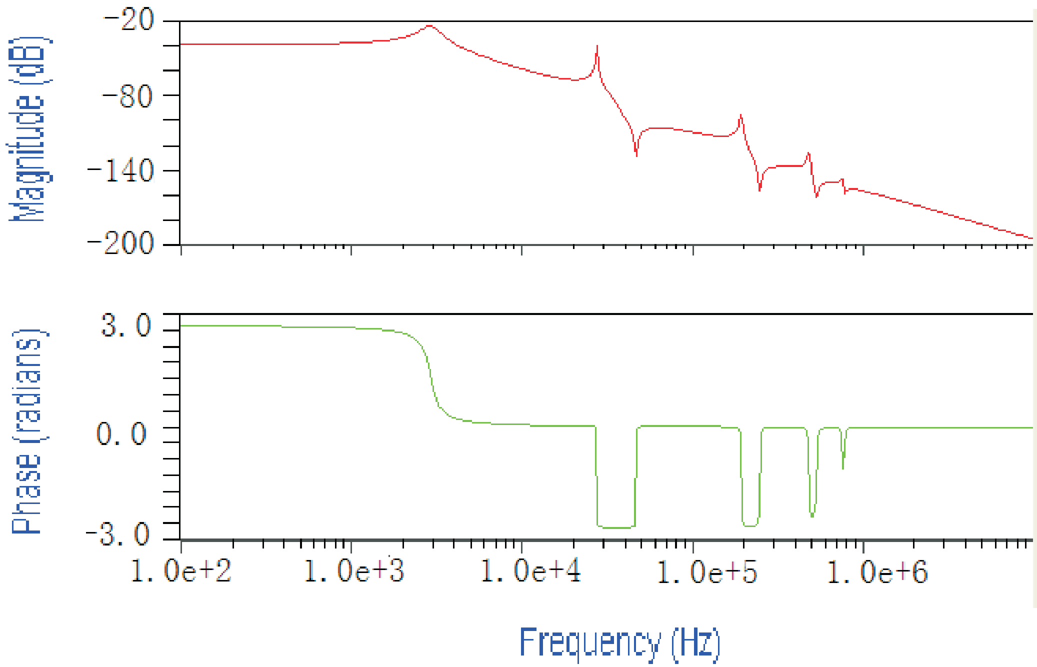


Simulation results - Distributed model

- **Finger dimensions are**

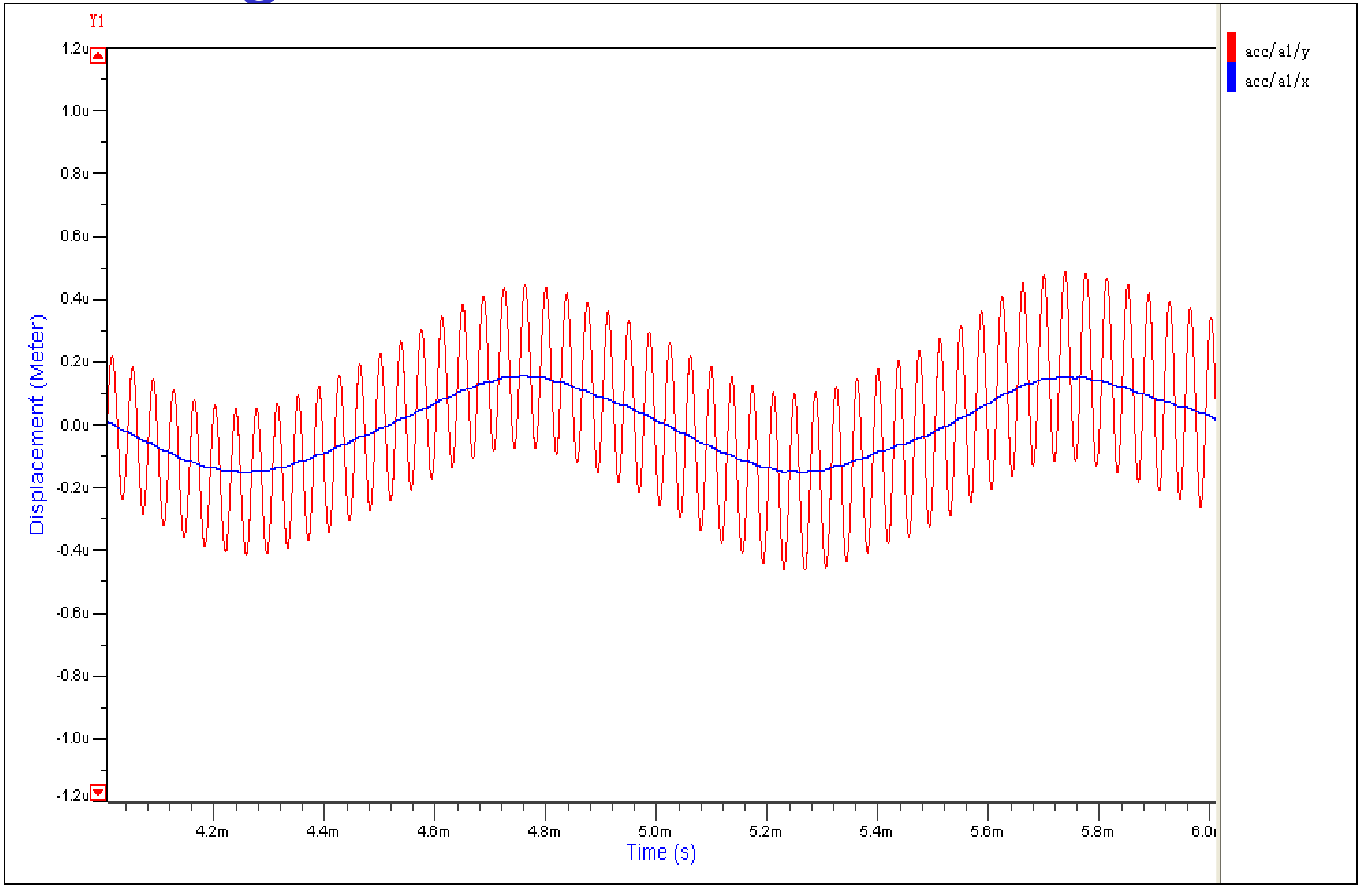
***L* = 200*μm*, *W* =2*μm*,**

T* = 1*μm



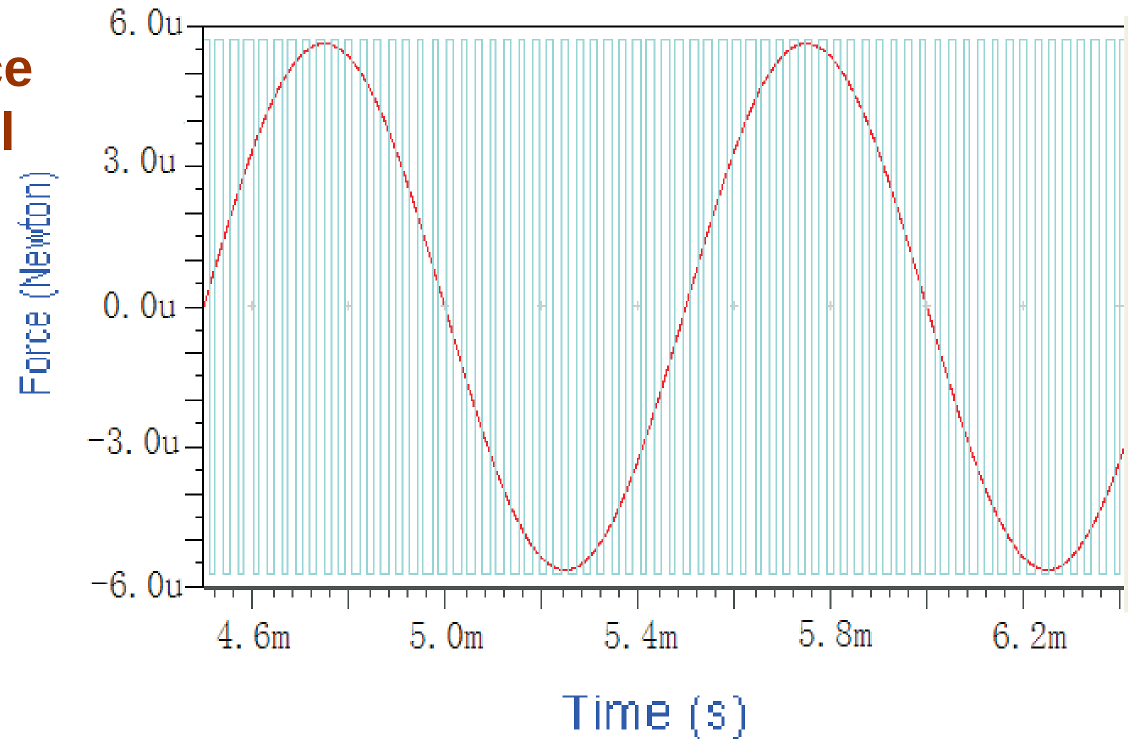
Movement of the sense fingers

- **Displacement of the lumped-mass (Blue) and average position of the sense finger (Red)**
- **Sense fingers bend significantly at their resonant frequency**



