Issues in MEMS Macromodeling

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Embedded Tutorial

BMAS Workshop
October 8, 2003

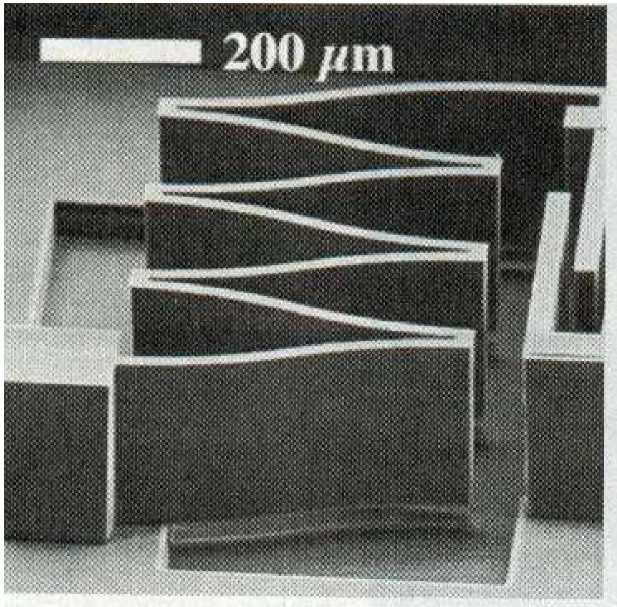
Issues in MEMS Macromodeling

- Processes and Applications
- Design Views
- Natures and Disciplines
- Hierarchy
- Interoperability
- Beam Element Example
- Filter Example
- Language Extension

Sampling of MEMS Processes

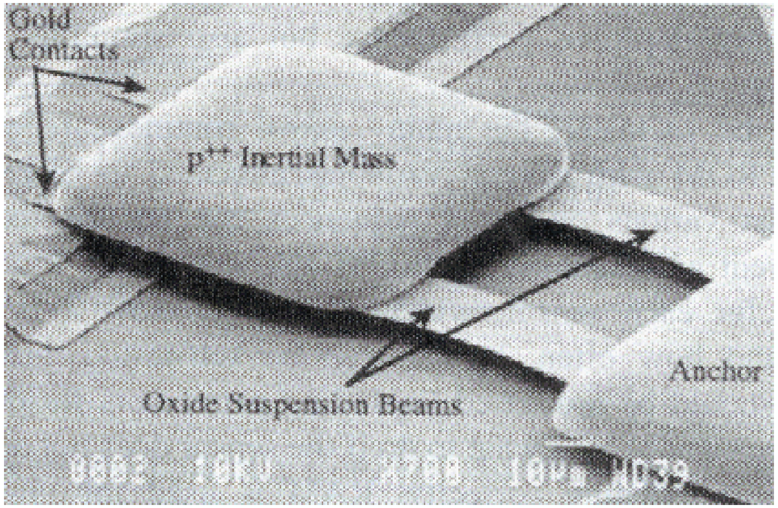
- **Bulk Silicon Micromachining**
  - **Dry and wet etched silicon microstructures**

Deep RIE
(Lucas Novasensor)



Kovacs, Proc. of IEEE, 98

Dissolved wafer
(Najafi, *et al.*)

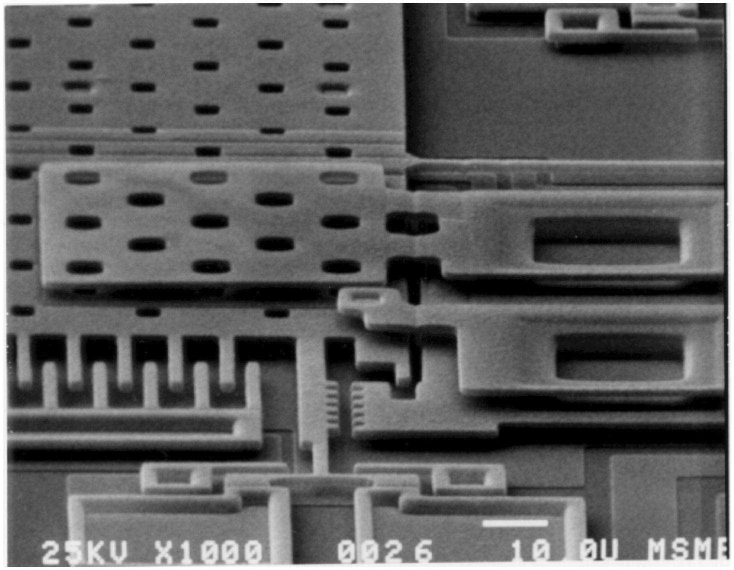


Selvakumar, et al., JMM 3/01

Sampling of MEMS Processes

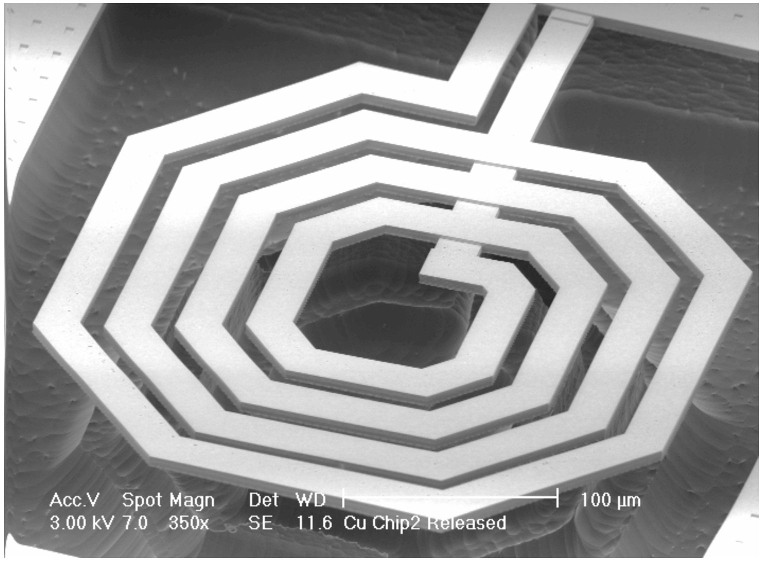
- **Surface Micromachining**
  - **Thin film microstructures**

Multi-level polysilicon (Howe, *et al.*)



Fedderet al., IEEE JMEMS 95

CMOS MEMS (Fedder, *et al.*)

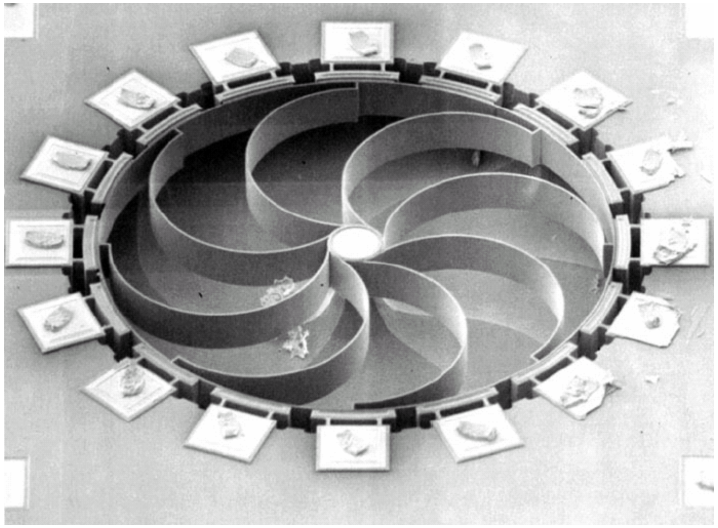


Lakdawala, et al, JSSC 3/02

Sampling of MEMS Processes

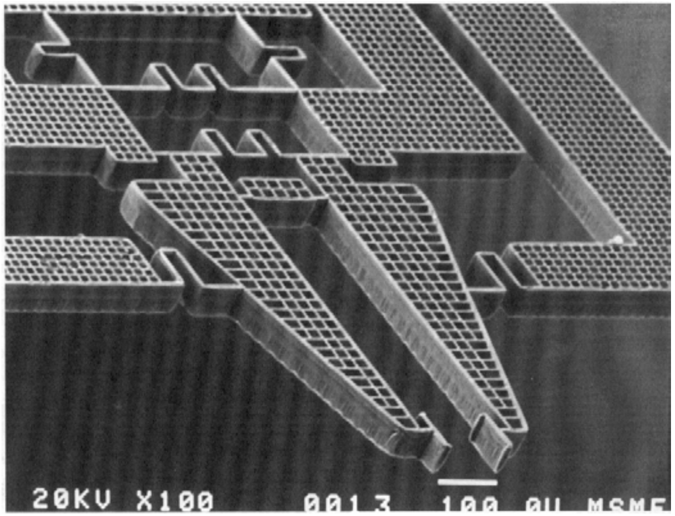
- Micromolding
  - Thin film microstrutcures
  - Shaped within micromachined mold

