UNIVERSITY OF Southampton
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and Computer Science

AUTOMATED PERFORMANCE OPTIMISATION AND LAYOUT SYNTHESIS OF MEMS ACCELEROMETER WITH SIGMA-DELTA FORCE-FEEDBACK CONTROL LOOP

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Outline

- **Main contribution**
- **Mixed-technology model of the Sigma-Delta accelerometer system**
- **Accurate model with sense finger dynamics**
- **Performance optimisation algorithm**
- **Exprements and results**
- **Conclusion**

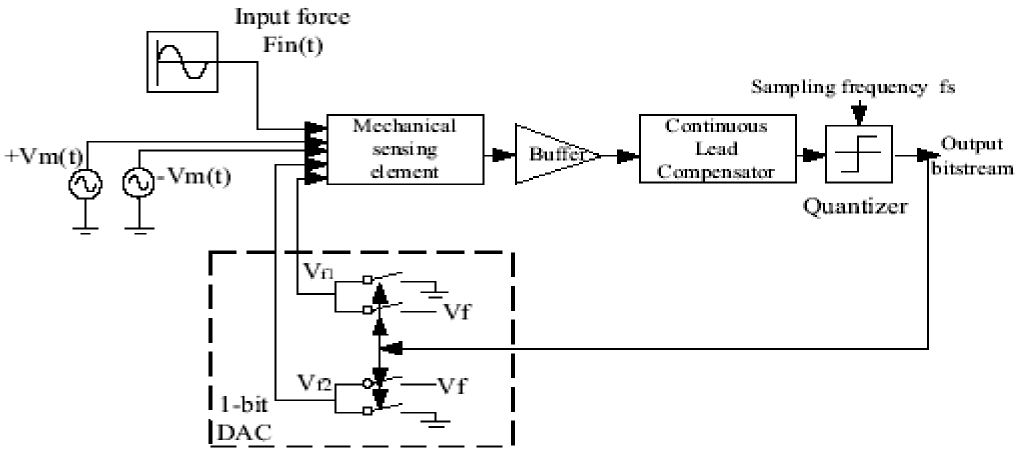
Main contribution

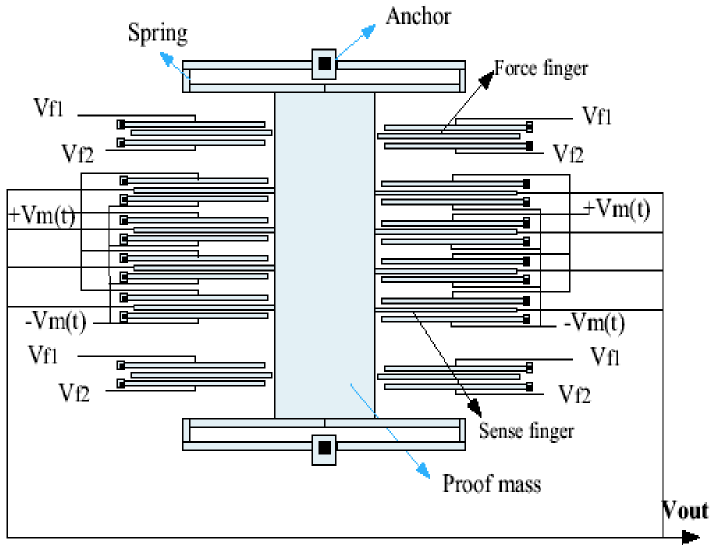
- Novel methodology for automated optimal design of a MEMS accelerometer with Sigma-Delta force-feedback control loop

- Based on a simulation-based optimisation technology using a genetic algorithm implemented in a dedicated C++ tool

- Improved accelerometer model using accurate distributed model for sense finger dynamics

