Modeling Nonlinear Communication ICs Using a Multivariate Formulation†

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ABSTRACT

This paper presents a technique for modeling nonlinear distortion of multirate time-varying communication circuits. To properly consider the weakly nonlinear distortion effects in circuits with multiple large-signal excitations, we capture the quasiperiodic boundary condition of the system Volterra kernels using a multivariate formulation. We then extend the model order reduction work of [8][9] to reduce this large multivariate representation for compact modeling. The proposed approach is demonstrated on a heterodyne front-end receiver.

1. INTRODUCTION

Detailed transistor-level simulations of mixed-signal communication integrated circuits often consume significant CPU time and result in lengthy design cycles. This simulation bottleneck makes it highly desirable to build efficient models that can facilitate system-level verification and design-exploration.

Recently, several projection-based nonlinear model reduction schemes have been proposed for analog circuits that can capture distortion in terms of system-level representations [6]-[9]. In [8], a weakly nonlinear model reduction algorithm NORM was proposed to properly choose projection vectors in order to reduce the model size. NORM was extended into a hybrid approach to more efficiently handle high Q circuits in [9].

Although all the prior work has focused on the reduction of periodically time-varying circuits corresponding to the existence of single periodic large excitation (e.g. a single LO) in the circuit, the extension to quasiperiodic circuits (e.g. containing multiple LOs) is useful and offers several benefits and tradeoffs. As more than one time-varying component of the system is considered, the modeling problem size increases correspondingly. However, the ability to macromodel combinations of multiple circuit components/blocks not only improves the system level simulation efficiency, but also overcomes the difficulty of capturing couplings across the circuit block boundaries.

As an example, a heterodyne RF receiver is shown in Fig. 1, where two LO signals of different frequencies are used. If the weakly nonlinear distortion due to the RF input is to be captured in the system-level macromodel, the circuit should be analyzed as a time-varying system with a operating condition based on the large-signal LO inputs. Modeling the receiver as a whole can easily incorporate any feedbacks or couplings between various blocks, and does not require any assumption on the signal isolation between circuit blocks associated with different LO signals.

To consider the distortion effect of these quasiperiodically time-varying circuits, we must describe the system using certain quasiperiodic boundary conditions. For these weakly nonlinear systems it is possible to apply time-varying Volterra series as an extension to the standard description, as in [4][5], where no time-varying aspect is considered. The quasiperiodic boundary conditions for a Volterra description can, in principle, be obtained using a method very similarly to either of the two methods proposed for steady-state simulation; namely [1][3], or [2]. In this paper, we choose the latter approach since it provides a more straightforward finite-difference formulation as well as other benefits [2]. Once the multivariate Volterra nonlinear transfer functions are formulated into a proper matrix form, they are reduced using an extended model order reduction approach of [8][9].

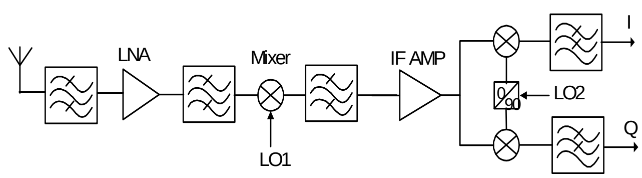

Fig. 1. A heterodyne receiver incorporating two large LO signals

2. BACKGROUND ON VOLTERRA SERIES

Volterra series has been widely used to characterize weakly nonlinear systems [4][5]. For a circuit with input $u(t)$, the response (circuit unknowns) $x(t)$ can be expressed using the expansion

$$x(t) = \sum_{n=1}^{\infty} x_n(t), \qquad (1)$$

where $x_n(t)$ is the nth order response, which is related to the input via convolution in time domain:

$$x_n(t) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} h_n(\tau_1, \dots, \tau_n) u(t-\tau_1) \dots u(t-\tau_n) d\tau_1 \dots d\tau_n. \qquad (2)$$