HARPSS
(Ayazi, Najafi)



IEEE JMEMS 9/00

HEXSIL
(Keller, *et al.*)



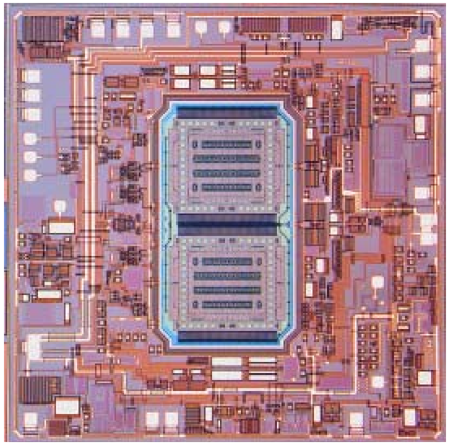
Madou, Fundamentals of Microfabrication, CRC Press

Why so many kinds of processes?

- Functionality
  - Special sensing or actuation properties
  - Meet performance specifications
  - Need nm, µm or mm size
  - Enable packaging
- Reliability
  - Environmentally (shock, temperature) tolerant
  - No fracturing, sticking, deformation during lifetime
- Cost reduction
  - Improved manufacturing yield

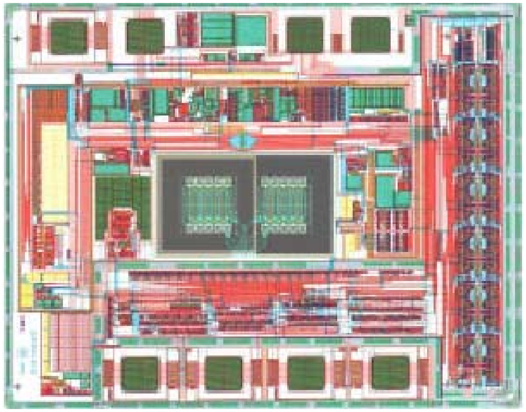
Selected MEMS Applications

- Sensors of numerous modalities
- Accelerometers and gyroscopes
- Pressure sensors
- RF switches, passives, resonant filters
- Read-write heads, AFM/STM probes
- Optical switches, displays
- (Ink) jet heads
- Biochemical labs-on-chip



Analog Devices Inc. Gyroscope

Texas Instruments Digital Micromirror Device



Motorola Tire Pressure Monitor

Mapping Applications Onto Domains

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electrical

mechanical

thermal

optical

fluidic

chemical

biological

magnetic

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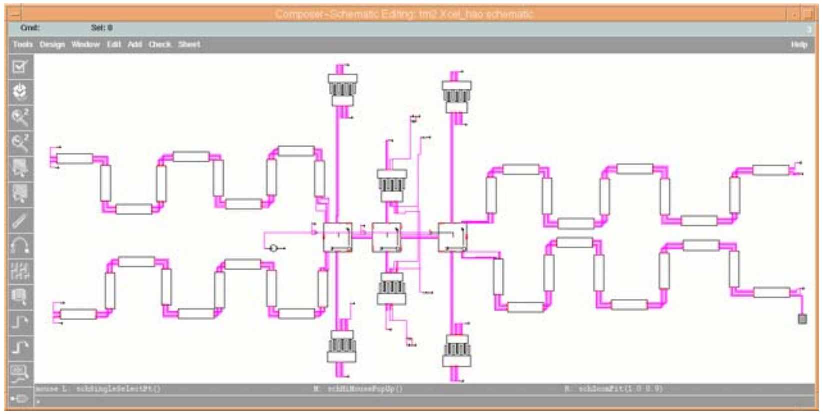
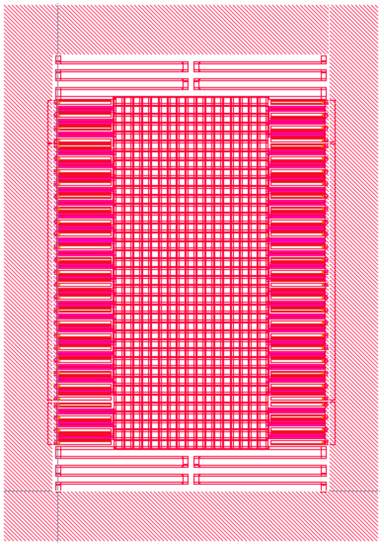
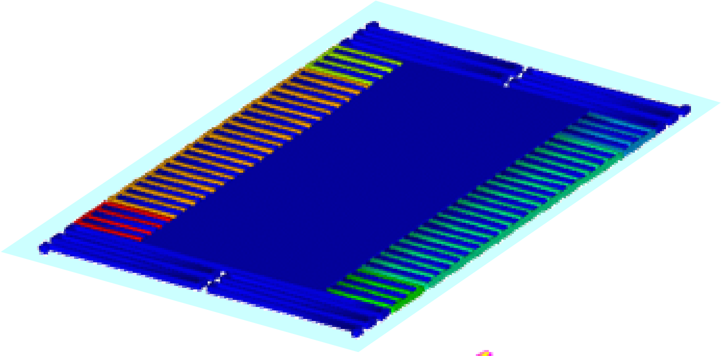
MEMS Design Representations

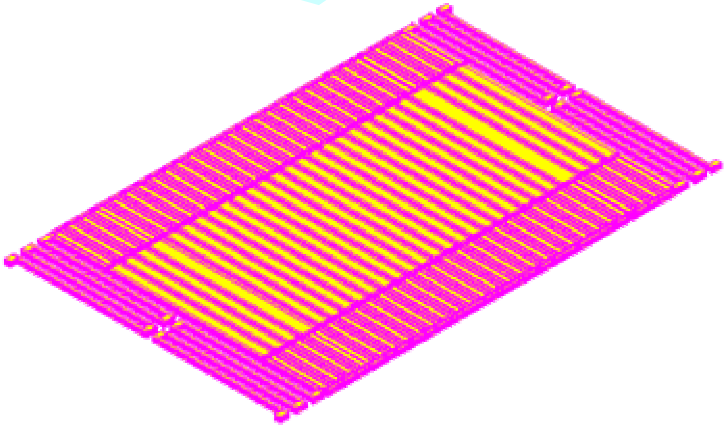
Views:

