Predicting Total Harmonic Distortion (THD) in ADSL Transformers using Behavioural Modeling

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ABSTRACT

As a design application, the ADSL broadband line transformer offers interesting challenges somewhat different to those in Power Supply design. In this paper the aspect of Total Harmonic Distortion (THD) as affected by the non-linear behaviour of the magnetic material in a line transformer is analysed using simulation. A mixed technology model of the line transformer is constructed and circuit simulation used to predict the level of THD for a variety of core types and configurations. The simulation results are compared with measured THD figues to demonstrate the accuracy of the models used.

INTRODUCTION

Despite the ever increasing integration of broadband modem integrated circuits for technologies such as ADSL (Asymmetric Digital Subscriber Line), there is still a need for a line transformer to isolate the line from the device at either the Central Office (CO) or Customer Premises (CPE).

A crucial design problem for the manufacturers of such components is the Total Harmonic Distortion (THD) that will result when the component is placed in the complete system, and how much is due to the non-linearities introduced by the magnetic component itself. As a result, it has become highly desirable to apply modeling techniques commonly used in power applications, such as switch mode power supplies, i.e. the use of non-linear magnetic material models, to assess the potential THD for a particular design.

In this paper a commonly used non-linear magnetic material model, the Jiles-Atherton model, is used to estimate the THD performance of a variety of line transformers.

BACKGROUND

A. Introduction to ADSL

Asymmetric Digital Subscriber Line (ADSL) technology is used as a high-speed modem link with asymmetric up- and down-stream data rates as the name suggests. The current primary application is for very high-speed internet access, but the goal of the technology is not restricted to purely internet applications, but also video-streaming (for commercial video-on-demand), fast data transfer, and fully integrated network services for companies and home use.

A significant differentiator between ADSL and current cable television access is the ability to utilise the existing infrastructure (i.e. copper twisted-pair wires) of the telephone network. Figure 1 shows the outline of a typical network configuration, with a Central Office (CO) connection at the service provider, and a number of remote Customer Premises Equipment (CPE) installations.

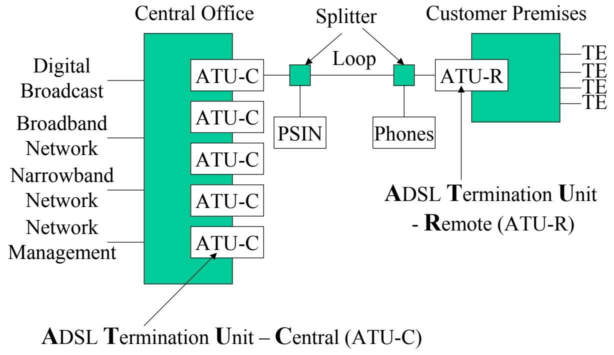

Figure 1: ADSL Network Configuration Outline

ADSL has proved a significant step in the development of Digital Subscriber Line (DSL) technology, as it has probably the best defined standard (IEEE T1.413-1995 [1]) that is actually implemented de facto in the telecommunications industry, by both service providers and chip manufacturers alike. Bingham [2], Cioffi [3] describe the development of the philosophy for ADSL and similar broadband approaches and Goralski [4] or Chen [5] provide background to the technology area to high-speed communications. It is beyond the scope of this paper to go into detail of the mechanism of the ADSL system, but it is useful to describe some details of the transmission scheme and the analogue interface to understand the requirements for ADSL line transformers.

ADSL Modulation Scheme

ADSL is based on a broadband modulation scheme, with multiple carriers placed at 4.3125kHz intervals. With 256 carriers a 1.1MHz bandwidth is required. These sub-carriers may also be referred to as sub-channels. Figure 2 shows how the bandwidth is divided into areas for the standard telephone communications (POTS), up-stream and down-stream bandwidth. In figure 2, with the distinct gap in the bandwidth between the up- and down-streams, there is no requirement for echo cancellation. If echo cancellation is used, then a more efficient use of the bandwidth occurs where the up- and down-streams overlap.

