VHDL-AMS Modeling of VCSEL including Noise

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Abstract

This paper presents a VCSEL diode model including electrical, thermal and optical behaviors where noise is present in the rate equations. VHDL-AMS language is used to write this model. Langevin forces are introduced in equations and generated in Mentor Graphics ADVanceMS© by white noise signal generators, using the UNIFORM statement. The comparison of transient results with Spice ones published in literature shows a good agreement.

Introduction

In the last decade, the Vertical Cavity Surface Emitting Laser (VCSEL) appeared as a reliable low-cost high-speed solution for data communication applications and interconnects. It started to challenge the well-established edge-emitting lasers in telecom and data storage applications.
This paper recalls the mathematical modeling described and presents the VHDL-AMS modeling of the proposed VCSEL and comparison with Spice models results published in literature.

VCSEL Model development

The fundamental difference between an edge-emitting laser and VCSEL is the fact that the laser oscillation as well as the out-coupling of the laser beam occur in a direction perpendicular to the gain region and the surface of the laser chip.
The advantages of VCSEL is that the surface emission and the small size make it possible to fabricate very dense two-dimensional arrays of VCSELs, suitable for multi-channels parallel transmission modules.
Figure 1 shows the schematic of an GaAs processed at TRT (France) which was used to compare some results.

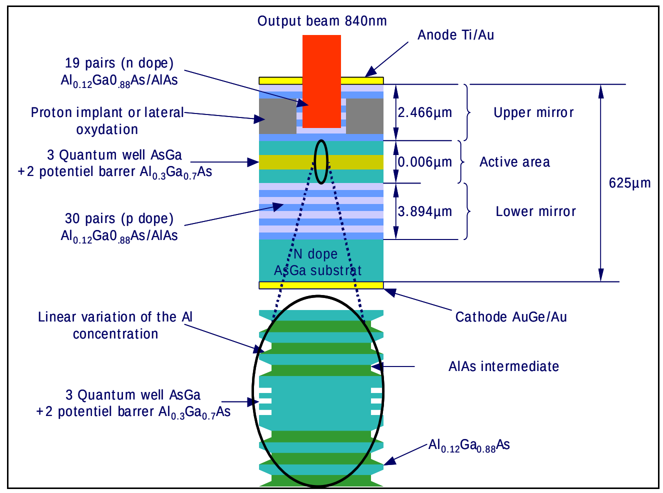

Figure 1 : VCSEL structure.

VCSEL modeling is based on the resolution of semiconductor laser rate equations [1], expressed for single mode VCSEL operation as function of photon, carrier numbers and phase modulation: S, N and ψ , respectively.

$$\frac{dS}{dt} = -\frac{S}{\tau_p} + \beta \frac{N}{\tau_n} + G_N(N - N_0)\frac{S}{1+\varepsilon S} \tag{1}$$

$$\frac{dN}{dt} = \eta_i \frac{I}{q} - \frac{N}{\tau_n} - G_N(N - N_0)\frac{S}{1+\varepsilon S} \tag{2}$$

$$\frac{d\psi}{dt} = -\frac{\alpha_h}{2}\left[G_N(N - N_0)\frac{1}{1+\varepsilon S} - \frac{1}{\tau_p}\right] \tag{3}$$

Parameters defined in these well known equations are physical internal parameters in general not available. They are defined on Table 1 with their derivation from system parameters [2]. With η_i is the current injection efficiency and I is the injected current.
The noise is an important factor when laser diode are used for signal transmission in equipment and in systems such as optical fiber communication. These equations can be rewritten as following:

Tableau 1: VCSEL System-Physical Parameters Conversion [2].