- Solid Model
- Mesh
- Layout
- A-HDL Model
- Schematic





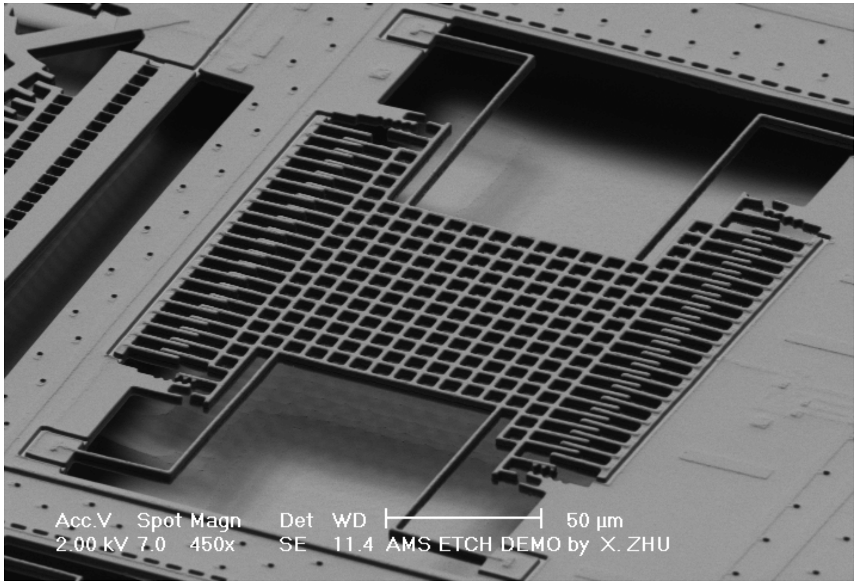




```
module beam(phim, phip, vm, vp, xm,
xp, ym, yp);
…
analog begin
r = `poly1_rho * (l/w);
ms = `poly1_den*w*l*poly1_t;
ii = poly1_t*(w*w*w)/12;
…
I(vp, vm) <+ V(vp, vm)/r ;
F(xm) <+ -Fxdm*cos_dc + Fydm*sin_dc;
F(ym) <+ -Fydm*cos_dc - Fxdm*sin_dc;
F(xp) <+ -Fxdp*cos_dc + Fydp*sin_dc;
F(yp) <+ -Fydp*cos_dc - Fxdp*sin_dc;
Tau(phim) <+ -Tq_m;
Tau(phip) <+ -Tq_p;
end
endmodule
```

3D Geometric Models

- Geometry
  - Mesh: Continuum Simulation
  - Solid Model: Visualization
- Example Tools
  - HKS's ABAQUS
  - ANSYS
  - Coventor's ANALYZER
  - CFDRC's CFD-ACE



- Note: holes are removed in solid model
- Simulation speed improved

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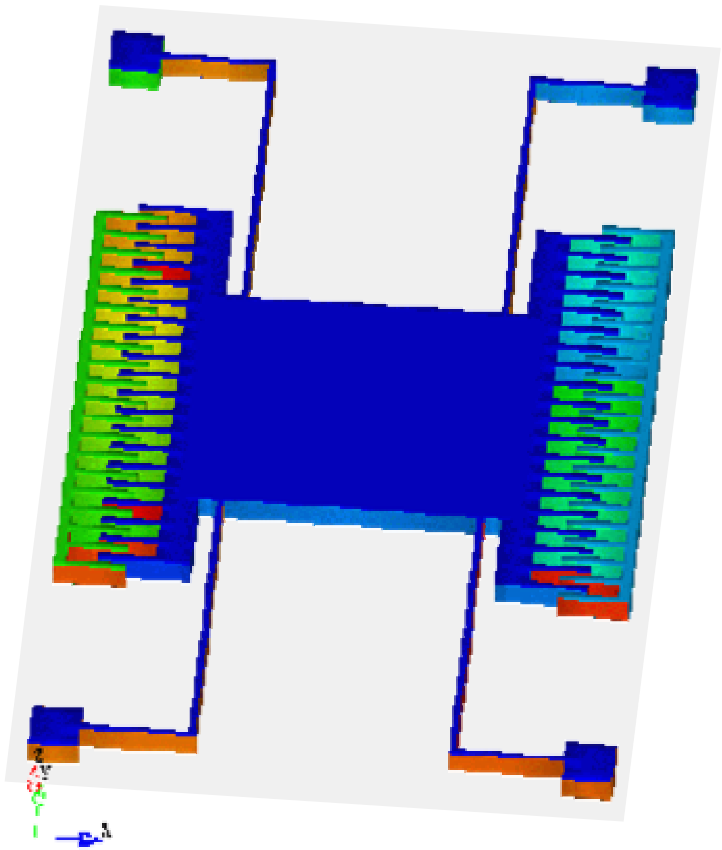
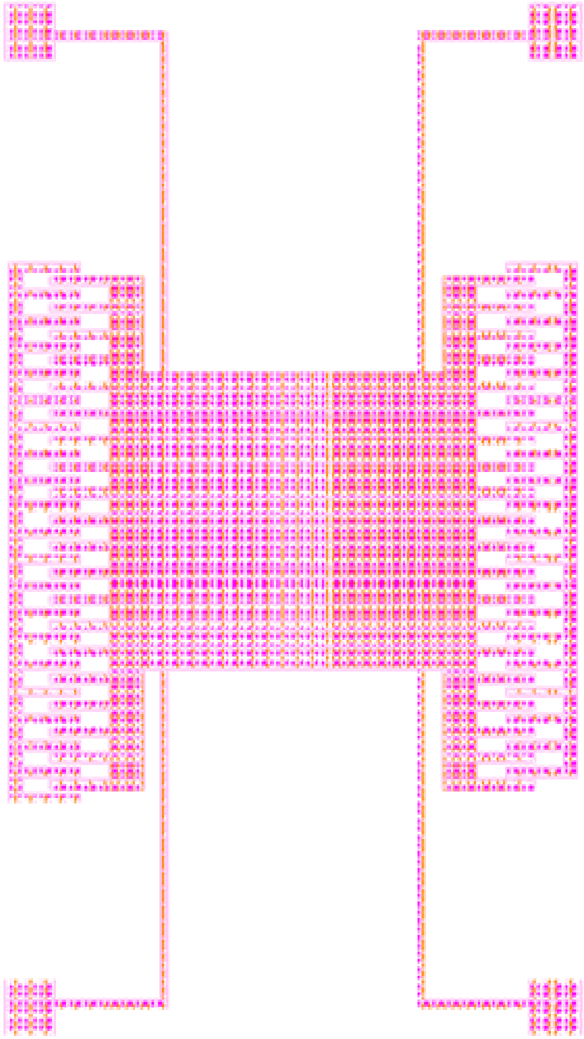


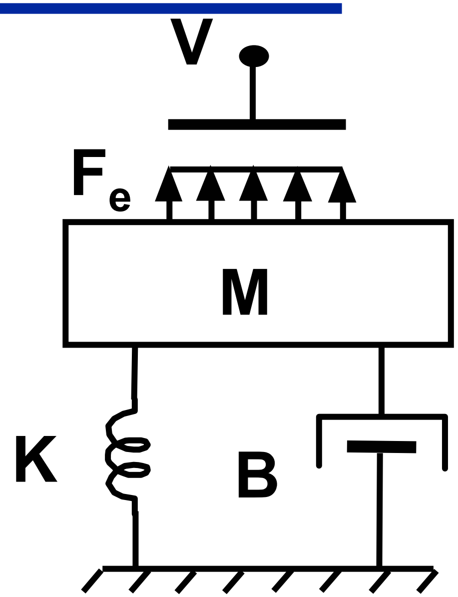
Layout

- Manufacturing blueprint
- Not intrinsically simulatable
- Layout $\rightarrow$ 3D view tools
  - CFDRC's MICROMESH
  - Coventor's DESIGNER
  - MEMSCAP's XPLORER



Behavioral Model – Symbol and Code

- Imported from network theory and macro-scale electromechanics
- Lumped parameter models
- Ordinary differential-algebraic equations



