

# Distributed Electrothermal Modeling in VHDL-AMS

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**Abstract**— This paper demonstrates how to apply VHDL-AMS to modeling a coupled distributed electrothermal problem. We present an example of a simple system where two resistors on top of a silicon substrate are thermally coupled. Thermal exchange is described with a heat balance equation discretized and solved using finite difference method. The entire system is modeled in VHDL-AMS. Our work is a tutorial demonstration of VHDL-AMS capability to model coupled electrothermal systems beyond the traditional equivalent thermal network representation.<sup>1</sup>

## I. INTRODUCTION

A general thermal problem can be modeled by using an appropriate simulator that provides a numerical solution of the heat balance equation for a given set of geometry, sources, and boundary conditions [1]. Modeling a coupled electrothermal problem is more challenging due to the interaction between electrical and thermal processes. Necessity to carry out a concurrent circuit simulation leads to coupling of a thermal simulator and a circuit simulator [2]. Examples and overviews of the work in this area can be found in [3], [4].

Another approach is to use an equivalent thermal network that consists of sources, capacitors, and resistors. These elements represent the effects of heatflow, self-heating, and mutual coupling between electrothermal devices that dissipate power or are sensitive to temperature. In this approach, only the temperature of these devices is of an importance. All other thermal effects are reduced to equivalent thermal network representation [5].

While in many cases the second approach provides an adequate accuracy, reducing a complex 3-D thermal problem to an equivalent thermal circuit network of reasonable size is not trivial, especially for complicated systems. If one wishes to use the first approach, the process of connecting two simulators challenging and custom in each case as well due to the lack of commonly accepted modeling environment. The fact that the time scales for thermal and electric processes are usually quite different presents an additional difficulty in coordinating the operation and interaction of circuit and thermal simulators.

VHDL-AMS (an IEEE standard hardware description language for modeling mixed-signal systems [6]) is an example of

the natural environment where modeling coupled electrothermal problems becomes straightforward. A thermal node can be added to any element in the circuit. Two quantities associated with each thermal node, temperature and heatflow, adequately describe electrothermal effects of the device. Thermal nodes of various elements are plugged into the heat exchange environment. The heat exchange can be modeled either using an equivalent thermal network or by solving the heat balance equation in the volume of interest.

Due to the fact that VHDL-AMS multi-physics capability is currently limited to differential and algebraic equations (DAE's) or equivalent circuits [7], an equivalent thermal network method remains the most popular approach for VHDL-AMS modeling of electrothermal problems. Representative works using this approach (also referred to as thermal impedance matrix approach [8]) include modeling of MMIC array [9], MOST transistor [10], self-heating diode [11] and resistor [12].

Up to date, there has been very little work related to distributed electrothermal modeling in VHDL-AMS because of the lack of support for partial differential equations (PDE's) in the current language standard [13]. For example, [14] deals only with the 2-D thermal problem without any electrical circuits.

In this paper, we present an example of a distributed problem: a simple circuit where two resistors are electrically independent but thermally coupled via a silicon substrate. Both resistors dissipate power to the substrate and are sensitive to temperature changes. Thermal exchange is described with a 2-D heat balance equation solved on the rectangular mesh using a finite difference spatial discretization method. This approach has already been applied to modeling PDE's in VHDL-AMS [15]. The entire system is modeled purely in VHDL-AMS using freely available simulator *Hamster*<sup>2</sup>.

The remainder of the paper is organized as follows. Section II describes a coupled electrothermal circuit example. Thermal modeling is described in Section III. VHDL-AMS implementation is presented in Section IV. Section V contains simulation results. Section VI contains discussion. Conclusions are given in Section VII.

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<sup>2</sup>*Hamster* is now part of *Simplorer*, trademark of *Ansoft Corp.*

## II. EXAMPLE

Consider a simple circuit system shown in Fig. 1. The system consists of two circuits, which are decoupled electrically but coupled thermally due to the fact that both  $R_2$  and  $R_3$  are located on the same silicon substrate. Circuit 1 can represent a pulse generator in the digital part of a mixed-signal system whereas circuit 2 can represent a transistor amplifier biasing circuit in the analog part of a mixed-signal system.