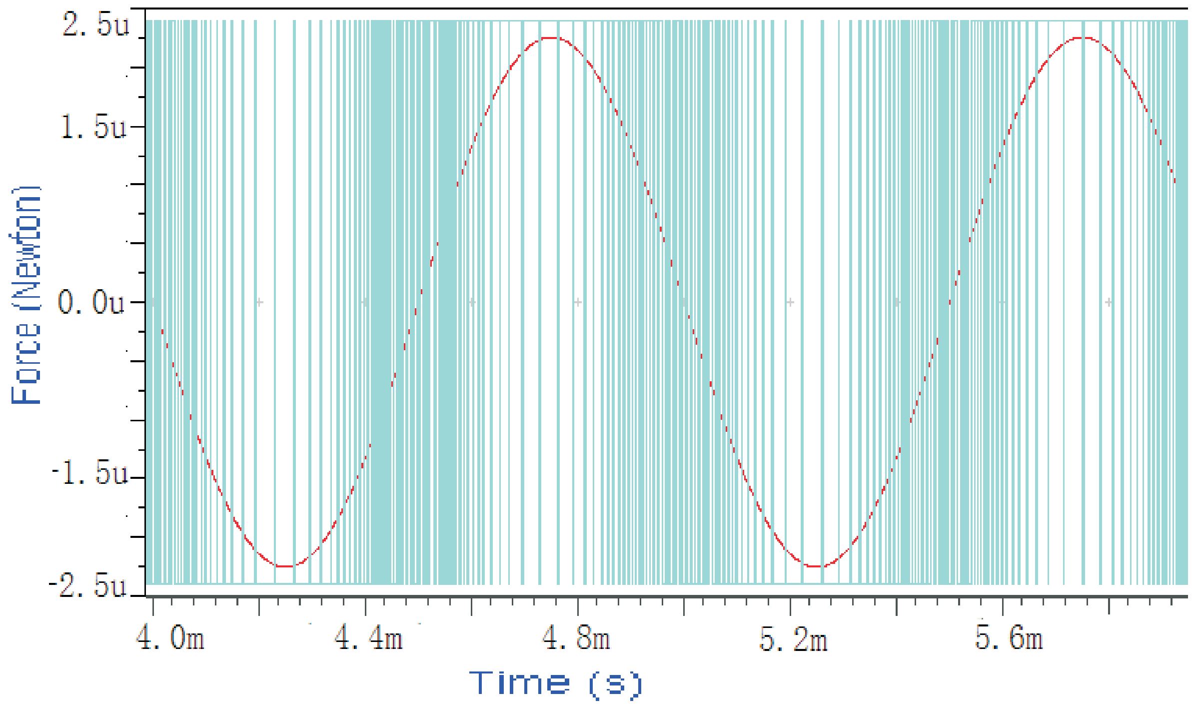
Simulation results of the distributed model

- Sense finger resonance can affect the performance of the Sigma-Delta control loop
- Here Sigma-Delta control breaks down - proposed model reflects this behavior correctly



Simulation results of conventional model

- Output bitstream of the conventional model.
- The Sigma-Delta control loop still appears to work but in reality it fails



CPU time comparison

Model	CPU time (SystemVision)
Conventional Model	9s 766ms
Distributed Model	21s 812ms

- SystemVision v.4 from Mentor Graphics was used to carry out simulation experiments

Conclusion

- An accurate distributed MEMS accelerometer model with sense finger dynamics has been developed and implemented in VHDL-AMS.
- The proposed model correctly reflects the way in which finger dynamics affect the performance of the control loop.