Issues in MEMS Macromodeling

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Embedded Tutorial
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Issues in MEMS Macromodeling

- Processes and Applications
- Design Views
- Natures and Disciplines
- Hierarchy
- Interoperability
- Beam Element Example
- Filter Example
- Language Extension
Sampling of MEMS Processes

- **Bulk Silicon Micromachining**
  - **Dry and wet etched silicon microstructures**
    - Deep RIE
      (Lucas Novasensor)
    - Dissolved wafer
      (Najafi, et al.)

*Kovacs, Proc. of IEEE, 98*

*Selvakumar, et al., JMM 3/01*
Sampling of MEMS Processes

- Surface Micromachining
  - Thin film microstructures

Multi-level polysilicon
(Howe, et al.)

Fedder et al., IEEE JMEMS 95

CMOS MEMS
(Fedder, et al.)

Lakdawala, et al, JSSC 3/02
Sampling of MEMS Processes

- Micromolding
  - Thin film microstructures
  - Shaped within micromachined mold

HARPSS
(Ayazi, Najafi)

HEXSIL
(Keller, et al.)

IEEE JMEMS 9/00

Madou, Fundamentals of Microfabrication, CRC Press
Why so many kinds of processes?

- **Functionality**
  - Special sensing or actuation properties
  - Meet performance specifications
  - Need nm, μm or mm size
  - Enable packaging

- **Reliability**
  - Environmentally (shock, temperature) tolerant
  - No fracturing, sticking, deformation during lifetime

- **Cost reduction**
  - Improved manufacturing yield
Selected MEMS Applications

- Sensors of numerous modalities
- Accelerometers and gyroscopes
- Pressure sensors
- RF switches, passives, resonant filters
- Read-write heads, AFM/STM probes
- Optical switches, displays
- (Ink) jet heads
- Biochemical labs-on-chip
Mapping Applications Onto Domains

- Sensors of numerous modalities
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modalities:
- electrical
- mechanical
- thermal
- optical
- fluidic
- chemical
- biological
- magnetic
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Mapping Applications Onto Domains

- Sensors of numerous modalities
- Accelerometers and gyroscopes (electrical)
- Pressure sensors (mechanical)
- RF switches, passives, resonant filters (thermal)
- Read-write heads, AFM/STM probes (optical)
- Optical switches, displays (optical)
- (Ink) jet heads (fluidic)
- Biochemical labs-on-chip (chemical, biological, magnetic)
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MEMS Design Representations

Views:
- Solid Model
- Mesh
- Layout
- A-HDL Model
- Schematic

module beam(phim, phip, vm, vp, xm, xp, ym, yp);
  ...
  analog begin
  r = `poly1_rho * (l/w);
  ms = `poly1_den*w*l*poly1_t;
  ii = poly1_t*(w*w*w)/12;
  ...
  I(vp, vm) <+ V(vp, vm)/r ;
  F(xm) <+ -Fx*m*cos_d + Fy*m*sin_d;
  F(ym) <+ -Fy*m*cos_d - Fx*m*sin_d;
  F(xp) <+ -Fx*p*cos_d + Fy*p*sin_d;
  F(yp) <+ -Fy*p*cos_d - Fx*p*sin_d;
  Tau(phim) <+ -Tq_m;
  Tau(phip) <+ -Tq_p;
  end
endmodule
3D Geometric Models

- **Geometry**
  - Mesh: Continuum Simulation
  - Solid Model: Visualization

- **Example Tools**
  - HKS’s ABAQUS
  - ANSYS
  - Coventor’s ANALYZER
  - CFDRC’s CFD-ACE

- **Note:** holes are removed in solid model

- **Simulation speed improved**
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- Note: holes are removed in solid model
- Simulation speed improved
Layout

- Manufacturing blueprint
- Not intrinsically simulatable
- Layout → 3D view tools
  - CFDRC’s MICROMESH
  - Coventor’s DESIGNER
  - MEMSCAP’s XPLORER
Imported from network theory and macro-scale electromechanics