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In the above equation, $h_n(\cdot)$ is the *n*th order Volterra kernel, and can be thought as an extension to the impulse response function of a linear system. The Laplace transform of the *n*th order Volterra kernel, $H_n(s_1, s_2, \ldots, s_n)$, is often referred to as the *n*th order nonlinear transfer function. Consider the following ordinary differential-algebraic (DAE) description of a weakly nonlinear circuit that is biased at a fixed operating point $x = x_0$

$$f(x(t)) + \frac{d}{dt}q(x(t)) = bu(t). \tag{3}$$

The first order linear transfer function is given by

$$(G_1 + sC_1)H_1(s) = b\,. \tag{4}$$

The symmetrized second order nonlinear transfer function is determined by [8]

$$[G_1 + \bar{s}C_1]H_2(s_1, s_2) = -[G_2 + \bar{s}C_2] \cdot \overline{H_1(s_1) \otimes H_1(s_2)}\,, \tag{5}$$

where $\bar{s} = s_1 + s_2$,

$$G_i = \frac{1}{i!}\frac{\partial^i f}{\partial x^i}\bigg|_{x = x_0} \in R^{n \times n^i},\ C_i = \frac{1}{i!}\frac{\partial^i q}{\partial x^i}\bigg|_{x = x_0} \in R^{n \times n^i},$$

$$\overline{H_1(s_1) \otimes H_1(s_2)} = \frac{1}{2}(H_1(s_1) \otimes H_1(s_2) + H_1(s_2) \otimes H_1(s_1))\,.$$

3. MULTIVARIATE VOLTERRA FORMULATION

For many communication circuits there exist one or more large excitations such as clock or LO. To apply Volterra series for these circuits, one can consider the nonlinear circuit as a weakly nonlinear system w.r.t. the small input signal that is being processed, even though the circuit responds to the clocks or LO's in a highly nonlinear way. This requires analysis of the small input signal upon a time-varying operating condition due to large system excitations. Correspondingly, instead of using a time-invariant Volterra description $H_n(s_1, s_2, \ldots, s_n)$, time-varying transfer functions $H_n(t, s_1, s_2, \ldots, s_n)$ are applied

$$G_1(t)H_1(t, s) + \frac{d}{dt}(C_1(t)H_1(t, s)) + sC_1(t)H_1(t, s) = b \tag{6}$$

$$\begin{aligned}&[G_1(t) + \bar{s}C_1(t)]H_2(t, s_1, s_2) + \frac{d}{dt}[C_1(t)H_2(t, s_1, s_2)]\\ &= -[G_2(t) + \bar{s}C_2(t)]\overline{H_1(t, s_1) \otimes H_1(t, s_2)}\\ &-\frac{d}{dt}([C_2(t)\overline{H_1(t, s_1) \otimes H_1(t, s_2)}])\end{aligned}\ . \tag{7}$$

In (6)-(7), the system conductance and capacitance coefficient matrices vary as time. Consequently, the first and second order nonlinear transfer functions are also function of time. For simplicity, let us assume that there are two large excitations in the circuit that create a 2-tone quasiperiodic operating point. Instead of solving (6)-(7) with a quasiperiodic boundary condition directly, we follow the multivariate approach in [2], and consider the bivariate version of equations (here the number of variables refers only to the number of time variables)

$$\begin{aligned}&\hat{G}_1(t_1, t_2)\hat{H}_1(t_1, t_2, s) + \frac{\partial}{\partial t_1}(\hat{C}_1(t_1, t_2)\hat{H}_1(t_1, t_2, s)) +\\ &\frac{\partial}{\partial t_2}(\hat{C}_1(t_1, t_2)\hat{H}_1(t_1, t_2, s)) + s\hat{C}_1(t_1, t_2)\hat{H}_1(t_1, t_2, s) = b\end{aligned}\ , \tag{8}$$