Mixed-technology model of the Sigma-Delta accelerometer system

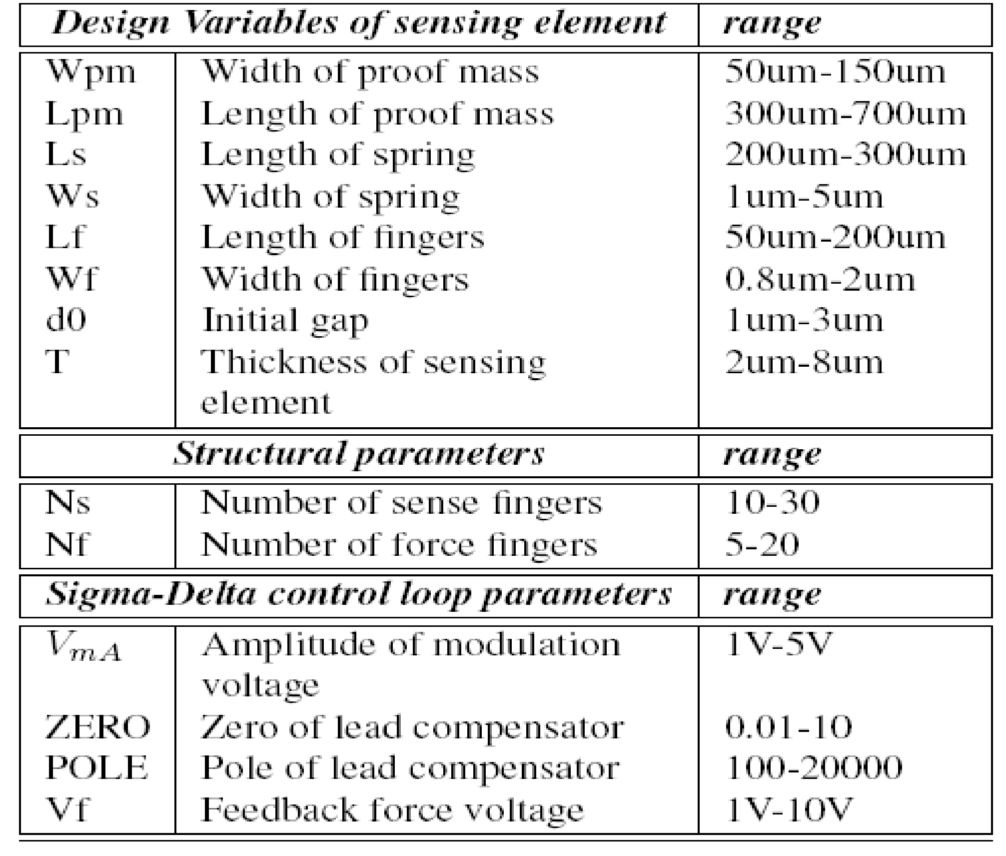




- High-performance MEMS usually employ an electromechanical Sigma-Delta modulation feedback control

- Mechanical sensing element is an integral part of the system and capacitive sensing element is used in this project

Design Parameters of Sigma-Delta accelerometer



Design Variables of sensing element		*range*
Wpm	Width of proof mass	50um-150um
Lpm	Length of proof mass	300um-700um
Ls	Length of spring	200um-300um
Ws	Width of spring	1um-5um
Lf	Length of fingers	50um-200um
Wf	Width of fingers	0.8um-2um
d0	Initial gap	1um-3um
T	Thickness of sensing element	2um-8um
Structural parameters		*range*
Ns	Number of sense fingers	10-30
Nf	Number of force fingers	5-20
Sigma-Delta control loop parameters		*range*
V_{mA}	Amplitude of modulation voltage	1V-5V
ZERO	Zero of lead compensator	0.01-10
POLE	Pole of lead compensator	100-20000
Vf	Feedback force voltage	1V-10V

ACCURATE MODEL WITH SENSE FINGER DYNAMICS

- Motion of fingers could be modeled by following partial differential equation (PDE):

$$\rho S \frac{\partial^2 y(x,t)}{\partial t^2} + C_D I \frac{\partial^5 y(x,t)}{\partial x^4 \partial t} + EI \frac{\partial^4 y(x,t)}{\partial x^4} = F_e(x,t)$$