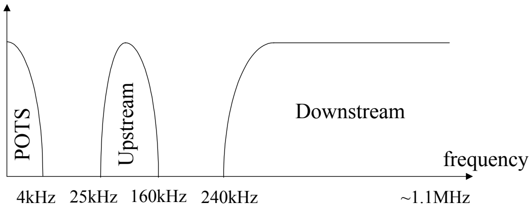

Figure 2: ADSL Bandwidth Allocation

Two methods of modulation can be used in ADSL, CAP and DMT. Carrier-less Amplitude/Phase Modulation (CAP) is very similar to QAM (Quadrature Amplitude Modulation), except that the carrier itself is not transmitted. The carrier does not actually hold any information and is therefore recreated at the receiver for efficiency. The phase is synchronised using phase sequences in the data.

Discrete Multi-Tone Modulation (DMT) is the method described in the ANSI standard T1.413, and uses intelligent allocation of the different channels to send the appropriate number of bits to achieve maximum overall throughput. Each individual channel uses QAM, so although both CAP and DMT use QAM at the lowest level, the difference is the dynamic allocation of data to individual channels using DMT that provides a potentially better data rate.

Overview of the Analogue Interface and Modeling Line Transformers

As has been described in the previous section, the signals to be transmitted between two ADSL modems are modulated using some form of QAM, with an interface between the modem hybrid and the line of a 1:1 line transformer, as shown in figure 3.

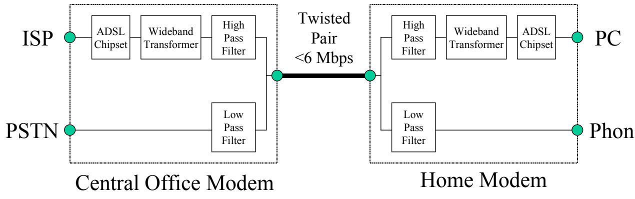

Figure 3: ADSL Analogue Interface

Day, Wurcer and Hoffman [6] provide an excellent primer in the design of ADSL hybrids, and address some of the rudimentary aspects of the transformer design. Other useful and practical sources of design information can be found in Cornil [7] and Cabler [8]. The key issues are loss, matching, parasitic effects and distortion. The frequency response is limited at low frequencies by the magnetising inductance. The parasitic leakage inductance and winding capacitance reduce the bandwidth at higher frequencies. Resistive losses in the windings also provide a source of loss in the device across the whole range. The overall effect on the insertion loss is summarised in figure 4.

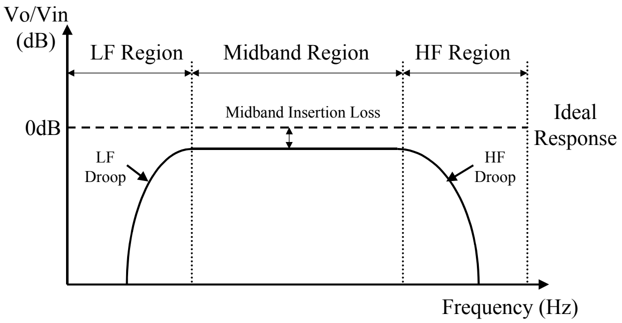

Figure 4: Transformer Insertion Loss with non-ideal model

Steffes [9] and Nash [10] demonstrate how simulating the line driver and its associated circuitry can predict the performance of the analogue interface in general. Day and Day *et al* [6] and Dean [11] have shown specifically how a simple model of a transformer, including the parasitics as shown in figure 5, can be used to predict the frequency response and insertion loss of the device. This is shown in figure 6, with a graph of the simulated insertion loss for a realistic ADSL device.

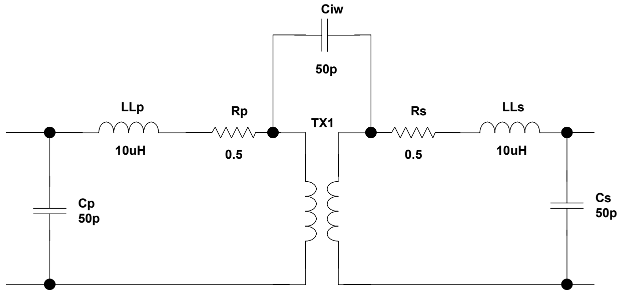

Figure 5 : Transformer Model Used for Simulation of Insertion Loss

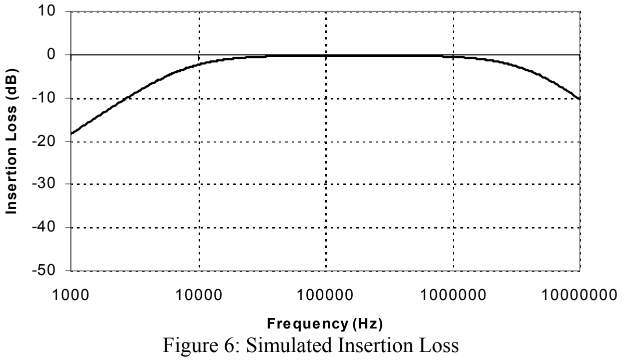

Figure 6: Simulated Insertion Loss

Using simulation in this way is an effective tool in identifying critical parameters, defining parametric limits and optimising device performance.