$$\begin{aligned}&[\hat{G}_1(t_1, t_2) + \bar{s}\hat{C}_1(t_1, t_2)]\hat{H}_2(t_1, t_2, s_1, s_2) +\\ &\frac{\partial}{\partial t_1}[\hat{C}_1(t_1, t_2)\hat{H}_2(t_1, t_2, s_1, s_2)] +\\ &\frac{\partial}{\partial t_2}[\hat{C}_1(t_1, t_2)\hat{H}_2(t_1, t_2, s_1, s_2)] =\\ &-[\hat{G}_2(t_1, t_2) + \bar{s}\hat{C}_2(t_1, t_2)]\overline{\hat{H}_1(t_1, t_2, s_1) \otimes \hat{H}_1(t_1, t_2, s_2)}\\ &-\frac{\partial}{\partial t_1}([\hat{C}_2(t_1, t_2)\overline{\hat{H}_1(t_1, t_2, s_1) \otimes \hat{H}_1(t_1, t_2, s_2)}])\\ &-\frac{\partial}{\partial t_2}([\hat{C}_2(t_1, t_2)\overline{\hat{H}_1(t_1, t_2, s_1) \otimes \hat{H}_1(t_1, t_2, s_2)}])\end{aligned}\ , \tag{9}$$

where t_1 and t_2 correspond to the time scales of two periodic large tones in the circuit. Each signal or matrix $X(t)$ in (6)-(7) can be related to the corresponding one, $\hat{X}(t_1, t_2)$, in (8)-(9), as $X(t) = \hat{X}(t, t)$. In addition, $\hat{X}(t_1, t_2)$ is biperiodic: $\hat{X}(t_1 + T_1, t_2 + T_2) = \hat{X}(t_1, t_2)$, where T_1 and T_2 are the periods of the two large excitations. If (8)-(9) can be solved with periodic boundary conditions in both t_1 and t_2, then the 2-tone quasiperiodic transfer functions can be easily obtained by setting the two time arguments equal in their bivariate version:

$$H_1(t, s) = \hat{H}_1(t, t, s) \text{ and } H_2(t, s_1, s_2) = \hat{H}_2(t_1, t_2, s_1, s_2)\,. \tag{10}$$

To solve (8) numerically, we discretize it using N_1 and N_2 time steps along t_1 and t_2 directions within a period of T_1 and T_2 respectively, on a 2d surface. Let us assume that uniform time steps $h_1 = \frac{T_1}{N_1}$ and $h_2 = \frac{T_2}{N_2}$ are used for t_1 and t_2, with corresponding sampling instants at $t_{1,i}, i = \overline{1, N_1}$, $t_{2,j}, j = \overline{1, N_2}$, respectively. Denoting the number of original circuit unknowns as M, we define the following matrices of dimension $N_2M \times N_2M$ for the consideration of discretization in t_2

$$\Delta_2 = \begin{bmatrix}\frac{1}{h_2}I & & & -\frac{1}{h_2}I\\ -\frac{1}{h_2}I & \frac{1}{h_2}I & & \\ & \ddots & \ddots & \\ & & -\frac{1}{h_2}I & \frac{1}{h_2}I\end{bmatrix}, \tag{11}$$

$$\tilde{G}_{1,i} = \begin{bmatrix}\hat{G}_1(t_{1,i}, t_{2,1}) & & \\ & \hat{G}_1(t_{1,i}, t_{2,2}) & \\ & & \ddots \\ & & & \hat{G}_1(t_{1,i}, t_{2,N_2})\end{bmatrix}, \tag{12}$$

$$\tilde{C}_{1,i} = \begin{bmatrix} \hat{C}_1(t_{1,i}, t_{2,1}) & & & \\ & \hat{C}_1(t_{1,i}, t_{2,2}) & & \\ & & \ddots & \\ & & & \hat{C}_1(t_{1,i}, t_{2,N_2}) \end{bmatrix}, \quad (13)$$

$$\tilde{J}_{1,i} = \tilde{G}_{1,i} + \Delta \tilde{C}_{1,i}. \quad (14)$$

To add the discretization along the t_1 direction, we define the following matrices of dimension $N_1 N_2 M \times N_1 N_2 M$

$$\tilde{G}_1 = \begin{bmatrix} \tilde{J}_{1,1} & & & \\ & \tilde{J}_{1,2} & & \\ & & \ddots & \\ & & & \tilde{J}_{1,N_1} \end{bmatrix}, \tilde{C}_1 = \begin{bmatrix} \tilde{C}_{1,1} & & & \\ & \tilde{C}_{1,2} & & \\ & & \ddots & \\ & & & \tilde{C}_{1,N_1} \end{bmatrix}, \quad (15)$$

$$\Delta = \begin{bmatrix} \frac{1}{h_1}I & & & -\frac{1}{h_1}I \\ -\frac{1}{h_1}I & \frac{1}{h_1}I & & \\ & \ddots & \ddots & \\ & & -\frac{1}{h_1}I & \frac{1}{h_1}I \end{bmatrix}, \tilde{J}_1 = \tilde{G}_1 + \Delta \tilde{C}_1. \quad (16)$$