Physical parameter	Quantity	Unit	Conversion towards physical parameters	Needed system parameter	Needed physical parameter
G_N	Differential gain	s^{-1}	$G_N \approx$ $[(e.\eta_{LI})/2P_0].(2\pi.f_{3dB})$	Slope η_{LI} Power P_0 Bandpass f_{-3dB}	
τ_p	Photon lifetime	s	$\tau_p \approx$ $\eta_{LI}.2q/(h\nu.v_g.\alpha_m)$	Slope η_{LI} Optical frequency ν	Cavity loss α_m
τ_n	Carrier lifetime	s	$\tau_n^{-1} \approx$ $(q/I).[N_0 + (\tau_s.G_N)^{-1}]$	Threshold current I_t Those of G_N and τ_p	Transparence number N_0
β	Spontaneous emission fraction	-	$\beta^{-1} \approx$ $(\tau_p/e).(I_t - e.N_0.\tau_n^{-1})$	Thresh current I_t Those of τ_p and τ_n	Transparence number N_0
ε	normalized gain compression factor	-	$\varepsilon \approx$ $G_N.[\xi..(2\pi.f_{3dB})^{-1} - \tau_s]$	Bandpass f_{3dB} Damping ξ in P function of mod. freq. f_m Those of τ_p	
α_H	Linewid. enhancement factor (Henry)	-	in the chirp band $\alpha_H \approx 2.[(\delta\nu/\delta I)/\varepsilon)]$ out of the band $\alpha_H \approx 2.[(\delta\nu/\delta I).f_m]/(\eta_{LI}.P)$	Chirp function of mod. freq. f_m Slope η_{LI} Damping ξ	

$$\frac{dS}{dt} = -\frac{S}{\tau_p} + \beta\frac{N}{\tau_n} + G_N(N-N_0)\frac{S}{1+\varepsilon S} + F_s \quad (4)$$

$$\frac{dN}{dt} = \eta_i\frac{I}{q} - \frac{N}{\tau_n} - G_N(N-N_0)\frac{S}{1+\varepsilon S} + F_n \quad (5)$$

$$\frac{d\psi}{dt} = -\frac{\alpha_h}{2}\left[G_N(N-N_0)\frac{1}{1+\varepsilon S} - \frac{1}{\tau_p}\right] + F_\psi \quad (6)$$

Where F_n, F_s and F_ψ are the Langevin Forces. They assumed to be Gaussian random process with zero as average value and have a correlation function of the form:

$\langle F_i(t) \quad F_j(t')\rangle = 2D_{ij}\,\delta(t-t')$,

where $i, j = S, N$ and D_{ij}, the diffusion coefficient. The dominant contribution to laser noise is supported by only two coefficients D_{ij} and D_{ij}; others can be assumed be nearly zero [3].

These different diffusion coefficients are given by (7), (8), (9):

$$D_{ss} = R_{sp} * S \quad (7)$$

$$D_{nn} = R_{sp} * S + \frac{N}{\tau_n} \quad (8)$$

$$D_{\psi\psi} = \frac{R_{sp}}{4S} \quad (9)$$

and $D_{ij} = 0$ for $i \neq j$

where R_{sp} is the rate of spontaneous emission and given by (10) :

$$R_{sp} = \frac{n_{sp}}{\tau_p}, \quad (10)$$

and n_{sp} is known as the spontaneous-emission factor and given by (11) : $n_{sp} = \dfrac{N}{N-N_0}$ (11)

To generate the Langevin forces we use random signal generators. Only white noise sources have been considered.

To generate a white noise source, we use the function *UNIFORM* provided by the library *math_real* of Mentor Graphics ADVanceMS©. This function returns a pseudo-random number x with uniform distribution. Next figure shows an example of a random signal generator process written with VHDL-AMS and using the *UNIFORM* statement.

```
Detection:process
        variable seed1:integer:= 3456 ;
        variable seed2:integer:= 4563 ;
        variable unf : real;
     begin
                    wait for tau;
     uniform(seed1,seed2,unf);
                    If unf>0.0 Then  out1<= unf;
                                            ELSE  null;
                    End If;
     uniform(seed1,seed2,unf);
                    If unf>0.0 Then   out2<= unf;
                                            ELSE            null;
                    End If;
 end process;
```

Figure 2 : VHDL-AMS Random Signal Generator.

To obtain a white noise with Gaussian distribution, we use Box-Muller transformation, it looks like:

$$y_1 = \sqrt{-2\ln(x_1)}\cos(2\pi x_2)$$

$$y_2 = \sqrt{-2\ln(x_1)}\sin(2\pi x_2)$$

We start with two independent random numbers, x1 and x2, which come from a uniform distribution (in the range from 0 to 1). Above transformations applied to get two new independent random numbers which have a Gaussian distribution with zero average value and a standard deviation of one.
The Langevin Forces in equations 4,5 and 6 are written using x_1 and x_2 by:[4]

$$\begin{cases} F_n = \sqrt{D_{nn}}.x_{1n} \\ F_p = \sqrt{D_{nn}}.x_{1p} \\ F_\psi = \sqrt{D_{\psi\psi}}.x_{1\psi} \end{cases}$$

there different x_{ij} are modeled with the same process only *timeout-clause* of *wait* statement is modified.