- E, I, C$_D$, ρ, S are all physical properties of the beam.
- Fe(x, t) - distributed electrostatic force along the beam:

$$F_e(x,t) = \frac{1}{2} \varepsilon A [\frac{V_0^{\,2}}{(d_0 - y(x,t))^2} - \frac{V_0^{\,2}}{(d_0 + y(x,t))^2}]$$

Distributed model for sense finger

- Partial derivatives wrt position can be replaced with:

$$\frac{\partial y_n(t)}{\partial x} = \frac{y_n(t) - y_{n-1}(t)}{\Delta x} \qquad n = 0, 1, 2 \ldots N$$

- Series of ODE (Ordinary Differential Equations) converted from the PDE by Finite Difference Approximation (FDA):

$$\rho S \frac{d^2 y_n}{dt^2} + \frac{C_D I}{(\Delta x)^4}\left(\frac{dy_{n+2}}{dt} - 4\frac{dy_{n+1}}{dt} + 6\frac{dy_n}{dt} - 4\frac{dy_{n-1}}{dt} + \frac{dy_{n-2}}{dt}\right) + \frac{EI}{(\Delta x)^4}\left(y_{n+2} - 4y_{n+1} + 6y_n - 4y_{n-1} + y_{n-2}\right) = \frac{f_{E_n}(t)}{\Delta x}$$

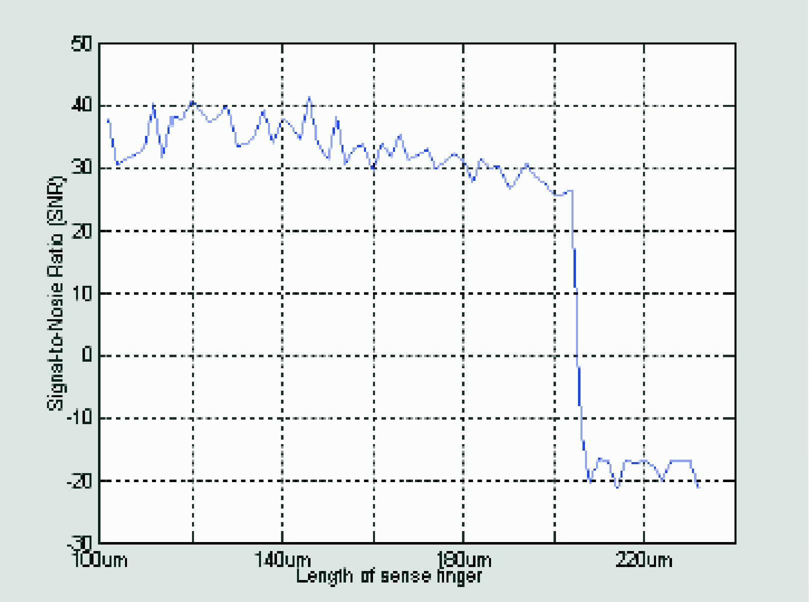
$$n = 0, 1, 2 \ldots N$$

Effect of sense finger dynamics

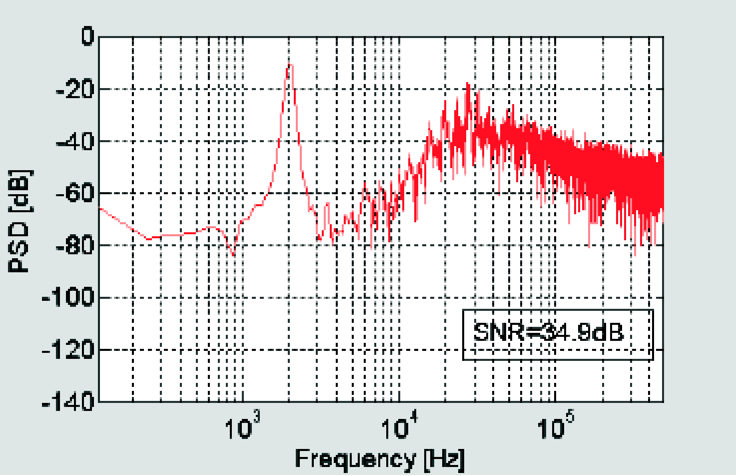
- Sense finger resonances affect the performance of the Sigma-Delta control loop
- Resonant frequencies