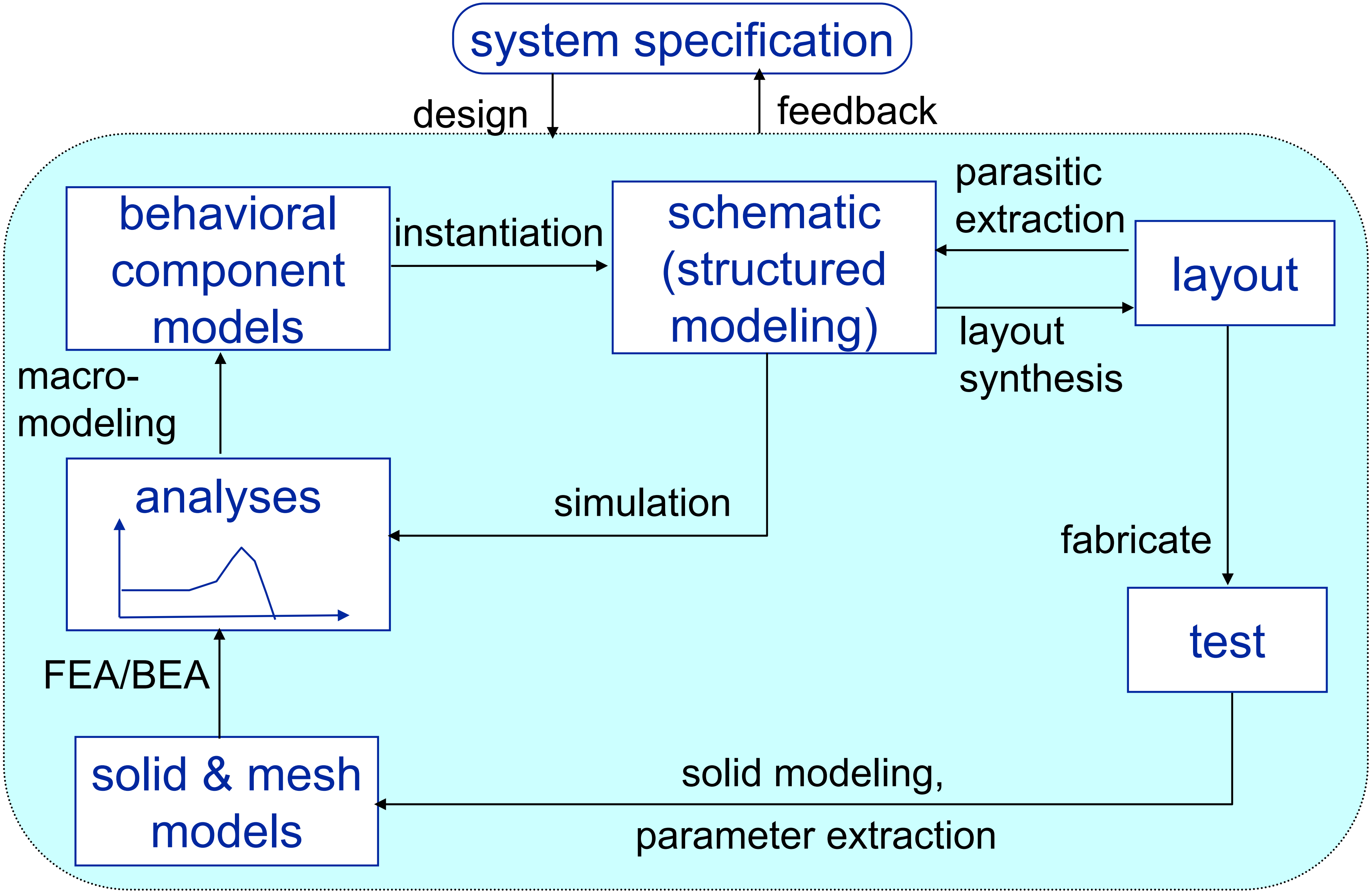
- Hand coded
- No AHDL automated coding tools for MEMS - yet

```
module resonator(vin);
…
parameter real  K = 1 ;
parameter real  B = 1e-7 ;
…
analog begin

Pos(Vtop) <+  ddt(Pos(top));
Pos(Atop) <+  ddt(Pos(Vtop));
Fe = (V(vin)*V(vin))*area*`eps0/2.0/
      ((z0-Pos(top))*(z0-Pos(top)));
F(top) <+ Fe -
   (K*Pos(top) + ms*Pos(Atop) +
B*Pos(Vtop));

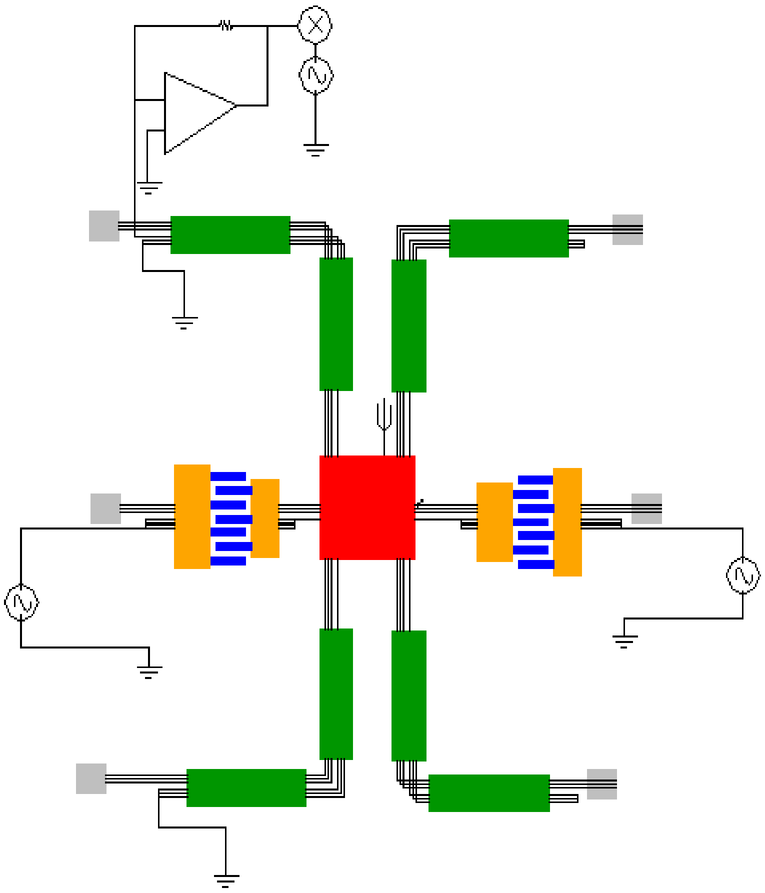
end
endmodule
```

MEMS Design Flow



MEMS Circuit Schematic

- Schematic composition of behavioral models
- Tools:
  - Coventor's ARCHITECT
  - MEMSCAP's MEMSPro
  - UC Berkeley's SUGAR
  - Carnegie Mellon's NODAS



- Components:
  - Mechanical
  - Electromechanical
  - Fluidic (ARCHITECT)

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Choice of Mechanical Discipline