Lumped parameter models

Ordinary differential-algebraic equations

Hand coded

No AHDL automated coding tools for MEMS - yet

```vhd
module resonator(vin);
  ...
  parameter real  K = 1 ;
  parameter real  B = 1e-7 ;
  ...
  analog begin
    Pos(Vtop) <+  ddt(Pos(top));
    Pos(Atop) <+  ddt(Pos(Vtop));
    Fe = (V(vin)*V(vin))*area*eps0/2.0/
      ((z0-Pos(top))*(z0-Pos(top)));
    F(top) <+  Fe -
      (K*Pos(top) + ms*Pos(Atop) +
       B*Pos(Vtop));
  end
endmodule
```
MEMS Circuit Schematic

- Schematic composition of behavioral models
- Tools:
  - Coventor’s ARCHITECT
  - MEMSCAP’s MEMSPRO
  - UC Berkeley’s SUGAR
  - Carnegie Mellon’s NODAS
- Components:
  - Mechanical
  - Electromechanical
  - Fluidic (ARCHITECT)
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Choice of Mechanical Discipline

- Flow = Force in all cases

- Choice #1
  - Potential = Velocity
  - Potential × Flow = Velocity × Force = Power
  - Displacement must be extracted

- Choice #2
  - Potential = Position
  - Potential × Flow = Position × Force is not meaningful

- Choice #3
  - Potential = Displacement
  - Potential × Flow = Displacement × Force = Energy

\[ x = \int_{0}^{t} v \, dx \]
Position vs. Displacement

- Positions are non-zero at rest (i.e., layout)
- Displacements can be smaller than $10^{-6} \times$ position
- Unnecessary offset may lead to numerical error
Inertial and Chip Frames of Reference

- Accelerometer example
- Position in chip frame is large relative to displacement
- Position in inertial frame becomes extremely large!

![Diagram showing inertial and chip frames of reference with distances labeled in micrometers and nanometers.](image)
Displacement, Chip Frame of Reference

- Layout location is natural reference for displacement
- Displacement = 0 at rest
- Displacements remain small signal
MEMS Translational Discipline

- Current *kinematic* conservative discipline
  - Across: Position
  - Through: Force

- New *kinematic_translational* conservative discipline:
  - Across: Displacement
  - Through: Force

```
  nature Displacement
  units = "m";
  access = Disp;
  ddt_nature = Velocity;
  abstol = 1e-12;
  endnature

  nature Force
  units = "N";
  access = F;
  abstol = 1e-12;
  endnature

  discipline kinematic_translational
  potential Displacement;
  flow Force;
  enddiscipline
```
MEMS Rotational Discipline

- Current rotational conservative discipline
  - Across: Angle
  - Through: Angular_Force

- New kinematic rotational conservative discipline:
  - Across: Angular_Displacement
  - Through: Angular_Force

```plaintext
nature Angular_Displacement
  units = "rad";
  access = Phi;
  ddt_nature = Angular_Velocity;
  abstol = 1e-7;
endnature

nature Angular_Force
  units = "N-m";
  access = Tau;
  abstol = 1e-16;
endnature

discipline kinematic_rotational
  potential Angular_Displacement;
  flow Angular_Force;
enddiscipline
```
MEMS Velocity Discipline

- velocity and angular_velocity signal-flow disciplines
- Needed to store kinematic states

```plaintext
nature Velocity
  units = "m/s";
  access = Vel;
  ddt_nature = Acceleration;
  idt_nature = Displacement;
  abstol = 1e-9;
endnature
discipline velocity
  potential Velocity;
enddiscipline

nature Angular_Velocity
  units = "rad/s";
  access = Omega;
  ddt_nature = Angular_Acceleration;
  idt_nature = Angular_Velocity;
  abstol = 1.0;
endnature
discipline angular_velocity
  potential Angular_Velocity;
enddiscipline
```
Abstol Settings for MEMS

