# Determining the Fidelity of Hardware-In-the-Loop Simulation Coupling Systems

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# Hardware-in-the-Loop-Simulation - I

Hardware-in-the-Loop-Simulation - a widely used concept, especially within the automotive industry.

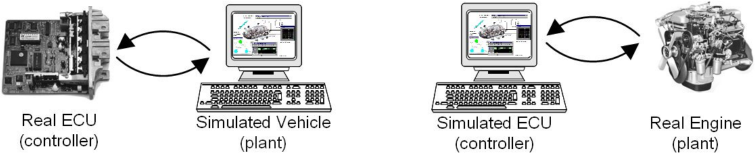


Figure: Hardware-in-the-Loop System Examples

Hardware-in-the-Loop-(HIL)-Simulation: one part of a real system or the system environment is replaced by a numerical model and interfaced to the remaining hardware via sensors and actuators.

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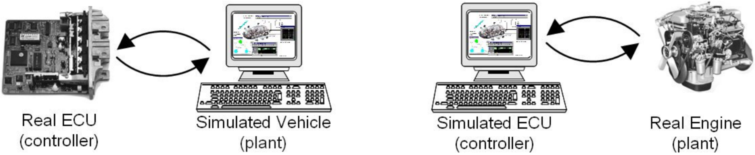


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Hardware-in-the-Loop-(HIL)-Simulation: one part of a real system or the system environment is replaced by a numerical model and interfaced to the remaining hardware via sensors and actuators.

# Hardware-in-the-Loop-Simulation - II

HIL simulation is used for:

- System simulation
- (Rapid) prototyping
- Component test
- System optimization
- ...

# Hardware-in-the-Loop-Simulation - III

Where is the problem?

- HIL Simulation systems are often designed straight-forward
- no deeper analysis of different system setups

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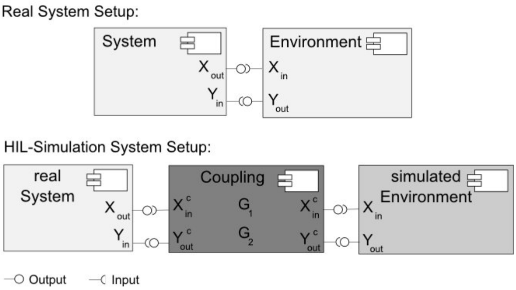


Figure: HIL System

$$X_{in} = X_{out}$$
$$Y_{in} = Y_{out} \quad (1)$$

Transformation functions $G_1(t)$ and $G_2(t)$:

$$X_{in}(t) = G_1(t) * X_{out}(t)$$
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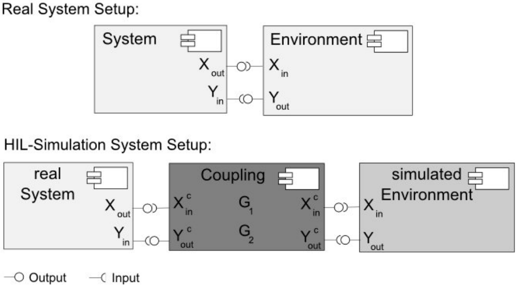


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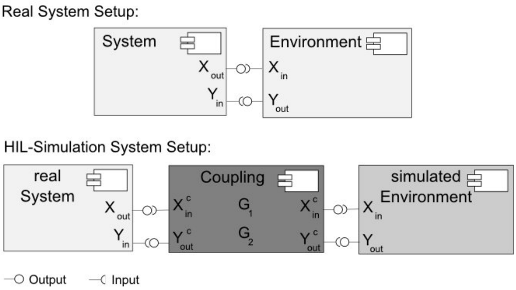


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- Term 'transparency' will be used [1]

# Hardware-in-the-Loop-Simulation System - II

A nearly transparent (ideal) coupling system:

$$\begin{bmatrix} G_1(t) & 0 \\ 0 & G_2(t) \end{bmatrix} \approx \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{3}$$

# Question

How we can measure the transparency of the coupling system?