In order to achieve effective and accurate simulation results, it is necessary to characterise these parameters accurately as has been discussed previously by Wilson [38] and Wilson *et al* [31]. The range of methods employed range from analytical approaches, such as those proposed by Dowell [12], Dauhajre [13] & [14] and Snelling [15], through to integrated modeling procedures involving multiple Finite Element Analyses [FEA], such as those described by Asensi *et al* [16]-[18], Wilson [19], Lopera *et al* [20] and Prieto *et al* [21]-[24].

The other issue with the transformer is its effect on signal distortion. If distortion occurs in a sub-carrier or channel, then degradation of the signal may occur, giving rise to loss of signal integrity and data errors. A further side effect of non-linearities in the core is the possible introduction of inter-modulation products in other sub-carriers or channels. Even if the transformer is designed to operate largely in a linear region, due to the nature of the modulation scheme used in ADSL, large Peak-to-Average Ratios (PAR) may occur giving rise to significant magnetic offsets and possible non-linear behaviour. Tellado [25] discusses the potential for large PAR in ADSL systems in some detail, and also methods for its reduction. Due to the broadband nature of the signals, phase shifts may also occur in the transmitted signal, as a result of different frequencies across the range having a different phase response in the core material. This may also result in phase shifts in the recovered ADSL signals.

It is important therefore in a simulation model to include the non-linear core model to ensure that the transformer does not degrade the performance of the system as a whole. This paper concentrates on this aspect of the transformer design, with investigations carried out into the trade-offs of material characteristics and air gaps, the effect of core size and number of windings on performance, and comparisons between different winding configurations and core types. Previous work [26] describes the basic design procedure for DSL transformers, but relies on 'rule of thumb' calculations and simplification of the THD (Total Harmonic Distortion) figures. It is proposed in this paper to apply circuit simulation to more accurately predict the effect of non-linear ferrite cores on the transformer's performance. The proposed model will have a similar parasitic model to that described previously, but will include an accurately characterised non-linear core model. In this case the original Jiles-Atherton [27]-[29] model has generally been used. The modifications to the Jiles-Atherton model that have been proposed previously by Wilson, Ross and Brown [30]-[33] generally refer to heavily saturated cores, and in general the ADSL transformers operate at low signal levels implying minor BH loops.

Distortion Performance Criteria

There are several criteria that are used to measure the performance of DSL transformers including Total Harmonic Distortion (THD) and Signal to Noise Ratio (SNR) This section defines these criteria as they have been used in this work. The THD figure is of primary interest to ADSL system designers and is usually the only distortion figure given on a data sheet.

Total Harmonic Distortion (THD) is used to establish the effect on an ideal sinusoidal test tone of non-linearities in the transformer. The THD is calculated using expression 1, where V_1 is the peak harmonic, the voltage at the ideal tone frequency, and V_2 , V_3 , V_4, V_5 are the second to fifth harmonics respectively.

$$THD = \frac{\sqrt{\sum_{i=2}^{5} V_i}}{V_1} \tag{1}$$

DISTORTION IN TRANSFORMERS DUE TO NON-LINEAR MAGNETIC MATERIALS

Introduction

Having defined the measurement criteria for distortion, it is useful at this stage to consider the sources of distortion, and how simulating transformers could predict the level of distortion. Due to the non-linear nature of the transformer ferrite core, the voltage waveforms on the windings will have a non-linear relationship with respect to the current through the driving (primary) windings. This is due to the non-linear relationship between B and H in the core. Figure 7 illustrates how the BH characteristic changes as the applied field strength (H) is increased, introducing non-linear behaviour.