$$\omega_i = \alpha_i^{2} \frac{W}{L^2}\sqrt{\frac{E}{12\rho}} \qquad \alpha_1 = 1.875, \alpha_2 = 4.694$$

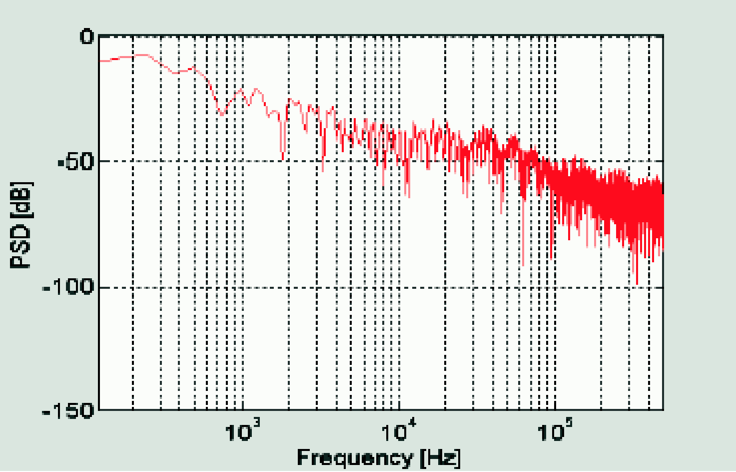
- Figure shows a control loop failure when finger length is excessive (cannot be captured by conventional sensing element model)



Power Spectral Density (PSD)



(a) Power spectrum density of output bitstream (sense finger length=120um)



(b) Power spectrum density of output bitstream (sense finger length=210um)

a) correctly working control loop

b) failing control loop when finger length is 210um.

Performance optimisation algorithm

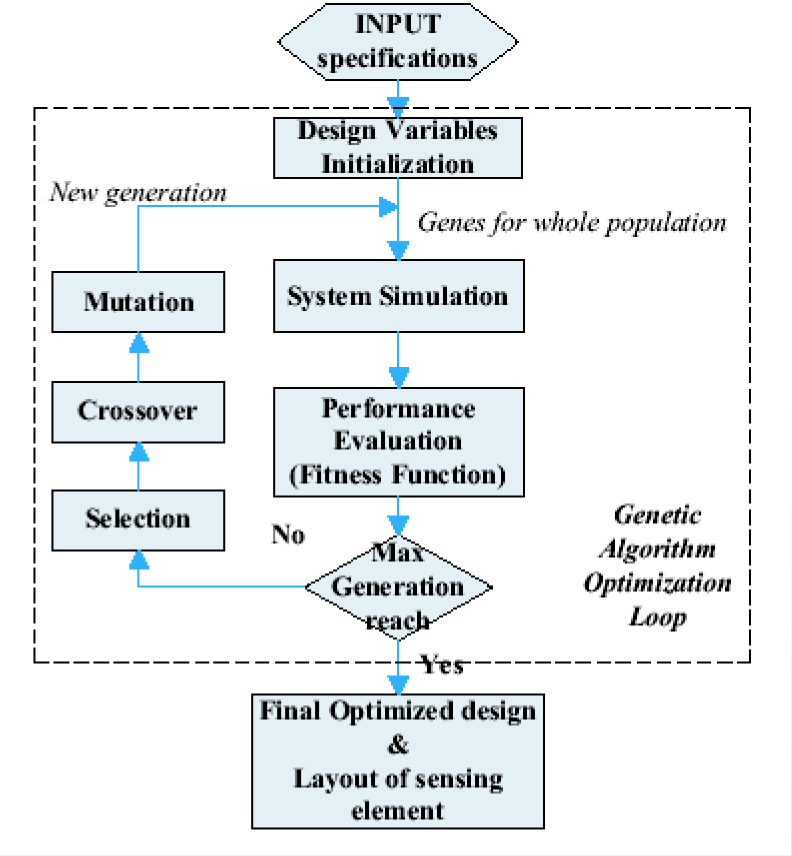
- **Genetic algorithm is used in conjunction with dedicated behavioral MEMS accelerometer model**
- **Objective function:**