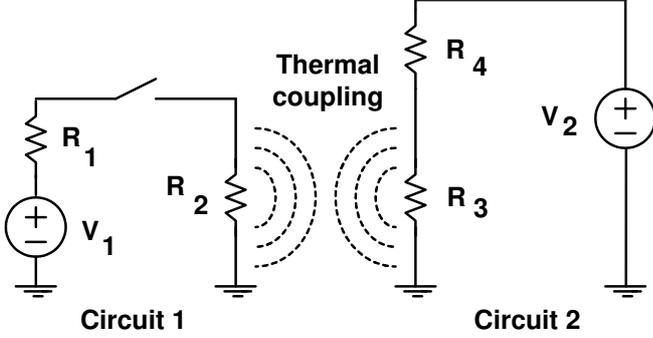


Fig. 1. Example of a circuit system with electrothermal coupling.

The electrothermal interaction that takes place in this system is illustrated in Fig. 2, where  $P$  is dissipated power,  $T$  is temperature,  $I$  is current, and  $V$  is voltage. Other devices are either not temperature sensitive, dissipate negligible power, or located off-chip. For the electrothermal resistors  $R_2$  and  $R_3$ ,

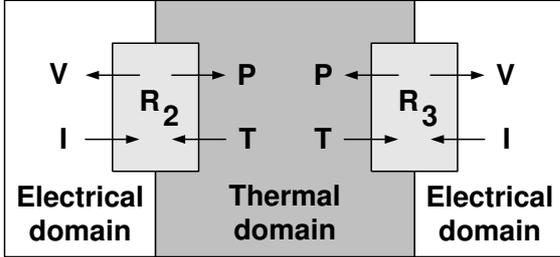


Fig. 2. Electrothermal interaction in the system shown in Fig. 1.

we will assume that their resistance is given by the following function of temperature [12]:

$$R = R_o [1 + \alpha(T_r - T_o)], \quad (1)$$

where  $R_o$  is the nominal resistance at a normal temperature of  $T_o = 300$  K,  $T_r$  is the resistor temperature, and  $\alpha$  is the temperature coefficient of resistance.

While the example presented here is rather trivial and somewhat artificial, it is a good conceptual demonstration of distributed electrothermal modeling in VHDL-AMS. We intentionally keep it simple to demonstrate clearly all steps involved into application of VHDL-AMS to an electrothermal problem. More complicated examples may include 3-D thermal problem and electrothermal semiconductor devices (capacitors, diodes, or transistors) but can be treated in a similar fashion.

## III. THERMAL MODELING

### A. Geometry

Consider a rectangular substrate shown in Fig. 3 with two resistors  $R_2$  and  $R_3$  located on top of it. The width of the substrate is  $W$ , and the height of the substrate is  $H$ . If resistor length  $L$  is much greater than their width and mutual separation, this three-dimensional problem can be treated as two-dimensional. Both resistors dissipate power and generate

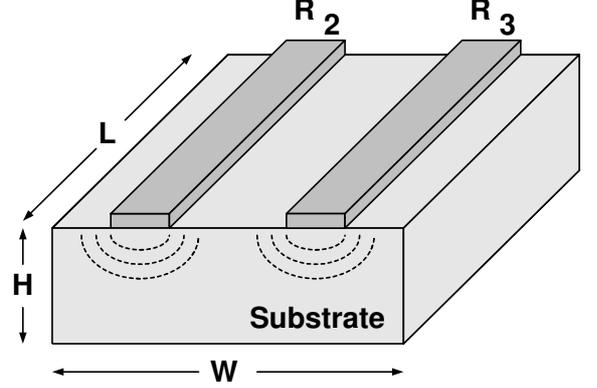


Fig. 3. Geometry of the thermal problem: resistors on top of a substrate.

heat flux into the substrate. The currents through these resistors depend on their temperatures and change as the substrate temperature changes.

### B. Heat Balance Equation

Assuming that there is no heat generated inside the substrate material, the temperature  $T$  inside the substrate can be found using the following heat balance equation:

$$\rho C \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T), \quad (2)$$

where  $\rho$  is the material density,  $C$  is the specific heat capacity, and  $k$  is the thermal conductivity.

Assume that the substrate material is uniform, and its properties are temperature-independent. Then we can rewrite (2) in Cartesian coordinates as:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \quad (3)$$

where  $x$  and  $y$  are the transversal coordinates.