- My rule of thumb:
  - Estimate “typical” forces and displacements
  - Set abstol at least to $10^{-3} \times \text{<typical value>}$
Comb Drive Force Estimate

\[ F_{e,x} = \frac{1}{2} \frac{dC}{dx} V^2 \approx N \frac{\varepsilon_o h_o}{g_o} V^2 \]

- Number of comb fingers = \( N = 6 \)
- Finger thickness = \( h_o = 2 \, \mu m \)
- Finger gap = \( g_o = 2 \, \mu m \)
- Voltage across fingers = \( V = 10 \, V \)

\[ F_{e,x} \approx 5.3 \, \text{nN} \]

\[ \text{abstol} \approx 1 \, \text{pN} \]
Cantilever Displacement Estimate

\[ y(L) = \frac{FL^3}{3EI_x} = \frac{4FL^3}{Ehw^3} \]

- Apply comb force, \( F = 5.3 \) nN
- Young’s modulus, \( E = 165 \) GPa

\[ y(L) = 8 \text{ nm} \]

\[ \text{abstol} \approx 1 \text{ pm} \]
Cantilever Angular Displacement Estimate

Apply comb force, $F = 5.3 \text{ nN}$

Young’s modulus, $E = 165 \text{ GPa}$

\[
\theta(L) = \frac{FL^2}{2EI_x} = \frac{6FL^2}{Ehw^3}
\]

\[
\theta(L) = 0.12 \text{ mrad}
\]

$\text{abstol} \approx 0.1 \mu\text{rad}$
Resonator Velocity Estimate

$L_b = 100 \mu m$
$L_t = 10 \mu m$
$w = 10 \mu m$

- $E = 165 \text{ GPa}$
- $h = 2 \mu m$
- $Q = 10$
- Apply comb force, $F = 5.3 \text{ nN}$

$k_x = 2Eh \left( \frac{w_b}{L_b} \right)^3 \frac{\tilde{L}_t^2 + 14\tilde{L}_t + 36}{4\tilde{L}_t^2 + 41\tilde{L}_t + 36}$; \quad \tilde{L}_t = \frac{L_t}{L_b}$

$\omega_x \approx \sqrt{\frac{k_x}{m}} \quad v_x \approx \omega_x Q x$

$k_x = 4.9 \text{ N/m}$
$f_x = 52 \text{ kHz}$
$x = 11 \text{ nm}$

$V_x = 3.5 \text{ mm/s}$

abstol $\approx 1 \mu m/s$
Crab-Leg Angular Force (Moment) Estimate

\[ F = 5.3 \text{ nN} \]

\[ M = 0.53 \text{ pN-m} \]

\[ L = 100 \mu m \]

Apply comb force, \( F = 5.3 \text{ nN} \)

\[ M = 0.53 \text{ pN-m} \]

\[ \text{abstol} \approx 0.1 \text{ fN-m} \]
Angular Velocity Estimate

- Static angular displacement \( \theta(L) = 0.12 \text{ mrad} \)
- Resonance = \( f_r = 276 \text{ kHz} \)
- Quality factor = \( Q = 45 \)
- Resonant angular displacement \( \theta(L) = 5.43 \text{ mrad} \)

\[ \omega_r = 1.03 \frac{w}{L^2} \sqrt{\frac{E}{\rho}} \]

\[ \frac{d\theta}{dt}(L) = 9.4 \text{ krad/s} \]

\( abstol \approx 1 \text{ rad/s} \)
MEMS Abstol – Summary of Examples

Much lower than in macro-scale mechanics
(Except angular velocity)