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# System Model

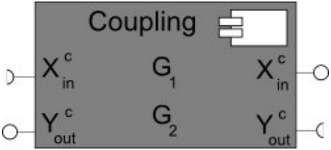


Figure: HIL Coupling System

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- Not necessary to model the complete system (unlike other approaches [1], [2])

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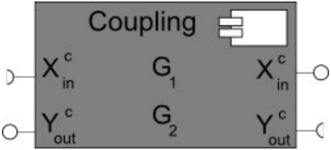


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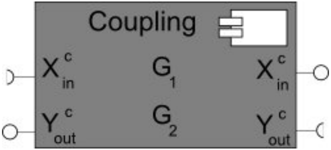


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# HIL Coupling System Model - I

The coupling is assumed to be representable as a lineare time invariant system (LTI system).

**Definition**

A LTI system can be described by the convolution of the input signal with the impulse response $y(t) = g(t) * x(t)$.

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# HIL Coupling System Model - II

## Definition

The transfer function (frequency domain) is defined as

$$y(s) = h(s)x(s) \quad \text{and so} \quad h(s) = \frac{y(s)}{x(s)} \tag{4}$$

$$h(s) = \frac{b_0 + b_1 s^1 + \cdots + b_m s^m}{a_0 + a_1 s^1 + \cdots + a_m s^m} \tag{5}$$

# Signal transformation

- Transfer function describes the coupling system
- Difference between the polynomials $y(s)$ and $x(s)$ represent the transparency of the signal transformation!

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- Idea: calculate the distance of the polynomials!

# Polynomial difference - I

The difference between two polynomials $y(s)$ and $x(s)$ can be defined as the distance of the polynomials within the $m+1$-dimensional space $\prod^{m}$ over polynomials of the grad m.

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# Polynomial difference - II

## Definition

A weighted distance $d_w(x(s), y(s))$ with $x(s) = a_0 + \cdots + a_m s^m$ and $y(s) = b_0 + \cdots + b_m s^m$ is defined as

$$d_w(x(s), y(s)) = \left| \begin{pmatrix} a_0 \\ \vdots \\ a_m \end{pmatrix} - \begin{pmatrix} b_0 \\ \vdots \\ b_m \end{pmatrix} \right|_w \qquad (6)$$

with the weighted norm

$$|\ldots|_w = \sqrt{w_0(a_0 - b_0)^2 + \cdots + w_m(a_m - b_m)^2} \qquad (7)$$

# Polynomial difference - III

- The polynomial difference is a measurement function for the transparency of single input/single output (SISO) systems.
- Now we need an extension for multiple input/multiple output (MIMO) systems!

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# MIMO systems - I

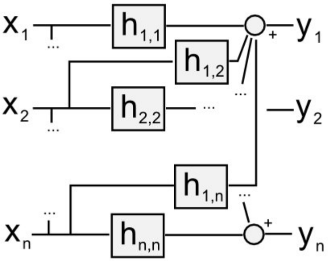


Figure: MIMO system

# MIMO systems - II

## Definition

MIMO systems can be described by a matrix of SISO transfer functions

$$H(s) = \begin{bmatrix} h_{1,1}(s) & \cdots & h_{n,1}(s) \\ \vdots & \ddots & \vdots \\ h_{1,n}(s) & \cdots & h_{n,n}(s) \end{bmatrix} \tag{8}$$

$$\text{with } Y(s) = \begin{pmatrix} y_0(s) \\ \vdots \\ y_n(s) \end{pmatrix} \text{ and } X(s) = \begin{pmatrix} x_0(s) \\ \vdots \\ x_n(s) \end{pmatrix} \tag{9}$$

$$Y(s) = H(s)X(s) \tag{10}$$

# MIMO systems - III

## Definition

The ideal transfer function matrix has a main diagonal containing ones. The other matrix elements are zero.

$$\begin{pmatrix} y_1(s) \\ \vdots \\ y_n(s) \end{pmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix} \begin{pmatrix} x_1(s) \\ \vdots \\ x_n(s) \end{pmatrix} \quad (11)$$

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# Transparency measure

## Definition

A norm $\|h(s)\|_p$ over a polynomial quotient $h(s) = \frac{y(s)}{x(s)}$ can be defined over the distance of $x(s)$ and $y(s)$ in $\prod^m$.

$$\left\| \frac{y(s)}{x(s)} \right\|_p = d_w(x(s), y(s)) \quad (12)$$

## Definition

Additional, the difference between the upper polynomial $y(s)$ and a zero polynomial can be defined as norm $\left\| \frac{y(s)}{x(s)} \right\|_p^0$ over the distance of $y(s)$ and 0 in $\prod^m$.