Thermal dependence

Different parameters depend on temperature in VCSEL rate equation. That is the case of the threshold current and the output power.
The output power can written now by (12):

$$P_{opt} = \eta(T)\left[I - I_{th}(T)\right] \quad (12)$$

where the threshold current is given by (13):[5]

$$I_{th}(T) = a_0 + a_1.T + a_2.T^2 + a_3.T^3 + a_4.T^4 \quad (13)$$

where $a_1 \ldots a_4$ are temperature dependent parameters and can be deduced from experimental VCSEL curves.
Another parameter depends on temperature, it's the differential gain and it written as (14):[5]

$$G = G_0 \frac{ag0 + ag1 * T + ag2 * T^2}{bg0 + bg1 * T + bg2 * T^2} \quad (14)$$

VHDL-AMS Modeling

In this section we present different model written with VHDL-AMS language.
We use pseudo random bit sequence to modulate the VCSEL, we use a CMOS driver between the VCSEL and a pseudo random generator (Figure 3).
Figure 4 shown the VHDL-AMS model of the generator.

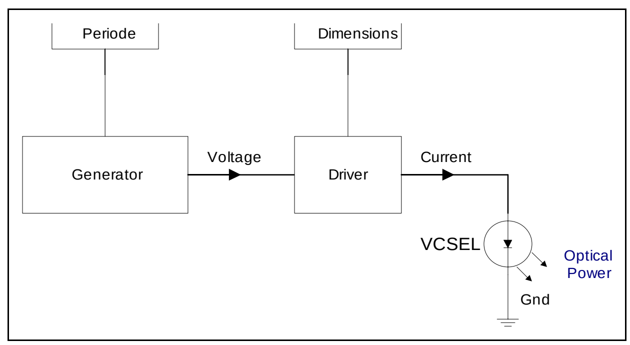

Figure 3 : Optical transmitters.

```
clock <= not clock after ( periode);
 PROCESS
  BEGIN
    wait until clock ='1';
  FOR i IN 1 TO n-1 LOOP
     SR(i+1) <= SR(i);
     IF SR(i) ='1'  THEN  Vout_int <=V_high;
                              ELSE Vout_int <= V_low;
      END IF;
    END LOOP;
    CASE n IS
       when 7 =>    SR(1) <= SR(3) xor SR(7);
       when 10 => SR(1) <= SR(3) xor SR(10);
       when 15 => SR(1) <= SR(1) xor SR(15);
END CASE;
 END PROCESS;
Vout == Vout_int'ramp;
END;
```

Figure 4 : Architecture of pseudo random generator.

Next Figure present a comparison between the Probability Density Function of Matlab© Random noise and VHDL-AMS one.

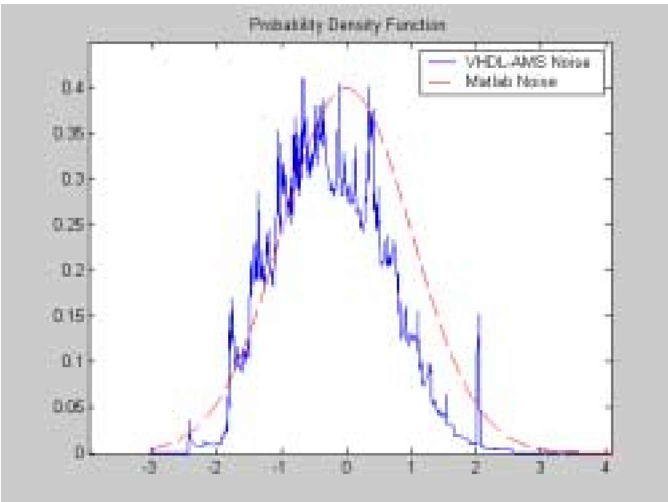

Figure 5 : PDF of two Gaussian Noise.