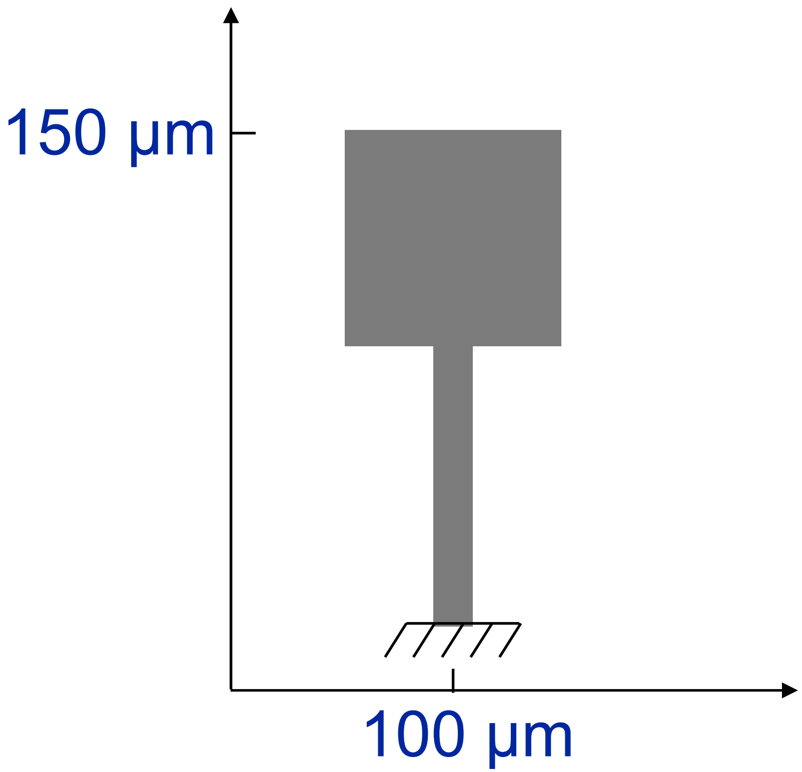
- Flow = Force in all cases
- Choice #1
  - Potential = Velocity
  - Potential × Flow = Velocity × Force = Power
  - Displacement must be extracted

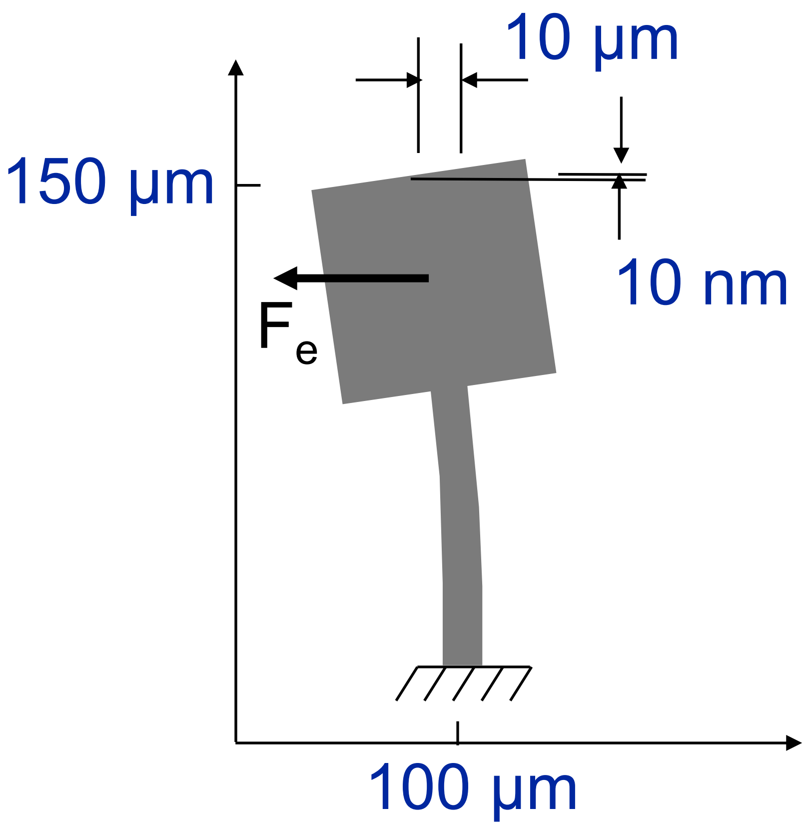
$$x = \int_0^t v_x \, dt$$

- Choice #2
  - Potential = Position
  - Potential × Flow = Position × Force is not meaningful
- Choice #3
  - Potential = Displacement
  - Potential × Flow = Displacement × Force = Energy

Position vs. Displacement

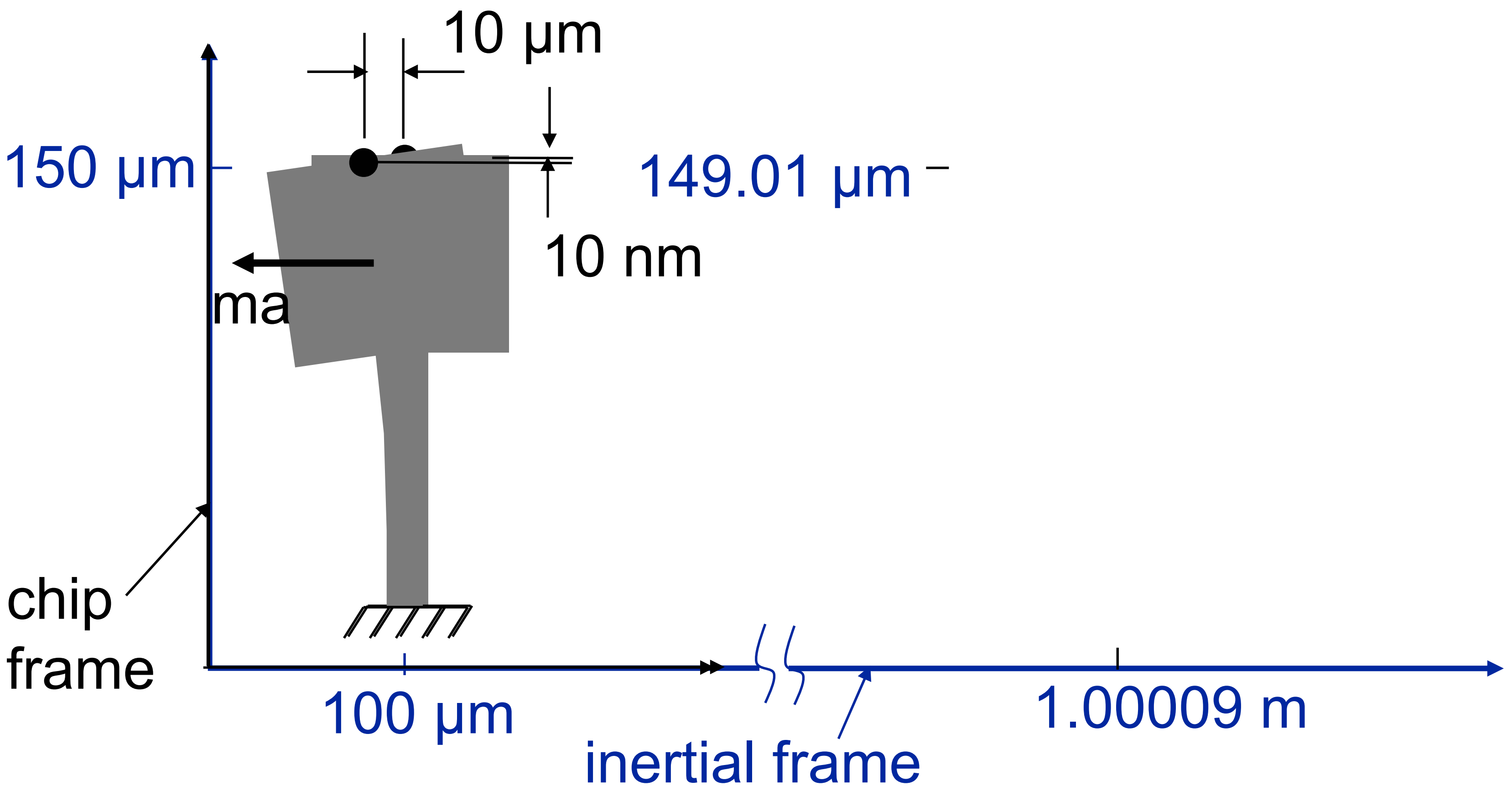
- Positions are non-zero at rest (i.e., layout)
- Displacements can be smaller than 10^{-6} × position
- Unnecessary offset may lead to numerical error