<table>
<thead>
<tr>
<th>Nature</th>
<th>Abstol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force</td>
<td>$10^{-12}$ N</td>
</tr>
<tr>
<td>Displacement</td>
<td>$10^{-12}$ m</td>
</tr>
<tr>
<td>Velocity</td>
<td>$10^{-6}$ m/s</td>
</tr>
<tr>
<td>Angular_Force</td>
<td>$10^{-16}$ N-m</td>
</tr>
<tr>
<td>Angular_Displacement</td>
<td>$10^{-7}$ rad</td>
</tr>
<tr>
<td>Angular_Velocity</td>
<td>$10^0$ rad/s</td>
</tr>
</tbody>
</table>
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MEMS Design Hierarchy

MEMS Behavioral Model Hierarchy

- System models
  - Increasing complexity
  - Decreasing reusability
  - Increasing design cycle time

- Block models
- Circuit models
- Atomic element models

Model Degrees of Freedom

■ 1-D, 1 DOF
  ■ Simplest to macromodel
  ■ mass-spring-damper system
■ Semi-analytic 2-D or 3-D
  ■ Assume lumped masses
  ■ 3 DOF (2-D) or 6 DOF (3-D) per mass
■ Lowest N modes
  ■ Extracted from numerical analysis
  ■ True modal decomposition requires self-consistent electromechanical simulation
■ DOF through structural modeling
Danger of Degrees of Freedom

- Choice of DOFs is dependent on
  - stimulus location
  - stimulus frequency

- Recommendation in MEMS community:
  - Always model to the lowest level possible (i.e. tolerable)

Example folded-flexure resonator:

- comb fingers can resonate!
  \( w = 1 \, \mu m \)
  \( L = 50 \, \mu m \)
  \( f_r = 550 \, kHz \)
Selected MEMS Flexures

- **Fixed-fixed flexure**
  - stiff in y
  - nonlinear stiffening

- **Crab-leg flexure**
  - less stiff in y
  - more linear
  - uses up area

- **Folded flexure**
  - more linear
  - area efficient

- **Serpentine flexure**
  - set stiffness in x, y
  - linear
  - area efficient
1-D Crab-Leg Resonator

Equation of motion:

\[ F_{\text{ext}} = m \ddot{y} + B \dot{y} + ky \]

\[ m \approx \rho h \left( L_p^2 + 4wL_a \right) \]

\[ B = \frac{\mu_{\text{air}} L_p^2}{d_{\text{eff}}} \]

\[ k = Eh \left( \frac{w}{L_b} \right)^3 \frac{4L_b + L_a}{(L_b + L_a)} \]
module crableg1D (dy);
inout dy;
kinematic_translational dy;
velocity vy;
parameter real La = 1e-5;
parameter real Lb = 1e-4;
real density = 2330, E = 165e9, viscosity = 1.79e-5, deff = 2e-6;
real h = 2e-6, w = 2e-6, Lp = 1e5;
real m, b, k;
analog begin
  m = density * h * (Lp^2 + 4 * w * La);
b = viscosity * Lp^2 / deff;
k = E * h * (w / Lb)^3 * (4 * Lb + La) / (Lb + La);
Vel(vy) <+ ddt(Disp(dy));
F(dy) <+ m * ddt(Vel(vy)) + b * Vel(vy) + k * Disp(dy);
end
endmodule

\[ F_{ext} = m\ddot{y} + B\ddot{y} + ky \]
1-D Parameterized Behavioral Model

module crableg1D (dy);
inout dy;
kineumatic_translational dy;
velocity vy;
parameter real La = 1e-5;
parameter real Lb = 1e-4;
real density = 2330, E = 165e9, viscosity = 1.79e-5, deff = 2e-6;
real h = 2e-6, w = 2e-6, Lp = 1e5;
real m, b, k;
analog begin
  m = density*h*(Lp^2+4*w*La);
  b = viscosity*Lp^2/deff;
  k = E*h*(w/Lb)^3*(4*Lb+La)/(Lb+La);
  Vel(vy) <+ ddt(Disp(dy));
  F(dy) <+ m*ddt(Vel(vy)) + b*Vel(vy) + k*Disp(dy);
end
endmodule
Functional-Level 2-D Models