The VCSEL is modeled with VHDL-AMS (Figure 6)and simulated with ADVanceMS©. Figure 7 presents some transient results, clock, injection current and optical power respectively. The comparison with other works [6] reported in figure 8 shows a good agreement in the output optical power signal.

```
ARCHITECTURE be_laser OF laser IS

--Laser rate equations
N'dot == etai * I/physical_Q - N/TauN - Go * (N - No) * S / (1.0 + eps * S ) + Fn ;
S'dot == - S/TauP + Beta * N / TauN + Go * (N - No) * S / (1.0 + eps * S ) + Fs;
Psi'dot ==AlphaH / 2.0 * ( Go * (N - No)/(1.0 + eps * S) - 1.0/TauP ) + Fpsi;
--Threshold current
Ith == a0 + a1 * Ti + a2 * Ti**2.0 + a3 * Ti**3.0 + a4 * Ti**4.0 ;
--Noise Source
Fn  == Vnoise  * sqrt( Dnn ) ;
Fs  == Vnoise2 * sqrt( Dss ) ;
Fpsi == Vnoise3 * sqrt( Dpsipsi ) ;
--Diffusion coefficient
Dnn    == Rsp * S+N / TauN;
Dss    == Rsp * S ;
Dpsipsi == Rsp / ( 4.0 * S ) ;
--Spontaneous emission
Rsp == nsp / TauP ;
nsp == N / ( N - No) ;
--Optical power
Popt == (physical_H * nu * Alpham * Vg / 2.0 ) * S ;

END ARCHITECTURE be_laser;
```

Figure 6 : VCSEL Architecture.

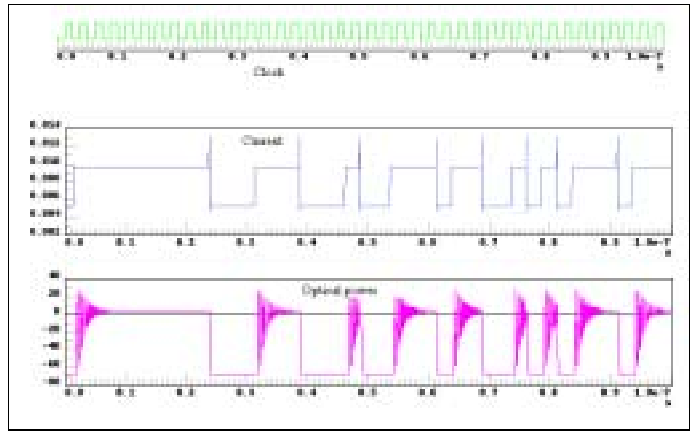

Figure 7 : Transient Results.

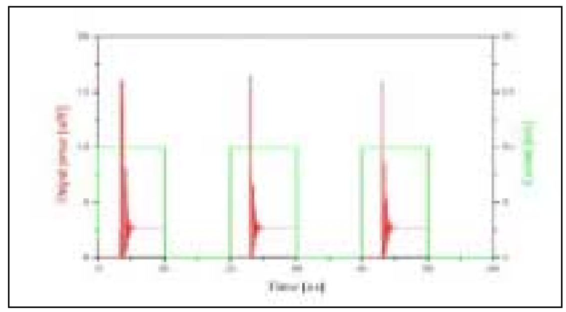

Figure 8 : Response to Square Current pulse. [6]

Conclusion

We have shown that VHDL-AMS is able to model advantageously VCSEL where different type of multi-technological quantities (here optical, thermal and electrical) are coexisting. A methodology for modeling the noise in the VCSEL in transient domain is presented, simulations are run with ADVanceMS©. Simulation results are positively compared with Spice models results published in literature.

References

[1] Z. Toffano and all. "Multilevel Behavioral Simulation of VCSEL based Optoelectronic Modules" , IEEE Journal of Selected Topics in Quantum Electronics Accepted Juin 2003.

[2] Z. Toffano, *Optoélectronique, composants photoniques et fibres optiques*, Paris, Editions Ellipses, 2001, chap.V, pp. 178-197.

[3] G.P Agrawal, Fiber-optic communication systems, Third Edition, 2002 Chapter3, pp. 77-127.

[4] S.F. Yu," Nonlinear Dynamics of Vertical-Cavity Surface-Emitting Lasers", IEEE Journal of Quantum Electronics. Vol.35, No.3, March 1999.

[5] P. V. Mena, "A Simple Rate-Equation-Based Thermal VCSEL Model", IEEE Journal of Lightwave Technology. Vol.17, No.5, May 1999.

[6] A. Przekwas and all, " Multi-level simulation of VCSEL devices and packages in CFD-ACE+ environment", International Workshop on Numerical Simulation of Semiconductor optoelectronic Devices Santa Barbara, March 25-27, 2001.