We further define

$$\tilde{H}_1(s) = [\hat{H}_1^T(t_{1,1}, t_{2,1}, s), \hat{H}_1^T(t_{1,1}, t_{2,2}, s), \ldots, \hat{H}_1^T(t_{1,1}, t_{2,N_2}, s), \ldots, \hat{H}_1^T(t_{1,N_1}, t_{2,N_2}, s)]^T, \quad (17)$$

$$\tilde{b} = [b^T, b^T, \ldots, b^T]^T. \quad (18)$$

to ultimately obtain

$$[\tilde{J}_1 + s\tilde{C}_1]\tilde{H}_1(s) = \tilde{b}. \quad (19)$$

Note that this finite difference formulation is identical to that used in [2] for steady-state analysis except that the conductance and capacitance coefficient matrices in the above formulation are known *a priori*. Similarly, for the second and the third order nonlinear transfer functions, a finite difference formula can be derived that also involves kronecker products.

4. MODEL ORDER REDUCTION

As shown in the previous section, the large finite difference equations arise in the multivariate formulation are in a problem size of $N_1 N_2 M \times N_1 N_2 M$, therefore they need to be properly reduced. Furthermore, these system equations are in a matrix form identical to that of (6)(7), thus they can be reduced using the projection-based nonlinear model order reduction technique of [8]. The NORM algorithm of [8] computes a projection matrix by explicitly considering moment matching of nonlinear transfer functions and employing multipoint expansions. As a result, it has an improved model compactness over earlier approaches such as [6][7], which is critical for nonlinear reduction problems.

A full projection-based approach, however, becomes less efficient for modeling high order nonlinear effects when they vary dramatically over the frequency band of interest (e.g., when the circuit under the modeling contains high-Q filtering blocks). The primary reason for this is that a large projection matrix is usually needed for capturing the sharp frequency domain characteristics, leading to large and dense reduced high order matrices. To overcome this difficulty, an adjoint-based hybrid approach was proposed in [8], where the first and second nonlinear effects at all nodes are approximated using projection, while high order effects (third order) at the specified output node are approximated via projection and the reduction of an adjoint network. The resulting model is further reduced by pruning nonlinear coefficient matrices in the original system coordinates using an idea similar to that of [10].

In this paper, the multipoint version of NORM algorithm of [8] is employed to efficiently approximate the first order and the second order system responses while the adjoint approach of [9] is chosen to approximate the third order distortion effects at the selected sidebands at the output.

5. RESULTS

A heterodyne front-end receiver in the architecture of Fig. 1 is used to test the proposed approach. The two LO frequencies are set at $f_{LO1} = 880MHz$ and $f_{LO2} = 70MHz$ respectively. The 2-tone quasiperiodic operating point of the receiver is computed via steady-state analysis, and the whole system is modeled as a weakly nonlinear system corresponding to the small RF input via the bivariate Volterra formation presented in Section 3. Up to the third order distortion effects are considered based on a set of finite difference equations that correspond to a matrix dimension of about 40,000.

To reduce this extracted full model, multipoint NORM is used to produce a reduced order model of size 36 for approximating the linear time-varying transfer functions as well as the second order nonlinear distortions. For the third order effects, we consider the amount of third order inter-