- Divide into smaller functional units
- Easier to analytically model each unit
- Greater DOFs

\[ \begin{align*}
\text{crableg\_spring2D} & : d_{xa}, d_{ya}, \phi_a \\
\text{plate\_2D} & : d_{xb}, d_{yb}, \phi_b \\
\text{anchor} & : d_{ul}, d_{ur}, \phi_{ul}, \phi_{ur}, \phi_{ll}, \phi_{lr}
\end{align*} \]
Chip and Local Frames of Reference

- External ports refer to chip frame
  - Direction of port is invariant with rotation
- Internal calculations refer to local frame
  - Must transform port in/out variables

\[ \text{rotation} = 0^\circ \]

\[ \text{rotation} = 90^\circ \]
Euler rotation angles specify layout position.

- \( \gamma \) about z-axis, then \( \beta \) about y'-axis, last \( \alpha \) about x''-axis.

\[
\begin{align*}
\gamma &= 0^\circ \\
\beta &= 0^\circ \\
\alpha &= 0^\circ \\
\gamma &= 90^\circ \\
\beta &= 0^\circ \\
\alpha &= 0^\circ \\
\gamma &= 0^\circ \\
\beta &= 0^\circ \\
\alpha &= 180^\circ \\
\gamma &= 0^\circ \\
\beta &= 180^\circ \\
\alpha &= 0^\circ
\end{align*}
\]
Can build any combination of crab-leg and plate

- $\gamma = 0^\circ$
- $\beta = 0^\circ$
- $\alpha = 0^\circ$

- $\gamma = 180^\circ$
- $\beta = 0^\circ$
- $\alpha = 0^\circ$

- $\gamma = 0^\circ$
- $\beta = 180^\circ$
- $\alpha = 0^\circ$

- $\gamma = -90^\circ$
- $\beta = 0^\circ$
- $\alpha = 180^\circ$

- $\gamma = 90^\circ$
- $\beta = 0^\circ$
- $\alpha = 180^\circ$
2-D Parameterized Behavioral Crab-Leg Model

module crableg_spring2D (dx, dy, phi_za, dx_b, dy_b, phi_zb);
    inout dx, dy, phi_za, dx_b, dy_b, phi_zb;
    kinematic_translational dx, dy, dx_b, dy_b;
    kinematic_rotational phi_za, phi_zb;

    parameter real alpha = 0.;
    parameter real beta = 0.;
    parameter real gamma = 0.;
    parameter real La = 1e-5;
    parameter real Lb = 1e-4;
    analog begin
        ...
        cos_a = cos(alpha*‘M_PI/180.);
        cos_b = cos(beta*‘M_PI/180.);
        cos_g = cos(gamma*‘M_PI/180.);
        ...
        dx_l = cos_b*cos_g*(Disp(dx_b)-Disp(dx)) + cos_b*sin_g*(Disp(dy_b)-Disp(dy));
        dy_l = -cos_a*sin_g*(Disp(dx_b)-Disp(dx)) + cos_a*cos_g*(Disp(dy_b)-Disp(dy));
        dphi = Phi(phi_zb) - Phi(phi_za);

        Convert to radians
        Transform displacements from chip frame to local frame
2-D Parameterized Behavioral Crab-Leg Model

//Calculate spring forces and moments
Fkx_l = 3/(La+Lb)*((4*La+Lb)/La^3*dx_l-3/La/Lb*dy_l-(2*La+Lb)/La^2*dphi);
Fky_l = 3/(La+Lb)*(-3/La/Lb*dx_l+(La+4*Lb)Lb^3*dy_l+3/b*dphi);
Mkz_b = 1/(La+Lb)*(-3*(2*La+Lb)/La^2*dx_l+3/Lb*dy_l+(4*La+3*Lb)/La*dphi);
...
Fx_bl = Fmx_bl+Fbx_bl+Fkx_l; // Fm are inertial forces
Fy_bl = Fmy_bl+Fby_bl+Fky_l; // Fb are damping forces
Fx_al = Fmx_al+Fbx_al-Fkx_l;
Fy_al = Fmy_al+Fby_al-Fky_l;