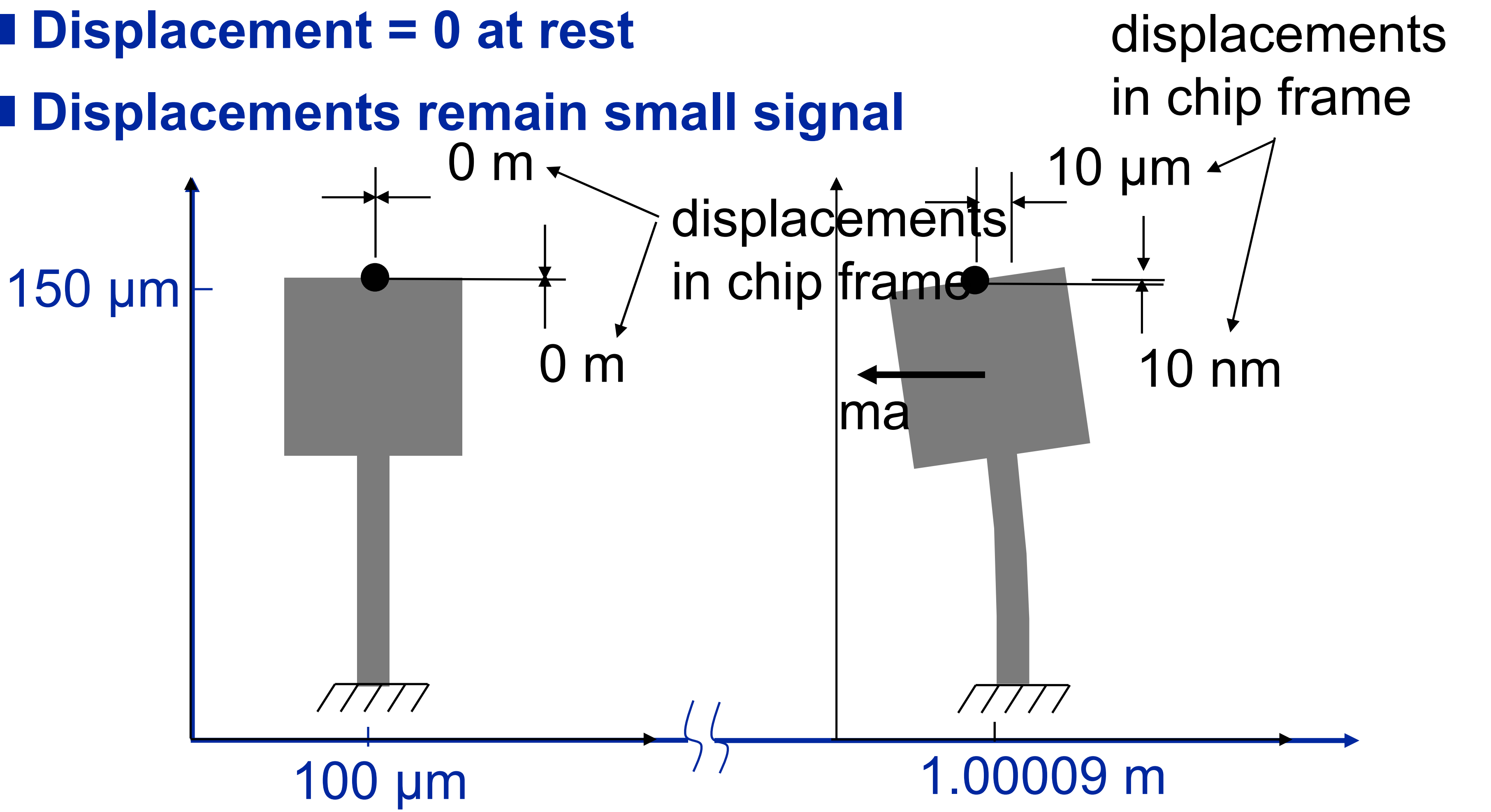
Inertial and Chip Frames of Reference

- Accelerometer example
- Position in chip frame is large relative to displacement
- Position in inertial frame becomes extremely large!



Displacement, Chip Frame of Reference

- **Layout location is natural reference for displacement**
- **Displacement = 0 at rest**
- **Displacements remain small signal**



MEMS Translational Discipline

- Current *kinematic* conservative discipline
  - Across: Position
  - Through: Force
- New *kinematic_translational* conservative discipline:
  - Across: Displacement
  - Through: Force

```
nature Displacement
    units = “m”;
    access = Disp;
    ddt_nature = Velocity;
    abstol = 1e-12;
endnature

nature Force
    units = “N”;
    access = F;
    abstol = 1e-12;
endnature

discipline kinematic_translational
    potential Displacement;
    flow Force;
enddiscipline
```

MEMS Rotational Discipline

- Current *rotational* conservative discipline
  - Across: Angle
  - Through: Angular_Force
- New *kinematic_rotational* conservative discipline:
  - Across: Angular_Displacement
  - Through: Angular_Force

```
nature Angular_Displacement
    units = “rad”;
    access = Phi;
    ddt_nature = Angular_Velocity;
    abstol = 1e-7;
endnature
nature Angular_Force
    units = “N-m”;
    access = Tau;
    abstol = 1e-16;
endnature
discipline kinematic_rotational
    potential
        Angular_Displacement;
    flow Angular_Force;
enddiscipline
```

MEMS Velocity Discipline

- *velocity* and *angular_velocity* signal-flow disciplines
- Needed to store kinematic states

```
nature Velocity
    units = “m/s”;
    access = Vel;
    ddt_nature = Acceleration;
    idt_nature = Displacement;
    abstol = 1e-9;
endnature
discipline velocity
    potential Velocity;
enddiscipline
```

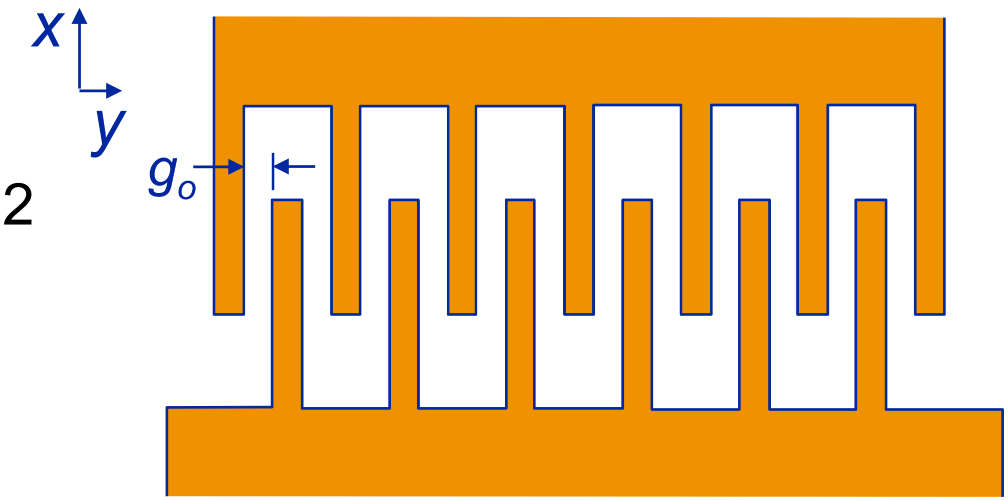
```
nature Angular_Velocity
    units = “rad/s”;
    access = Omega;
    ddt_nature =
    Angular_Acceleration;
    idt_nature = Angular_Velocity;
    abstol = 1.0;
endnature
discipline angular_velocity
    potential Angular_Velocity;
enddiscipline
```

Abstol Settings for MEMS

- My rule of thumb:
  - Estimate “typical” forces and displacements
  - Set abstol at least to $10^{-3} \times$ <typical value>