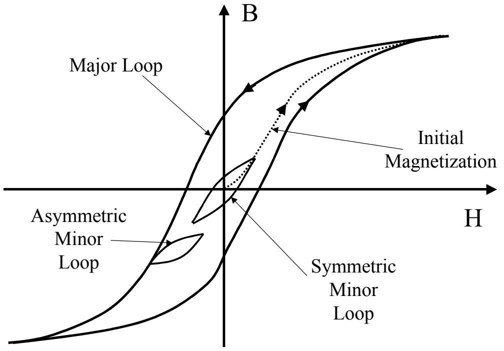

Figure 7: Non-Linear BH Behaviour in Transformer Core

Near the origin, the relationship between B and H is essentially linear, but as H increases, the slope of the BH curve steepens until saturation occurs, at which point the slope reduces once again (This is clearly seen on the initial magnetization curve). This illustrates how there are two potential sources of distortion, at low and high field strengths. Figure 7 also shows how any magnetic offset conditions may cause different slopes and sizes of loop, as shown with the minor loop variations.

Modeling Hysteresis using a mixed-technology approach

This behaviour can be modelled in transformers using a mixed-domain approach with the hysteresis modeled in the magnetic domain connected to the electrical domain using winding models. The resulting voltage waveforms reflect the effect of the non-linear behaviour of the core. This approach is well understood and has been described by Cherry [34], Laithwaite [35] and Carpenter [36]. More specifically the use of detailed hysteresis models in the magnetic domain in conjunction with electrical circuit models are described by Brown, Ross, Nichols and Penny [37] and Wilson [38]. As long as accurate models of the core non-linearity are used, then it is possible to estimate the effect on distortion of the voltage waveforms. A common assumption in the available design guides from manufacturers such as Ferroxcube [26] indicates that the 3rd harmonic is the only significant problem in DSL transformer design, but this assumes that hysteresis is not taking place in the core. If hysteresis is included, then the 2nd through to the 5th harmonic may have an impact on the transformer performance. In the testing and simulation that follows, all the harmonics are considered, not just the 3rd.

The general approach to modeling a transformer using mixed technology model is shown in figure 8, with a two winding transformer modeled using two winding models and a single non-linear core model in the magnetic domain.

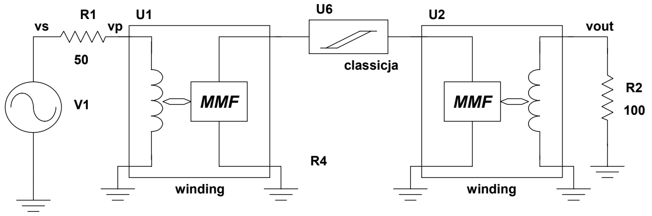

Figure 8: Transformer Mixed-Technology Model & Test Circuit

The crucial part of the model for accurate prediction of the non-linear behaviour at the device terminals is the non-linear core model in the magnetic domain. As has been discussed previously, the focus in ADSL applications is on low signal levels, with minimal distortion, and so accurate minor loop modeling is important. Carpenter [39] and Jiles [40] have described how a model can be produced, based on the original Jiles-Atherton model, that can accurately replicate the minor loop behaviour while Wilson and Ross [31] demonstrate how appropriate characterisation and optimization of the model can provide a best fit of the model's behaviour under these specific conditions.

Source of Distortion in the Electrical Domain

The source of the distortion apparent in the electrical domain is the non-linearities in the magnetic domain. This may be a problem in understanding the direct effect of a particular aspect of the magnetic material behaviour on the resulting electrical waveforms. This is due in part to the fact that the voltage across a winding is in fact the derivative of the flux (and flux density) in the magnetic material. In order to aid the understanding of the waveform shapes, a typical model is used and the applied field strength gradually increased so that the BH loop becomes progressively more saturated. The resulting waveforms can be analysed to observe the effect on the voltage across the terminals of the magnetic component in the electrical domain. By applying a fixed field strength with values of 1A/m , 10A/m and 50A/m for a core model characterized with the material 3E6, the BH curves, resulting voltage waveforms for an ideal winding with a single turn, and the Fourier coefficients can be calculated. The BH curve for Hmax=50A/m is given in figure 9.