  Maximize the signal to noise ratio (SNR)

- **Input specifications:**

  Geometrical constraints of the sensing element, control loop parameter ranges and performance specifications

- **Output:** optimized layout of sensing element and Sigma-Delta control loop parameters



Objective function (i.e. ‘fitness’)

- **Performance constraints:**

  1) Minimum *Signal-to-noise ratio: SNR >30dB*

  2) Minimum *Static sensitivity of mechanical sensing element: S>1.5fF/g*

- **Fitness function:**

$$F_{fit} = K * \frac{Objective}{Objective_r}$$

1) *K* is a penalty parameter whose value depends on whether the performance constraints are met. *K* is set to 100 if both constraints are met, otherwise K is 0.01.

2) *Objective* is the SNR obtained from simulation

3) *Objectiver* is a user defined reference value.

Experiments

- ***Experiment 1 : maximum SNR***

Fitness: $$F_{fitSNR} = K * \frac{SNR}{SNR_r}$$

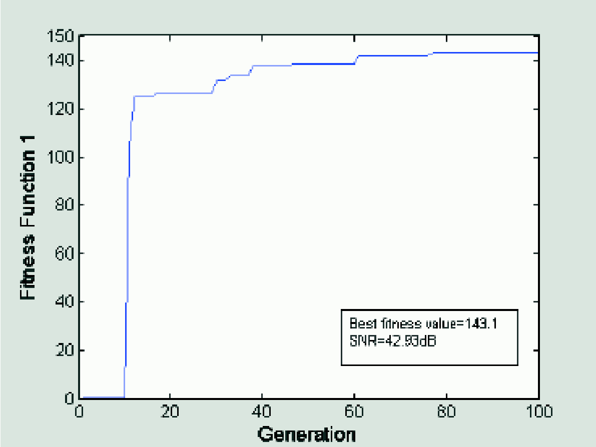
where reference SNR: $$SNR_r = 30dB$$

- ***Experiment 2 : additional experiment to obtain maximum static sensitivity of sensing element***

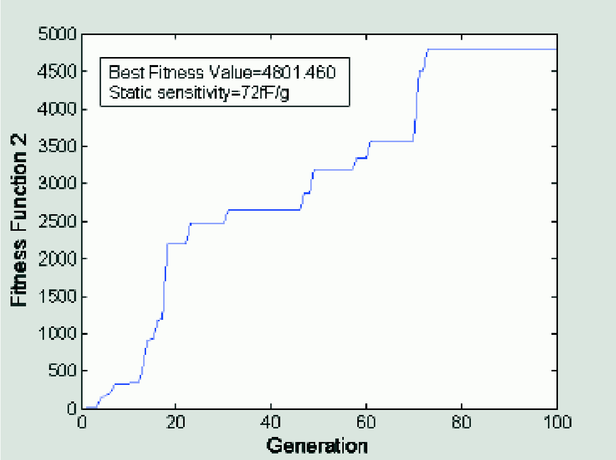
Fitness : $$F_{fitS} = K * \frac{S}{S_r}$$