F(dxb) <+ cos_b*cos_g*Fx_bl - cos_a*sin_g*Fy_bl;
F(dyb) <+ cos_b*sin_g*Fx_bl + cos_a*cos_g*Fy_bl;
F(dxa) <+ cos_b*cos_g*Fx_al - cos_a*sin_g*Fy_al;
F(dya) <+ cos_b*sin_g*Fx_al + cos_a*cos_g*Fy_al;

Tau(phi_zb) <+ Mm_b + Mb_b + Mkz_b;
Tau(phi_za) <+ Mm_a + Mb_a - Mkz_b - La*Fkx_l - Lb*Fky_l;
end
endmodule

Transform forces from local frame to chip frame
“Atomic” Elements

Circuit representations of suspended MEMS can be partitioned into four basic lumped-parameter elements: plates, beams, gaps, and anchors.

- **PLATE**
  - L: 50u
  - w: 100u
  - angle: 0

- **BEAM**
  - L: 100u
  - w: 4u
  - angle: 0

- **GAP**
  - g: 4u
  - Lo: 25u
  - angle: 0

- **ANCHOR**
Multi-Level Design Reuse

- Elements (symbols and models) can be reused in new designs
- Low-level elements are:
  - Anchor
  - Beam
  - Plate
  - Gap
  - Comb
Automated layout is hierarchically p-cell (parameterized cell) driven directly from elements.
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Port Interoperability

- Essential for composition
  - Atomic-level, functional-level schematic components

Three items must match at ports for meaningful interconnection:
1. Disciplines must match
   - choose displacements, forces
2. Reference frames must match
   - choose chip frame of reference
3. Associated reference directions must match
   - ?
Associated Reference Directions

- **Electrical:**
  - positive valued flow $\rightarrow$ positive charge flow
  - positive valued potential $\rightarrow$ positive voltage

- **What is convention for mechanical disciplines?**
  - choice is arbitrary, but
  - interpretation can be frustrating
CMU Mechanical Nodal Conventions

- **Across variables** \((x, y, \theta_z)\)
  - Positive valued displacements are in positive axial direction
  - Positive valued angles are counterclockwise around axis

- **Through variables** \((F_x, F_y, M_z)\)
  - Positive force flowing into node acts in positive axial direction
  - Positive moment flowing into node acts counterclockwise around axis
Beam in Tension

$F_{x,a}$ is negative; $F_{x,b}$ is positive

$x_a$ is negative; $x_b$ is positive

Equivalent schematic:
Beam in Compression

$F_{x,a}$ is negative; $F_{x,b}$ is positive

Equivalent schematic:

$F_{x,a}$  
$F_{x,b}$
Moving Beam

$F_{x,a}$ and $F_{x,b}$ are positive
$x_a$ and $x_b$ are positive

Equivalent schematic:
Rotated Beam

$y_a$ is negative; $y_b$ is positive

and both $\theta_a$ and $\theta_b$ are positive

Equivalent schematic:
Pure Bending

Equivalent schematic:
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Bus Terminals

- Individual pins

- Bus terminals
Linear Beam Model

- Captures linear beam mechanics, including:
  - Axial
  - Lateral
  - Torsional

- 12-DOF symmetric composable model

- Model assumptions
  - Small displacements
  - Axial, lateral and torsional motions are independent

\[
[F] = [m][\ddot{x}] + [B][\dot{\dot{x}}] + [k][\dot{x}]
\]

- Mass matrix
- Damping matrix
- Stiffness matrix
Nonlinear Beam Effect I: Large Geometric Deflection