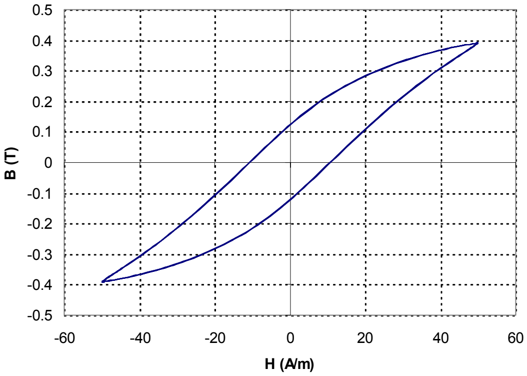

Figure 9: Simulated BH Curve for 3E6 material with Hmax = 50A/m

As the field strength increases from 1A/m to 50 A/m, the BH curve first develops a small hysteresis loop and as the the material nears saturation, there is the transition into a major loop shape with distortion at the loop tips. The effect this has on the resulting voltage on the winding terminals is shown figure 10 (Hmax=50A/m).

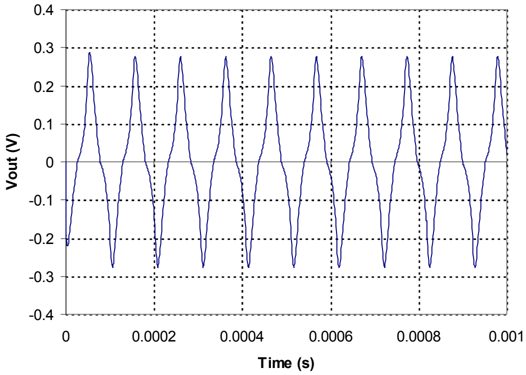

Figure 10: Simulated Voltage for 3E6 material with Hmax = 50A/m

It is clear that by the time the applied field strength has reached 50A/m there is significant distortion of the voltage waveform on the winding. In order to quantify the levels of distortion, and to compare between different levels of applied signal, the normalized harmonics were calculated using a Discrete Fourier Transform (DFT) and this is quantified in table 1, which shows the relationship between the applied field strength and the calculated THD level on the output voltage waveforms.

Hmax (A/m)	THD (%)
1	0.38
10	4.05
50	28.4

Table 1: Relationship between Applied Field Strength and THD

Using this general approach, specific cases of transformer can be tested and the THD compared with measured values.

Predicting Total Harmonic Distortion (THD) with Simulation

Introduction

In order to investigate the techniques required to simulate the THD performance of a line transformer for ADSL, a series of transformers were constructed, THD measured, models derived and simulated. Comparisons were made between the measured and simulated results to assess the accuracy of the simulation model and its usefulness for predicting the THD figure for a variety of transformer configurations. Gapped and un-gapped cores were chosen to establish the effect of air gaps on the performance, and similarly toroidal (ring) cores were compared with two-piece (ER11) cores. The types of core used in this section were a TN10/6/4 toroid and ER11 (gapped and un-gapped). All of the cores were made of the materials 3E5 or 3E6 (Philips), or T38 (Siemens).

Toroid TN10/6/4-3E5 Line Transformer

The first test case was a TN10/6/4 toroid made of 3E5 material. The component was a two-winding transformer, each winding wound with 60 turns of 28 s.w.g wire. To ensure accurate results, a 12-bit oscilloscope was used to enable more accurate definition of small signal harmonics (the Tektronix TDS220 oscilloscope used previously in this work has only 8-bit resolution) therefore the ADC-212 Picotech Virtual Instrument was used. To ensure that spectral leakage of the FFT results was minimized, the frequency of the source was specified at 9732Hz to produce a sampled waveform with an integral number of cycles for a sample size of 4096.

Using the optimization software developed by Wilson, Ross and Brown [33], the 3E5 ferrite material was characterized using the original Jiles-Atherton model, with the parameters given in table 2. The model was optimized to measured results obtained at approximately the same operating conditions as the THD test circuit to ensure a like-with-like comparison.

Parameter	Value
a	5.245
k	20.022
c	1.1675
α	4.1u
M_s	241k

Table 2: 3E5 model parameters

The transformer was tested using the circuit shown in figure 11, with the peak input voltage varied from 2V to 10V in 2V steps. The simulation model was based on the general structure shown in figure 8.

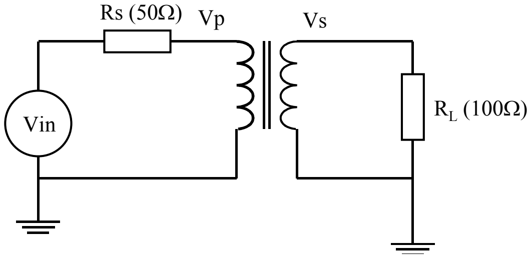

Figure 11: Transformer THD Test Circuit

The THD calculated for the measured and simulated test circuits is given in figure 12.