Comb Drive Force Estimate

$$F_{e,x} = \frac{1}{2}\frac{dC}{dx}V^2 \approx N\frac{\varepsilon_o h_o}{g_o}V^2$$

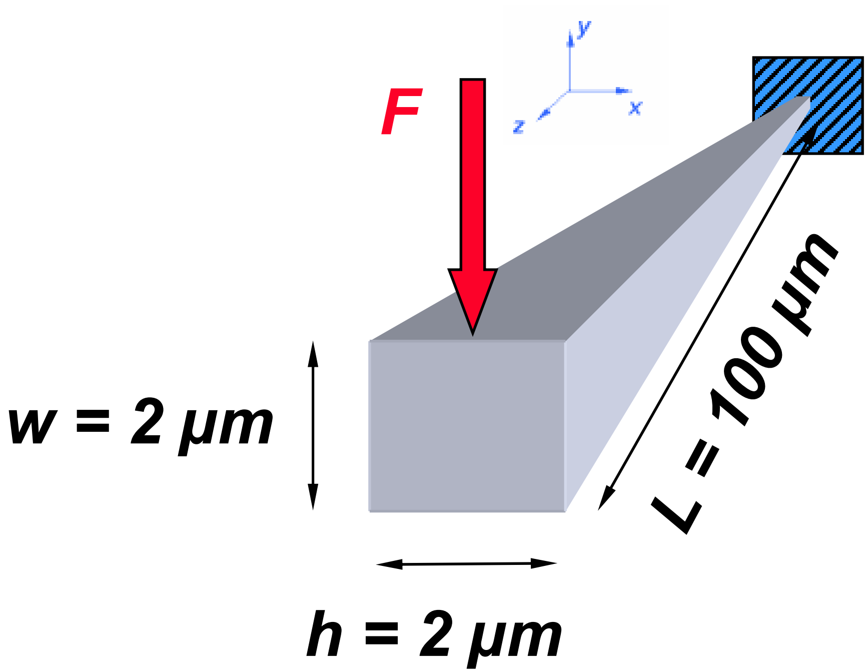


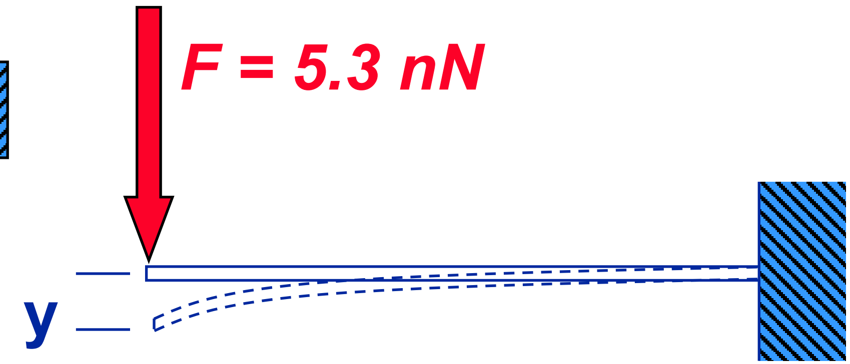
- Number of comb fingers = N = 6
- Finger thickness = h_o = 2 μm
- Finger gap = g_o = 2 μm
- Voltage across fingers = V = 10 V

$F_{e,x} \approx 5.3$ nN

abstol ≈ 1 pN

Cantilever Displacement Estimate





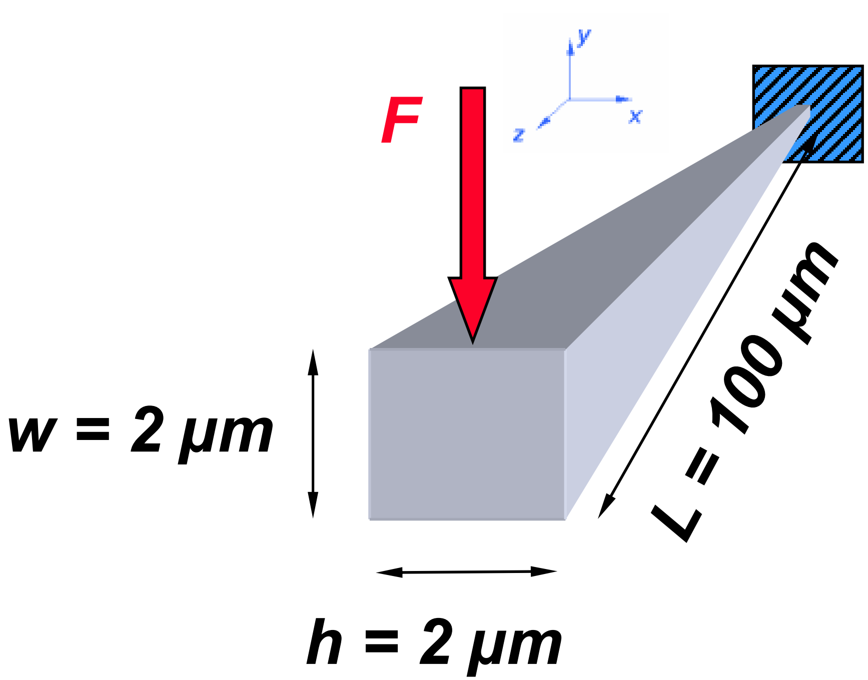
$$y(L) = \frac{FL^3}{3EI_x} = \frac{4FL^3}{Ehw^3}$$

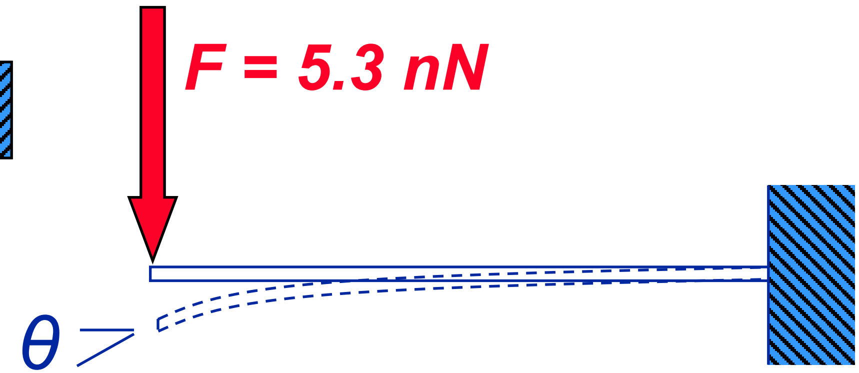
- Apply comb force, F = 5.3 nN
- Young's modulus, E = 165 GPa