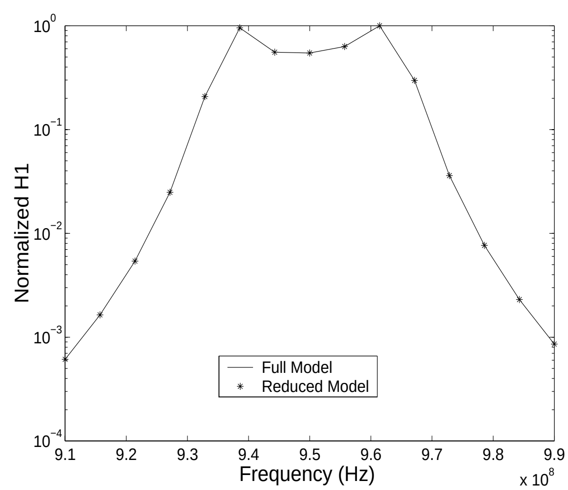

Fig. 2. The first order transfer function as a function of the RF frequency

modulation distortion translated by one f_{LO1} and one f_{LO2}, i.e. close to the base band. To model the propagation of the third order nonlinear distortions from various circuit nodes to the output, an adjoint network is formed with the proper 2-D DFT matrix (corresponding to the wanted third order transfer function sideband at the output) absorbed in its input vector. This linear adjoint network is reduced into a system of size 20. Fig. 2 shows the harmonic of the first order transfer function corresponding to the conversion gain of the receiver. As the output is fully differential, the second order nonlinear effect is ideally zero. To see the third order nonlinear distortion translated by the sum of two LO frequencies, the respective harmonic of the varying third order transfer function is plotted in Fig. 3. As can be seen, for both plots, the reduced order model agrees very well with the full model over a wide range of the input frequencies. Using the reduced order model, the simulation efficiency is improved by a factor of 13.

6. CONCLUSIONS

In this paper, a multivariate Volterra formulation is presented to characterize important weakly nonlinear effects in communication ICs. This choice of multivariate representation leads to a natural boundary condition formulation such that the system distortion under multiple circuit time variations can be well captured. The resulting set of large system equations are effectively reduced using nonlinear model order reduction techniques for improving simulation efficiency.

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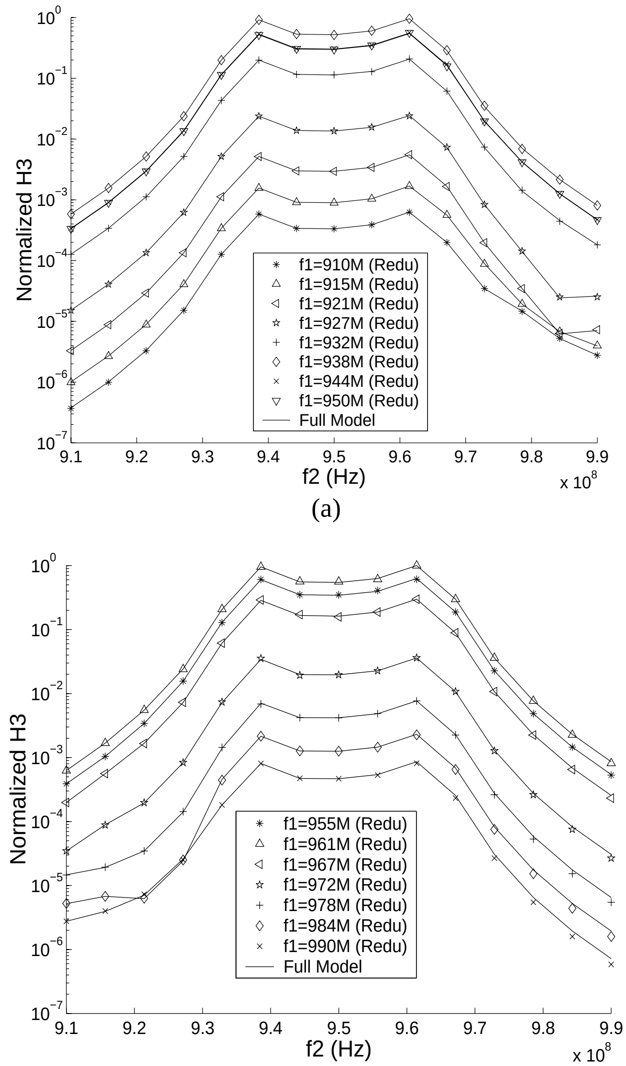

Fig. 3. The third order transfer function as a function of RF input tones. Two tone frequencies f1 and f1 vary from 910-990MHz while the third tone is fixed at -950MHz. (a) f1 varies from 910MHz to 950MHz, and (b) f1 varies from 955MHz to 990MHz.