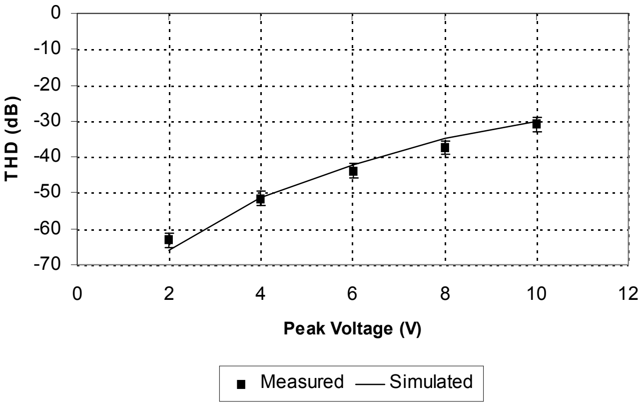

Figure 12: TN10/6/4-3E5 Comparison of Measured and Simulated THD

Using this approach it is clear that accurate estimates of the Total Harmonic Distortion for transformers can be obtained with the use of suitably characterized material models of the core. In this case, the difference between the measured and simulated results is within 4dB (worst case) and often within a fraction of a dB. It was found that if the material was not characterized for the general operating conditions of the transformer, that much less accurate results were obtained, highlighting the necessity for specific material models, or models that change behaviour accurately depending on the applied field strength levels.

ER11-3E6 Line Transformer

Obviously the toroid analysed in the previous section would not be used in practice, but rather a gapped core such as EP13 or ER11. The EP13 cores have been well used in commercial applications but are becoming a significant cost of the hybrid design due to their relatively large size and weight. It was therefore considered useful to investigate the performance of ER11 cores. These cores are significantly smaller, and also offer the potential to be used in planar or thick film constructions as well as the conventional wire wound form.

The first test case was a two winding transformer, wire wound on an ER11 two piece un-gapped core made of Philips 3E6. Each winding was wound with 9 turns of 28 s.w.g wire to give an inductance of 230uH. This transformer was tested using the same test circuit as used previously (shown in figure 11). The core was characterised in the same way as the toroid previously described, with the resulting model parameters given in table 3.

Parameter	Value
a	15.0
k	40.0
c	1.06
α	3.6u
M_s	224k

Table 3: ER11-3E6 model parameters

The parameters are in a similar range to those for the 3E5 toroid, but it is worth noting that the characterization includes the small gap between the two core halves. The THD of the transformer was measured using the circuit shown in figure 12, with the peak input voltage varied from 1V to 10V in 1V steps. The THD calculated for the measured and simulated test circuits is given in figure 13.

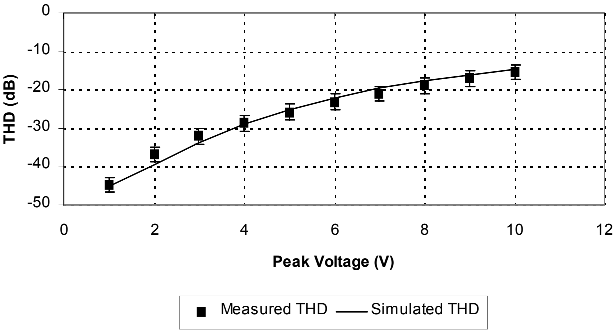

Figure 13: ER11-3E6 Comparison of Measured and Simulated THD

In this case, the core is un-gapped, so the THD is relatively poor for this range of input voltage. The addition of a core gap will have the effect of storing energy in the gap rather than the core and hence reducing the effect of the core material non-linearities on the signal through the transformer.

The use of the mixed technology models for measuring the THD from a simulation has been demonstrated to be of usefulness, with the added benefit of insight into the magnetic behaviour inside the core. As an illustration of this, figure 14 shows the simulated BH curves inside the gapped and un-gapped core models for the same applied current level (7mA).