- Example: Cantilever beam
- Beam foreshortening, $x$ and $y$ are coupled
- Force projection into axial stress
- Not symmetric formulation

$$y(x) = f_1(x)y_a + f_2(x)\phi_a + f_3(x)y_b + f_4(x)\phi_b$$

$f_i(x)$: cubic shape functions for small displacements
Coordinate Transformation

- **Chip frame:** specifies layout position
- **Local frame:** specific to each element
- **Displaced frame:** shape functions are applied

![Diagram of coordinate transformation]
Nonlinear Beam Effect II: Large Axial Stress Stiffening

- Example: Fixed-fixed beam
- Nonlinearity starts at small displacement
- Effective beam length, $L'$
- Axial force, $N$

![Graphs showing linear and nonlinear relationships between $y$-displacement and axial force](image-url)
Calculation of Axial Stiffening

- Use small angle shape function
- Refer to displaced frame
- Integrate shape to find displaced length

\[ y(x) = f_1(x)y_a + f_2(x)\phi_a + f_3(x)y_b + f_4(x)\phi_b \]

\[ L' = \int ds = \int_{x_a}^{L+x_b-x_a} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \, dx \]

\[ \delta L = L' - L \]
Geometric Stiffness Matrix

Linear stiffness matrix

\[
[F] = [K_0 + K_G(x)] [x]
\]

Geometric stiffness matrix

\[
N(x) \cdot [K_{G0}] = \frac{EA}{L} (x_a - x_b)
\]

**Ref:** Przemieniecki, Theory of Matrix Structural Analysis, 1968
Nonlinear Beam Code

```plaintext
rphix = (l_phixa+l_phixb)/2.0;
rx = rphix/rphi;
new_l1 = rt*rx*rx + rc;
...
new_l_xb = new_l1*(l_xb+l) + new_m1*l_yb + new_n1*l_zb - l;
...
lp = l+new_l_xb-new_l_xa;
l_eff = (lp*(15*pow(l_new,6)*(2+pow(new_l_phiya,2))... // calculate the axial force
F_axial = ea*(l_eff-l)/l;
// linear stiffness matrix [k0], with shear deformation
// variables for shear deformation
Asy = 2.0/3.0*area; //effective shear area
...
k0_1_1 = ea/l;
k0_2_2 = 12.0*E*Iz/pow(l,3);
...
k1_1_1 = 0;
k1_2_2 = F_axial*2.0*(6+10*Sy+5*pow(Sy,2))/(10*l*pow(1+Sy,2)); matrix
...
k_1_1 = k0_1_1 + k1_1_1;
k_2_2 = k0_2_2 + k1_2_2;
...
new_Fkxa = -F_axial;
new_Fkxb = F_axial;
new_Fkya = k_2_1*new_l_xa+k_2_2*new_l_ya+k_3_2*new_l_za...
...
Fkxa=inv_new_l1*new_Fkxa+inv_new_m1*new_Fkya+inv_new_n1*new_Fkza;
```

// dynamic rotation matrix from translated local frame to the displaced frame
// transform displacements into the displaced frame
// calculate effective beam length in the displaced frame
// geometric nonlinear stiffness matrix [kG], with shear deformation
// calculate the spring forces in the displaced frame
// transform spring forces from the displaced frame back to the local frame
Nonlinear Beam Model: Verification I

- Static analysis of a cantilever beam
- Compared to Elastica

$L = 100 \mu m$
$w = 2 \mu m$
$t = 2 \mu m$
$E = 165 \text{ GPa}$

![Diagram of a cantilever beam with normalized displacement and force plots showing comparison to Elastica with different numbers of elements.]
Nonlinear Beam Model: Verification II

- Static analysis of a fixed-fixed beam
- Compared to ABAQUS

$L = 100 \, \mu m$

$w = 2 \, \mu m$

$t = 2 \, \mu m$

$E = 165 \, \text{GPa}$
Issues in MEMS Macromodeling

- Processes and Applications
- Design Views
- Natures and Disciplines
- Hierarchy
- Interoperability
- Beam Element Example
- Filter Example
- Language Extension
Design Example: Third order Bandpass Filter