Boundary conditions on conductive (the contact surface between the substrate and the resistor) and convective (all surfaces that do not have a contact with the resistor) boundaries are given by [16]:

$$k \frac{\partial T}{\partial \vec{n}} = \begin{cases} \frac{P}{A}, & \text{conductive} \\ h(T_a - T), & \text{convective} \end{cases}, \quad (4)$$

where  $\vec{n}$  is the vector normal to the boundary surface,  $P$  is the power dissipated in the resistor,  $A$  is the area occupied by the resistor, and  $T_a$  is the ambient temperature.

### C. PDE Discretization

A variety of numerical methods are available for solving PDE's such as heat balance equation. For simplicity of implementation and clarity of illustration, we use a finite difference discretization with classical central difference formula [17]. Since VHDL-AMS does not currently support PDE's, we must discretize partial derivatives with respect to  $x$  and  $y$ .

Consider a rectangular  $N \times M$  mesh, whose nodes are the points where the temperature needs to be determined. The mesh and the node labels are shown in Fig. 4. The horizontal and vertical spacings between the mesh points are  $\Delta x = W/(N - 1)$  and  $\Delta y = H/(M - 1)$ .

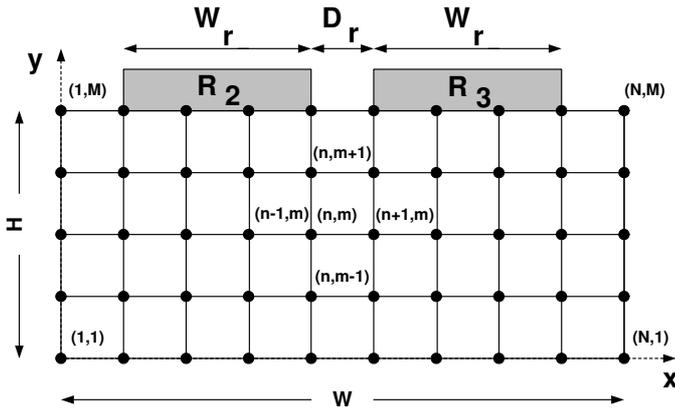


Fig. 4. Rectangular finite difference mesh.

Inside the material, (3) can be discretized to obtain:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C} \left[ \frac{T_{n+1,m} - 2T_{n,m} + T_{n-1,m}}{\Delta x^2} + \frac{T_{n,m+1} - 2T_{n,m} + T_{n,m-1}}{\Delta y^2} \right]. \quad (5)$$

One can rewrite (5) as:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C \Delta x \Delta y} \times \left[ \frac{T_{n+1,m} - T_{n,m}}{\Delta x} \Delta y - \frac{T_{n,m} - T_{n-1,m}}{\Delta x} \Delta y + \frac{T_{n,m+1} - T_{n,m}}{\Delta y} \Delta x - \frac{T_{n,m} - T_{n,m-1}}{\Delta y} \Delta x \right]. \quad (6)$$

In (6), inside the brackets one can identify the derivative terms which correspond to the horizontal and vertical temperature gradients. To take into account boundary conditions, the derivative terms which contain mode indices outside of  $[1..N, 1..M]$  range are replaced with

$$\pm \frac{h(T_a - T_{n,m})}{k} \quad (7)$$

on convective boundaries or with

$$\pm \frac{P}{kA} \quad (8)$$

on conductive boundaries.

### IV. VHDL-AMS IMPLEMENTATION

The discretization described in the previous section results in a system of  $N \times M$  ODE's that can be solved in VHDL-AMS concurrently with the circuit equations. The temperature of each electrothermal resistor is computed by averaging the temperature over all grid points that lie on the resistor contact surface area. Below, we provide VHDL-AMS codes for our circuit system, the electrothermal resistor, and the silicon substrate material.