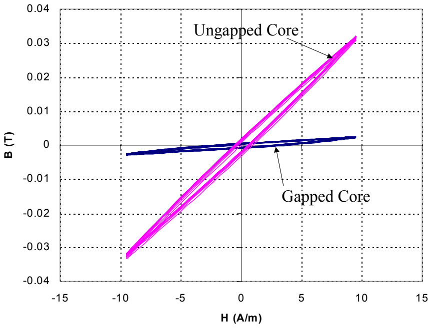

Figure 14: ER11-3E6 Gapped and Un-gapped Core Simulated BH Curves

As can be seen from the figure, the area of the ungapped core BH loop is greater and also steeper. This illustrates the effect of the air gap, effectively reducing the effective permeability of the overall core & gap combination. One effect of the introduction of a gap is to effectively reduce the distortion, as more of the energy is stored in the gap.

CONCLUSIONS

This paper has shown how the use of mixed-technology magnetic component models, accurate predictions can be made of the THD performance of the device, and that with the added insight that a mixed domain behavioural model brings, design decisions can be made about the geometry and winding configuration using these models with an increased level of understanding about the behaviour of the magnetic core.

This work has also demonstrated how effective use can be made of magnetic component models in analyzing the effects of non-linear magnetics on an electrical circuit or system's behaviour.

REFERENCES

[1] IEEE Standard T1.413-1995, "Network and Customer Installation Interfaces - Asymmetric Digital Subscriber Line (ADSL) Metallic Interface", August 1995, American National Standards Institute, Inc.

[2] J.A.C. BINGHAM, "Multicarrier Modulation for Data Transmission; An Idea whose Time Has Come", *IEEE Communications Magazine*, May 1990, pp5-14.

[3] J. M. CIOFFI, "A multicarrier primer", ANSI Document, T1413 Technical Subcommittee, (91-157).

[4] W. GORALSKI, "ADSL and DSL Technologies", McGraw-Hill, 1st Edition, 1998

[5] W.Y. CHEN, "DSL Simulation Techniques and Standards Development for Digital Subscriber Line Systems", Macmillan Technical Publishing, 1998

[6] B.D. DAY, S. WURCER & T. HOFFMAN, "Optimizing Line Driver Designs for Maximum Power; "Bridging" ADSL Line Driver Challenges", Analog Devices Appication Note, Analog Devices, Inc.

[7] J.P. CORNIL, "Building an ADSL Modem, the Basics", Analog Circuit Design - (X)DSL and other Communications Systems;RF MOST

Models;Integrated Filters and Oscillators, 1999, Kluwer Academic Publishers, pp3-48

[8] C.D. CABLER, "Survey of the State of the Art Analog Front End Circuit Techniques for ADSL", Analog Circuit Design - (X)DSL and other Communications Systems;RF MOST Models;Integrated Filters and Oscillators, 1999, Kluwer Academic Publishers, pp117-126

[9] STEFFES M., "Optimizing Performance in an xDSL Line Driver", *Electronic Design*, April 19, 1999, pp44-58

[10] NASH, E., "Line-Driver Design For Broadband Communications Applications", *Electronic Design*, December 1, 1997, pp81-94

[11] DEAN D., "Gain Confidence in Network Designs By Simulating the LAN Magnetics", Pulse, Inc. Technical Note.

[12] P.L. DOWELL, "Effects of eddy currents in transformer windings", *Proceedings of the IEE*, Vol. 113, No 8, 1966, p1387

[13] A. DAUHAJRE & R.D. MIDDLEBROOK, "Modeling and estimation of leakage phenomenon in magnetic circuits", *PESC '86*, 1986, pp213-226

[14] A. DAUHAJRE, "Modeling and Estimation of Leakage Phenomena in Magnetic Circuits", PhD Thesis, California Institute of Technology, Pasadena, California, 1986.

[15] E.C. SNELLING, Soft Ferrites, Butterworths, 2nd Edition, pp330-335

[16] R. ASENSI, J.A. COBOS, O. GARCIA, R. PRIETO & J UCEDA, "New Modeling Strategy for high frequency transformer windings", *IECON Proceedings*, 1994, pp246-251

[17] R. ASENSI, J.A. COBOS, O. GARCIA, R. PRIETO & J. UCEDA, "A full procedure to model high frequency transformer windings", *IEEE Power Electronics Specialist Conference Proceedings*, 1994, Vol. 2, pp856-863

[18] R ASENSI, J.A. COBOS, O. GARCIA, R. PRIETO & J UCEDA, "A CAD tool for magnetic components modeling", *American Power Electronics Conference (APEC*), 1996, Vol 1, pp427-433