where reference static sensitivity: $$S_r = 1.5\, fF/g$$

Fitness improvement process



(a) Experiment 1: maximum SNR



(b) Experiment 2: maximum static sensitivity

- **Optimizations were carried out using following design parameters:**

  1) Oversampling ratio: OSR=64

  2) Sampling frequency: *fs*=1MHz

  3) Input force: $F_{in}(t) = F_{Amp}\sin(2\pi f_{input} t)$

  where $F_{Amp} = 100g * M\,(1g = 9.8/s^2)$ $f_{input} = 2KHz$

Automated optimization results

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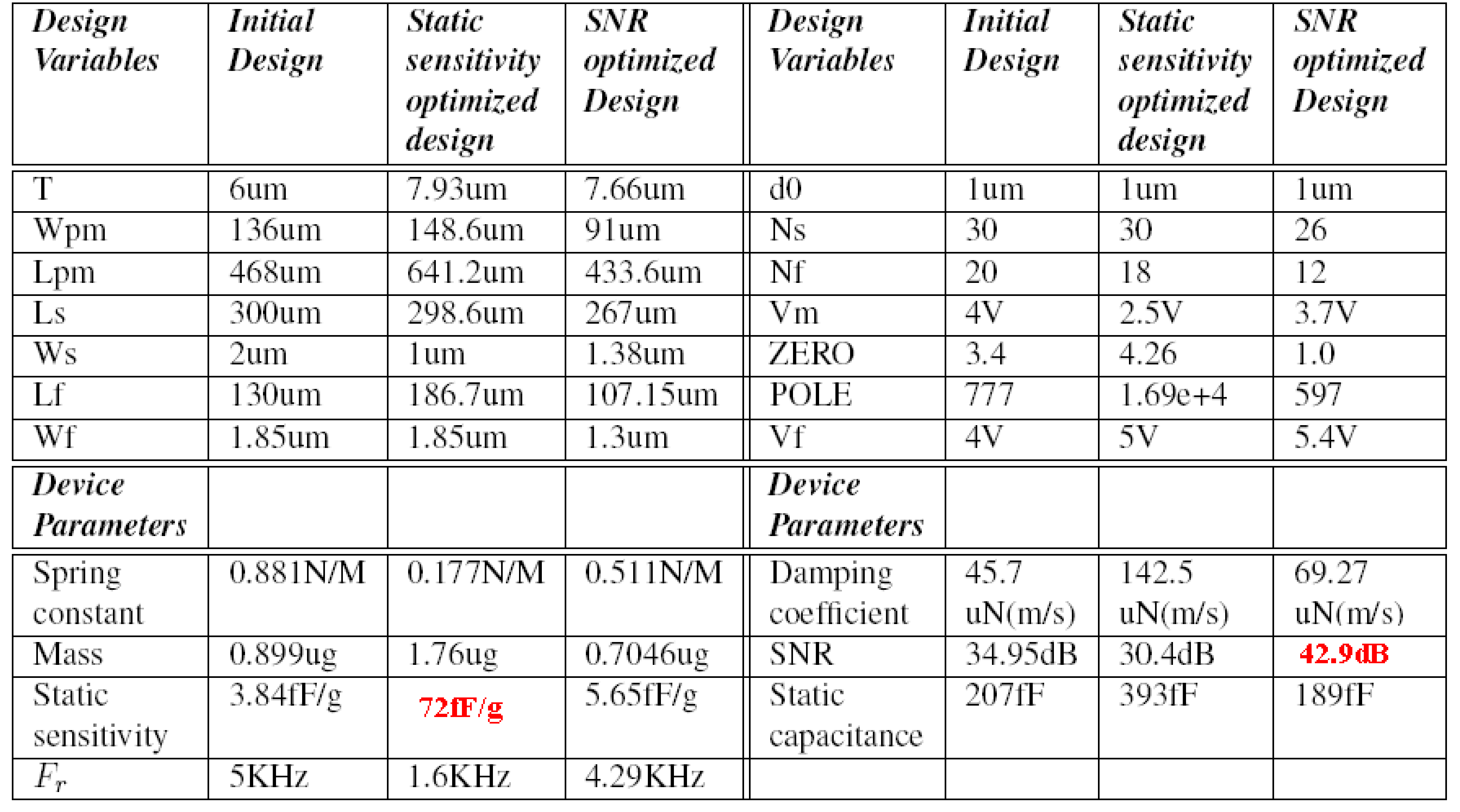
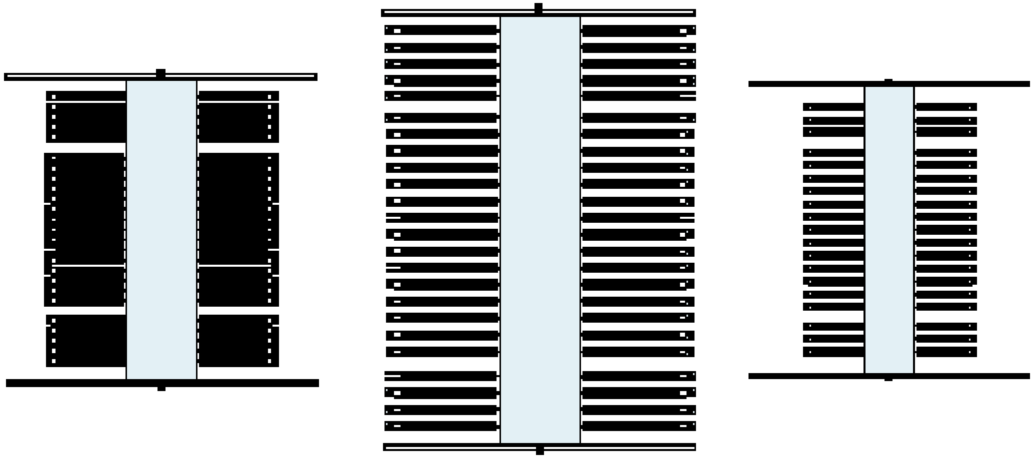