The parameters of the circuit elements were chosen such that the thermal coupling effect can be seen in the simulation:  $V_1 = 10$  V,  $R_1 = 10$  Ohm,  $R_2 = 10$  Ohm (at 300 K),  $V_2 = 2$  V,  $R_3 = 10$  Ohm (at 300 K),  $R_4 = 10$  Ohm. The VHDL-AMS model of the circuit system is shown below.

```

LIBRARY DISCIPLINES; LIBRARY IEEE;
USE DISCIPLINES.ELECTROMAGNETIC_SYSTEM.ALL;
USE DISCIPLINES.THERMAL_SYSTEM.ALL;
USE IEEE.MATH_REAL.ALL;

ENTITY system IS END;

ARCHITECTURE behav OF system IS
    TERMINAL n1,n2,n3,n4: ELECTRICAL;
    TERMINAL t1,t2: THERMAL;

BEGIN
    V1: ENTITY const_source (behav)
        GENERIC MAP (2.0)
        PORT MAP (n1,electrical_ground);
    R1: ENTITY resistor (behav)
        GENERIC MAP (10.0)
        PORT MAP (n1,n2);
    R2: ENTITY th_resistor (behav)
        GENERIC MAP (10.0,0.1)
        PORT MAP (n2,electrical_ground,t1);
    R3: ENTITY th_resistor (behav)
        GENERIC MAP (10.0,0.1)
        PORT MAP (n3,electrical_ground,t2);
    R4: ENTITY resistor (behav)
        GENERIC MAP (10.0)
        PORT MAP (n3,n4);
    V2: ENTITY pulse_source (behav)
        GENERIC MAP (10.0,25.0e-6,50.0e-6)
        PORT MAP (n4,electrical_ground);
    Si: ENTITY material (behav)
        GENERIC MAP (300.0)
        PORT MAP (t1,t2);
END behav;

```

The electrothermal resistor model is similar to [12] and is shown below. While in reality the temperature coefficient of resistance is on the order of 0.001-0.01 (CMOS poly and n-well resistors), we chose the value  $\alpha = 0.1$  K<sup>-1</sup> to have stronger temperature dependence and to emphasize the electrothermal effect.

```

LIBRARY DISCIPLINES; LIBRARY IEEE;
USE DISCIPLINES.ELECTROMAGNETIC_SYSTEM.ALL;
USE DISCIPLINES.THERMAL_SYSTEM.ALL;
USE IEEE.MATH_REAL.ALL;

ENTITY th_resistor IS
  GENERIC (r_o, alpha: REAL);
  PORT (TERMINAL a, b : ELECTRICAL)
  PORT (TERMINAL t: THERMAL);
END;

ARCHITECTURE behav OF th_resistor IS
  QUANTITY v_r ACROSS i_r THROUGH a TO b;
  QUANTITY t_r ACROSS h_r THROUGH t
    TO thermal_ground;
  CONSTANT t_o : REAL := 300.0;
BEGIN
  i_r == v_r/(r_o*(1.0+alpha*(t_r - t_o)));
  h_r == i_r * v_r;
END behav;

```

We used the following values for the properties of the silicon substrate and its boundaries:

$$k = 1.412 \left[ \frac{W}{K \cdot cm} \right], \quad (9)$$

$$\rho = 2.33 \left[ \frac{g}{cm^3} \right], \quad (10)$$

$$C = 0.7 \left[ \frac{J}{g \cdot K} \right], \quad (11)$$

$$h = 1000 \left[ \frac{W}{cm^2 \cdot K} \right]. \quad (12)$$

Above, high value of the convective heat transfer coefficient was chosen for illustrative purpose. The dimensions of the problem geometry were:

$$W = 45 \text{ mil}, \quad (13)$$

$$H = 20 \text{ mil}, \quad (14)$$

$$L = 150 \text{ mil}, \quad (15)$$

$$W_r = 15 \text{ mil}, \quad (16)$$

$$D_r = 5 \text{ mil}, \quad (17)$$

where  $W_r$  is resistor width,  $D_r$  is resistor spacing, and 1 mil = 0.0254 mm. The mesh has the dimensions of  $N = 10$  and  $M = 5$ .

Large geometry size was also chosen for illustrative purpose. In reality, the resistor size would be smaller and can be estimated from:

$$R = R_s L / W_r, \quad (18)$$

where  $R_s$  is the resistance per square, or sheet resistance. In CMOS process, sheet resistance would be on the order of 20 Ohm for poly and 2000 Ohm for n-well resistors. For example, the size of 200 Ohm poly resistor could be  $10 \mu\text{m} \times 100 \mu\text{m}$ . In our example, the sheet resistance was set to be  $R_s = 1 \text{ Ohm}$ .