[19] P.R. WILSON, "Advanced modeling and simulation techniques for magnetic components", *IEE Power Electronics and Variable Speed Drives Conference Proceedings*, Sep 1998, pp187-193

[20] J.M. LOPERA, M. PERNIA, J.M. ALONSO & F. NUNO, "A complete transformer electric model including frequency and geometric effects", *PESC '92 Record. 23rd Annual IEEE Power Electronics Specialists Conference (Cat. No.92CH3163-3)*, Vol. 2., pp 1247-52

[21] R. PRIETO, V. BATALLER, J.A. COBOS & J. UCEDA ,"Influence of the winding strategy in toroidal transformers", *IECON 1998*, Volume 1, pp359-64

[22] R. PRIETO, J.A. COBOS, O. GARCIA, P. ALOU & J. UCEDA, "Taking into account all the parasitic effects in the design of magnetic components", *APEC '98*, Vol. 1, pp400-406

[23] R. PRIETO, J.A. COBOS, O. GARCIA & J. UCEDA, "Interleaving techniques in magnetic components", *APEC '97*, Vol. 2, pp931-936

[24] R. PRIETO, J.A. COBOS, O. GARCIA & J. UCEDA, "Influence of the winding strategy on the parasitics of magnetic components", *EPE '97*, Vol. 2, pp38-43

[25] J. TELLADO, "Multicarrier Modulation with low PAR", Kluwer Academic Publishers, 2000.

[26] "The use of Ferrite Cores in DSL wideband Transformers", Application Note, Ferroxcube, Inc., 2001

[27] DC JILES & DL ATHERTON, "Theory of Ferromagnetic Hysteresis (invited)", *Journal of Applied Physics*, Vol. 55 , No 6, Mar 1984, 2115-2120

[28] D.C.JILES & D.L.ATHERTON, "Theory of Ferromagnetic Hysteresis", *Journal of Magnetism and Magnetic Materials*, 61, 1986, 48-60

[29] D.C. JILES & D.L. ATHERTON, "Ferromagnetic Hysteresis", *IEEE Transactions on Magnetics*, Vol 19, No. 5, Sep 1983, pp2183-2185

[30] P.R. WILSON, J.N. ROSS & A.D. BROWN, "Optimizing the Jiles-Atherton model of hysteresis using a Genetic Algorithm", *IEEE Transactions on Magnetics*, Vol. 37, No. 2, March 2001, pp989-993

[31] P.R. WILSON & J.N. ROSS, "Definition and Application of Magnetic Material Metrics in Modeling and Optimization", *IEEE Transactions on Magnetics*, Vol. 37, No. 5, Sep 2001, pp3774-3780

[32] P. R. WILSON, J. N. ROSS & A. D. BROWN, "Simulation of Magnetic Component Models in Electric Circuits including Dynamic Thermal Effects", *IEEE Transactions on Power Electronics*, Accepted for Publication 2001.

[33] P.R. WILSON, J.N. ROSS & A.D. BROWN, "Magnetic Material Model Characterization and Optimization Software", *IEEE Transactions on Magnetics*, Accepted for Publication 2001

[34] E. C. CHERRY, "The duality between inter-linked electric and magnetic circuits", *Proceedings of the Physics Society*, Volume 62, 1949 pp101-111

[35] E. R. LAITHWAITE, "Magnetic Equivalent Circuits for electrical machines", *PROC. IEE*, Vol. 114, November 1967, 1805-1809

[36] C.J. CARPENTER, "Magnetic Equivalent Circuits", *PROC. IEE.,*Vol 115,No 10, October 1968, 1503-1511

[37] AD BROWN, JN ROSS, KG NICHOLS AND MD PENNY, "Simulation of Magneto-Electronic Systems using Kirchoffian Networks", *European Conference on Magnetic Sensors and Actuators*, Sheffield, July 1998

[38] P.R. WILSON, "Modeling Magnetic Components In Electric Circuits", PhD Thesis, University of Southampton, UK, 2001.

[39] K.H. CARPENTER, "A differential equation approach to minor loops in the Jiles-Atherton hysteresis model", *IEEE Transactions on Magnetics*, Vol. 27, No. 6, Nov 1991, pp4404-4406

[40] D.C. JILES, "A self consistent generalized model for the calculation of minor loop excursions in the theory of hysteresis", *IEEE Transactions on Magnetics*, Vol. 28, No.5, Sep 1992, pp2602-2604