Design Variables	*Initial Design*	*Static sensitivity optimized design*	*SNR optimized Design*	*Design Variables*	*Initial Design*	*Static sensitivity optimized design*	*SNR optimized Design*
T	6um	7.93um	7.66um	d0	1um	1um	1um
Wpm	136um	148.6um	91um	Ns	30	30	26
Lpm	468um	641.2um	433.6um	Nf	20	18	12
Ls	300um	298.6um	267um	Vm	4V	2.5V	3.7V
Ws	2um	1um	1.38um	ZERO	3.4	4.26	1.0
Lf	130um	186.7um	107.15um	POLE	777	1.69e+4	597
Wf	1.85um	1.85um	1.3um	Vf	4V	5V	5.4V
Device Parameters				*Device Parameters*			
Spring constant	0.881N/M	0.177N/M	0.511N/M	Damping coefficient	45.7 uN(m/s)	142.5 uN(m/s)	69.27 uN(m/s)
Mass	0.899ug	1.76ug	0.7046ug	SNR	34.95dB	30.4dB	**42.9dB**
Static sensitivity	3.84fF/g	**72fF/g**	5.65fF/g	Static capacitance	207fF	393fF	189fF
F_r	5KHz	1.6KHz	4.29KHz				

Synthesized layouts

- Layout of the mechanical sensing element is automatically generated

- Presented optimization-based approach deals with design trade-offs effectively for given choice of design objectives.



(a) Initial Design
SNR=34.95dB
Static Sensitivity=3.84fF/g

(b) Optimization design of experiment 1
(Static sensitivity optimized design)
SNR=30.4dBdB
Static Sensitivity=72fF/g

(c) Optimization design of experiment 2
(SNR optimized design)
SNR=42.9dBdB
Static Sensitivity=5.65fF/g

Conclusion

- **Effective simulation-based approach to automated synthesis of MEMS accelerometer in Sigma-Delta control loop has been presented.**

- **Future work will extend digital MEMS accelerometer design to multi-objective optimisation and an automated synthesis of the control loop where higher order Sigma-Delta control loops will be used to maximise performance.**