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# Matrix of Transparency - I

## Definition

A matrix of transparency can be defined as follows. The main diagonal contains the elements $\|h_{i,i}(s)\|_p$ with $1 \leq i \leq n$, while the other positions are filled with elements $\|h_{i,j}(s)\|_p^0$ with $1 \leq i \leq n\,,\ 1 \leq j \leq n\,,\ i \neq j$.

$$\begin{bmatrix} \|h_{1,1}(s)\|_p & & \|h_{j,i}(s)\|_p^0 \\ & \ddots & \\ \|h_{i,j}(s)\|_p^0 & & \|h_{n,n}(s)\|_p \end{bmatrix} \tag{14}$$

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# Matrix of Transparency - II

## Definition

With a matrix norm we can now define a transparency function $\mathfrak{tr}$ for a MIMO system transfer matrix.

$$\mathfrak{tr}(H(s)) = \left\| \begin{bmatrix} \|h_{1,1}(s)\|_p & & \|h_{j,i}(s)\|_p^0 \\ & \ddots & \\ \|h_{i,j}(s)\|_p^0 & & \|h_{n,n}(s)\|_p \end{bmatrix} \right\| \tag{15}$$

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# Fidelity definition

## Definition

The fidelity function $\mathfrak{fd}$ of a coupling system can be now defined by the transparency of the transfer function.

$$\mathfrak{fd}(H(s)) = \frac{1}{1 + \mathfrak{tr}(H(s))} \tag{16}$$

Remark: The value of the fidelity ranges between zero and one.

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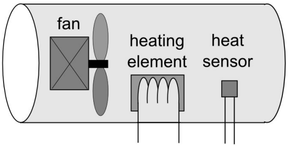
# Heat-Sensor-in-the-Loop

- Heat-sensor-HIL simulation (continuous system)
- Heating element + fan are the coupling system

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**Definition**

Heating element transfer function:

$$H_h(s) = K * \frac{1}{1 + Ts} \qquad (17)$$

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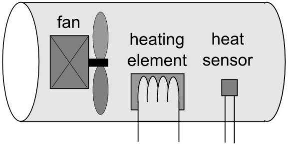


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# Transfer function

- Proportional coefficient $K$ and the time constant $T$ depending on environmental variables
- e.g. specific heat capacity, density and velocity of the transfer medium
- $K = \frac{1}{c_m \gamma_m A v}$
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$c_m$ - heat capacity of air
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# Results - I

System fidelity:

- air velocity $v = 1m/s$: $\mathfrak{fd}(H_h(s)) = 0.847$
- air velocity $v = 10m/s$: $\mathfrak{fd}(H_h(s)) = 0.982$

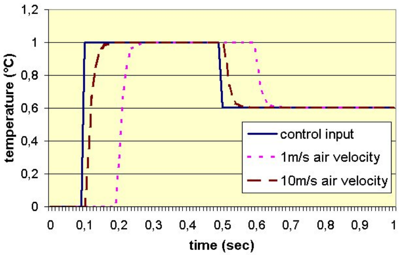


Figure: Heating system - different air velocities

# Results - II

- Obviously an increasing the air velocity leads to better results.
- But what about the influence of different heating element materials?

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## Results - II

- Obviously an increasing the air velocity leads to better results.
- But what about the influence of different heating element materials?

# Results - III

System fidelity:

- steal heating element: $\mathfrak{fd}(H_h(s)) = 0.847$
- copper heating element: $\mathfrak{fd}(H_h(s)) = 0.848$
- aluminum heating element: $\mathfrak{fd}(H_h(s)) = 0.843$

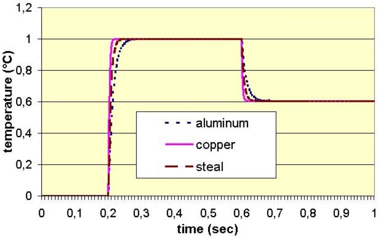


Figure: Heating system - different materials

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# Conclusion

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# Questions?

Any questions?

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