- Topology due to Wang & Nguyen (MEMS ’97)
- More complex than just a single resonator
  - Multi-domain interactions
  - Multiple functional devices
  - Coupling is crucial to the performance
Design Example:
Micromechanical Bandpass Filter

- Cascade of 3 coupled microresonators
MEMS Bandpass Filter

driving resonator  coupling resonator  sensing resonator

Vbias  coupling beam  Vbias

coupling beam  Vbias

coupling beam  Vbias  V_out

Q-adjust resistor

frequency tuning combs

V_in  V_tuning  Vbias

Q-adjustment resistor
Bandpass Filter Verification

- **NODAS MEMS Circuit**

- **Equivalent Linear SPICE Circuit**

Ref: Q. Jing et al., IEEE MEMS 2000
Parasitics in CMOS MEMS

- **Mechanical parasitics**
  - Parasitic mass
  - Holes
  - Routing metal
  - Parasitic joints
  - Varying beam widths
  - Similar length of beam and joints

- **Electrical parasitics**
  - Metal to substrate gap

- **Holes**
  - m3 = 57%
  - m2 = 34.2%
  - m1 = 35.9%

- **Routing metal**
  - w1 ≈ w2
  - w1 << w2

- **Parasitic joints**
  - w1 ≈ w2

- **Varying beam widths**

- **Similar length of beam and joints**

- **Additional substrate gap in MEMS areas**
Verification by Layout Extraction

Baidya, Gupta & Mukherjee, IEEE JMEMS, Feb 2002
Verification by Layout Extraction

Experimental results match extracted schematic

Baidya, Gupta & Mukherjee, IEEE JMEMS, Feb 2002
Issues in MEMS Macromodeling

- Processes and Applications
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- Interoperability
- Beam Element Example
- Filter Example
- Language Extension
Proposed Composite Discipline

multidiscipline mems2D
  discipline dx : kinematic_translational
    potential.access = "Dx";
    flow.access = "Fx";
  enddiscipline
  discipline dy : kinematic_translational
    potential.access = "Dy";
    flow.access = "Fy";
  enddiscipline
  discipline phi : kinematic_rotational
  enddiscipline
endmultidiscipline
Proposed Composite Discipline

multidiscipline mems2D
  discipline dx : kinematic_translational
    potential.access = "Dx";
    flow.access = "Fx";
  enddiscipline
  discipline dy : kinematic_translational
    potential.access = "Dy";
    flow.access = "Fy";
  enddiscipline
  discipline phi : kinematic_rotational
enddiscipline
endmultidiscipline
Desire equations that are concise, easy to read

Syntax looks like standard disciplines

Could use matrix notation...

```plaintext
inout a, b;
mems2D a, b;
...
real kxx, kyy, ktaa, ktab, ktbb, kxy, kxt, kyt;
analog begin
...
Fx(a, b) <+ kxx*Dx(a,b) + kxy*Dy(a,b) + kxt*Phi(a,b);
Fy(a, b) <+ kxy*Dx(a,b) + kyy*Dy(a,b) + kyt*Phi(a,b);
Tau(a) <+ kxt*Dx(a,b) + kyt*Dy(a,b) + ktaa*Phi(a) + ktab*Phi(b);
Tau(b) <+ kxt*Dx(a,b) + kyt*Dy(a,b) + ktab*Phi(a) + ktbb*Phi(b);
end
```
Conclusions

- MEMS behavioral modeling with A-HDLs is slowly being adopted
- Still far from the preferred starting point in design

- Accurate behavioral models exist for mechanics

- Near-term Work:
  - General electrostatic gap
  - Thermomechanics, electrothermal, piezo-effects

- Atomic-level models in other domains way behind

- Open standard cell library is needed
  - Need standard conventions adopted for interoperability
  - Provides templates for behavioral modeling
  - Promotes structured modeling
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