The VHDL-AMS model of the substrate is shown below. Because *Hamster* simulator does not support simultaneous *generate* statement, *Matlab*<sup>3</sup> was used to generate the VHDL-AMS code for the system of equations (6).

```

LIBRARY DISCIPLINES; LIBRARY IEEE;
USE DISCIPLINES.ELECTROMAGNETIC_SYSTEM.ALL;
USE DISCIPLINES.THERMAL_SYSTEM.ALL;
USE IEEE.MATH_REAL.ALL;

ENTITY material IS
  GENERIC (Ta: REAL);
  PORT (TERMINAL t1,t2: THERMAL);
END;

ARCHITECTURE behav OF material IS
  QUANTITY t_1 ACROSS h_1 THROUGH t1
    TO thermal_ground;
  QUANTITY t_2 ACROSS h_2 THROUGH t2
    TO thermal_ground;

  CONSTANT k : REAL := 1.412;
  CONSTANT rho : REAL := 2.33;
  CONSTANT C : REAL := 0.7;
  CONSTANT mil : REAL := 0.001*2.54;
  CONSTANT A1 : REAL := 15.0*150.0*mil*mil;
  CONSTANT A2 : REAL := 15.0*150.0*mil*mil;
  CONSTANT Width : REAL := 50.0*mil;
  CONSTANT Height : REAL := 25.0*mil;
  CONSTANT h : REAL := 1000.0;
  CONSTANT N : REAL := 10.0;
  CONSTANT M : REAL := 5.0;
  CONSTANT dx : REAL := Width/(N-1.0);
  CONSTANT dy : REAL := Height/(M-1.0);
  CONSTANT Const : REAL := k/(rho*C*dx*dy);
  CONSTANT hk : REAL := h*k;

  QUANTITY PkA1, PkA2: REAL;
  QUANTITY T11, T12, ... T104, T105: REAL;

BEGIN
  BREAK T11 => Ta;
  ...
  BREAK T105 => Ta;

  PkA1 == - h_1 / (k * A1);
  PkA2 == - h_2 / (k * A2);

  T15' dot == Const * ((T25-T15) * dy/dx - dy*hk*(T15-Ta)
    + dx*hk*(Ta-T15) - (T15-T14) * dx/dy);
  T25' dot == Const * ((T35-T25) * dy/dx - (T25-T15) *
    dy/dx + dx*PkA1 - (T25-T24) * dx/dy);
  ...
  T22' dot == Const * ((T32-T22) * dy/dx - (T22-T12) *
    dy/dx + (T23-T22) * dx/dy - (T22-T21) * dx/dy);
  ...
  T101' dot == Const * (dy*hk*(Ta-T101) - (T101-T91) *
    dy/dx + (T102-T101) * dx/dy - dx*hk*(Ta-T101));

  t_1 == (T25 + T35 + T45 + T55) / 4.0;
  t_2 == (T65 + T75 + T85 + T95) / 4.0;

END behav;

```

<sup>3</sup>Trademark of *Mathworks, Inc.*



## VII. CONCLUSIONS

In this paper, we demonstrated how to perform VHDL-AMS modeling of circuit systems with distributed electrothermal interaction. We presented an example where two resistors were thermally coupled via a silicon substrate. The 2-D heat balance PDE was discretized using a finite difference method, and the resulting system of ODE's was solved in VHDL-AMS together with the circuit equations using *Hamster* simulator.

In this example, we manually specified the geometry and the material properties, created a mesh, discretized the equations, and set boundary conditions. Ultimately, these tasks need to be automated, as it is done in many domain-specific simulators, such as ANSYS<sup>4</sup> or FEMLAB<sup>5</sup>. One can envision a graphical user interface that allows user to perform these functions for standard IC packages and to generate an appropriate set of VHDL-AMS codes. Having a PDE support in VHDL-AMS would further help to simplify this process.

While the example presented here may be simple, it nevertheless demonstrates the idea: complex coupled electrothermal systems can in principle be modeled and simulated in a single unified language environment, such as VHDL-AMS. One major advantage of using the standard language is the fact that the model does not depend on the underlying simulator. This can lead to high portability of electrothermal models and ease of sharing them among the designers.

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<sup>4</sup>Trademark of ANSYS, Inc.

<sup>5</sup>Trademark of The COMSOL group