y(L) = 8 nm

abstol ≈ 1 pm

Cantilever Angular Displacement Estimate





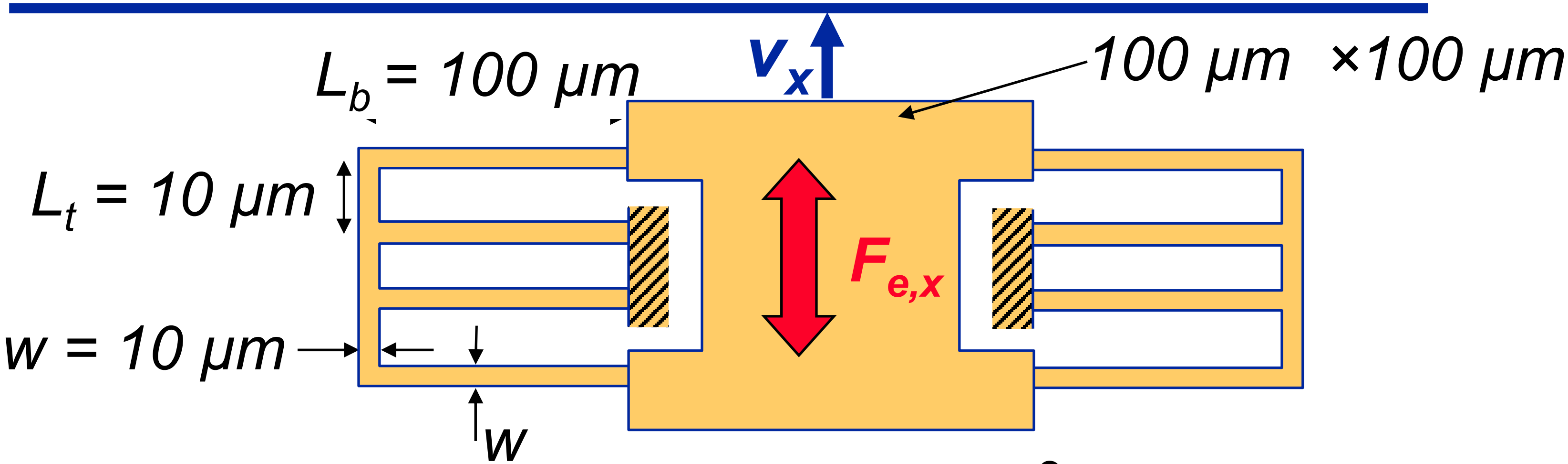
$$\theta(L) = \frac{FL^2}{2EI_x} = \frac{6FL^2}{Ehw^3}$$

- Apply comb force, F = 5.3 nN
- Young’s modulus, E = 165 GPa

$\theta(L)$ = 0.12 mrad

abstol ≈ 0.1 μrad

Resonator Velocity Estimate



- E = 165 GPa
- h = 2 μm
- Q = 10
- Apply comb force, F = 5.3 nN

$$k_x = 2Eh\left(\frac{w_b}{L_b}\right)^3 \frac{\tilde{L}_t^2 + 14\tilde{L}_t + 36}{4\tilde{L}_t^2 + 41\tilde{L}_t + 36}; \quad \tilde{L}_t = \frac{L_t}{L_b}$$

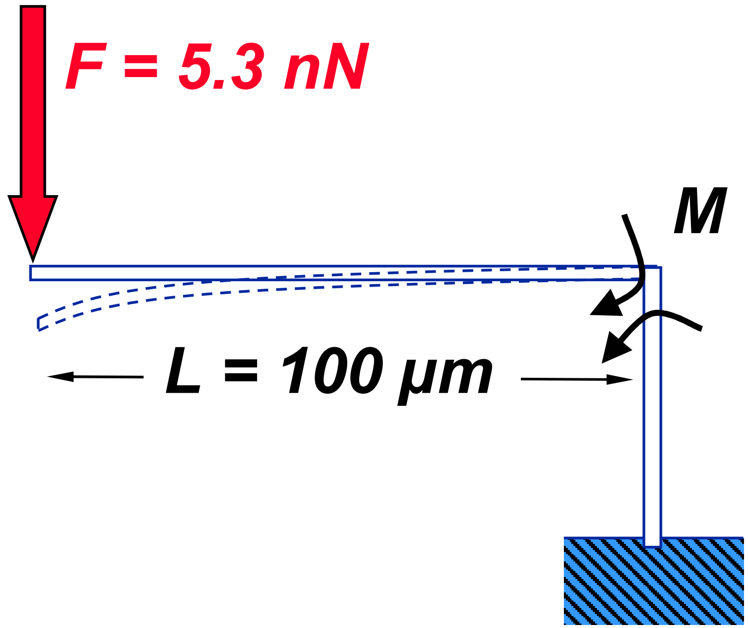
$$\omega_x \approx \sqrt{\frac{k_x}{m}} \quad v_x \approx \omega_x Q x$$

k_x = 4.9 N/m
f_x = 52 kHz
x = 11 nm

V_x = 3.5 mm/s

abstol ≈ 1 μm/s

Crab-Leg Angular Force (Moment) Estimate



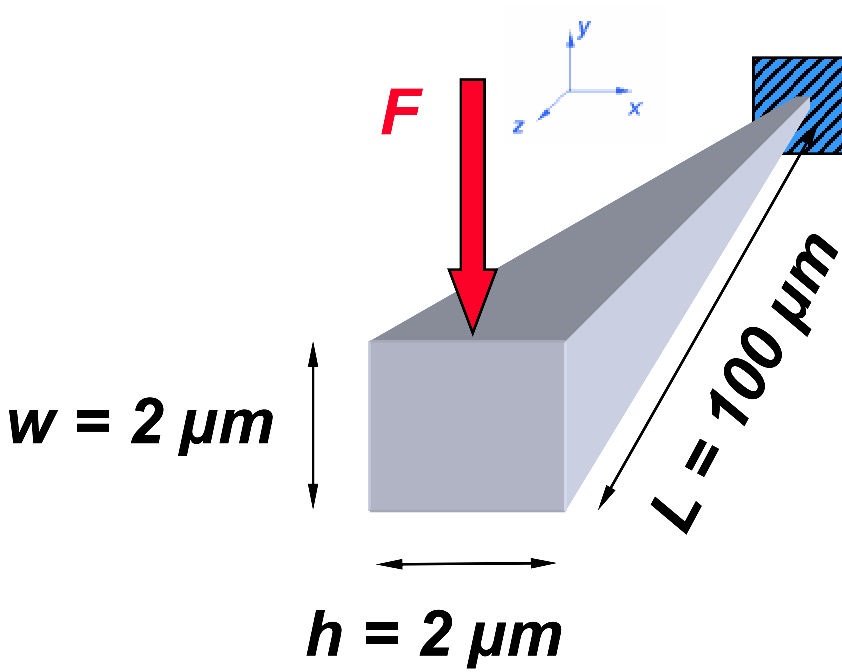
$$M = FL$$

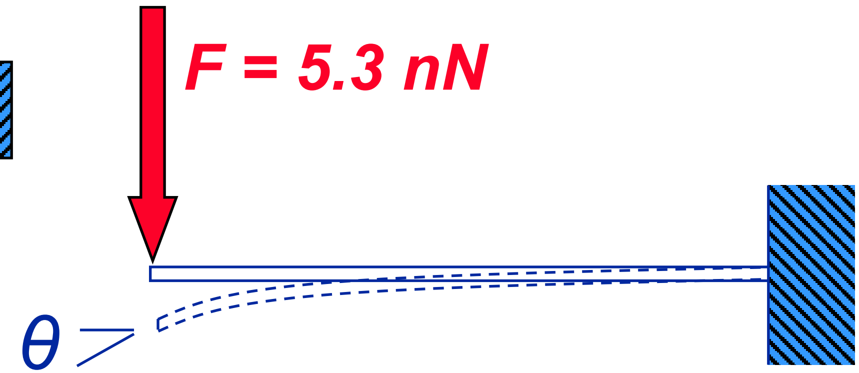
- Apply comb force, F = 5.3 nN

M = 0.53 pN-m

abstol ≈ 0.1 fN-m

Angular Velocity Estimate





$$\omega_r = 1.03 \frac{w}{L^2} \sqrt{\frac{E}{\rho}}$$

- **Static angular displacement**
  $\theta(L)$ = 0.12 mrad
- **Resonance = f_r = 276 kHz**
- **Quality factor = Q = 45**
- **Resonant angular displacement**
  $\theta(L)$ = 5.43 mrad

$d\theta/dt(L)$ = 9.4 krad/s

abstol ≈ 1 rad/s

MEMS Abstol – Summary of Examples

Much lower than in macro-scale mechanics

(Except angular velocity)

Nature	Abstol
Force	10^{-12} N
Displacement	10^{-12} m
Velocity	10^{-6} m/s
Angular_Force	10^{-16} N-m
Angular_Displacement	10^{-7} rad
Angular_Velocity	10^{0} rad/s

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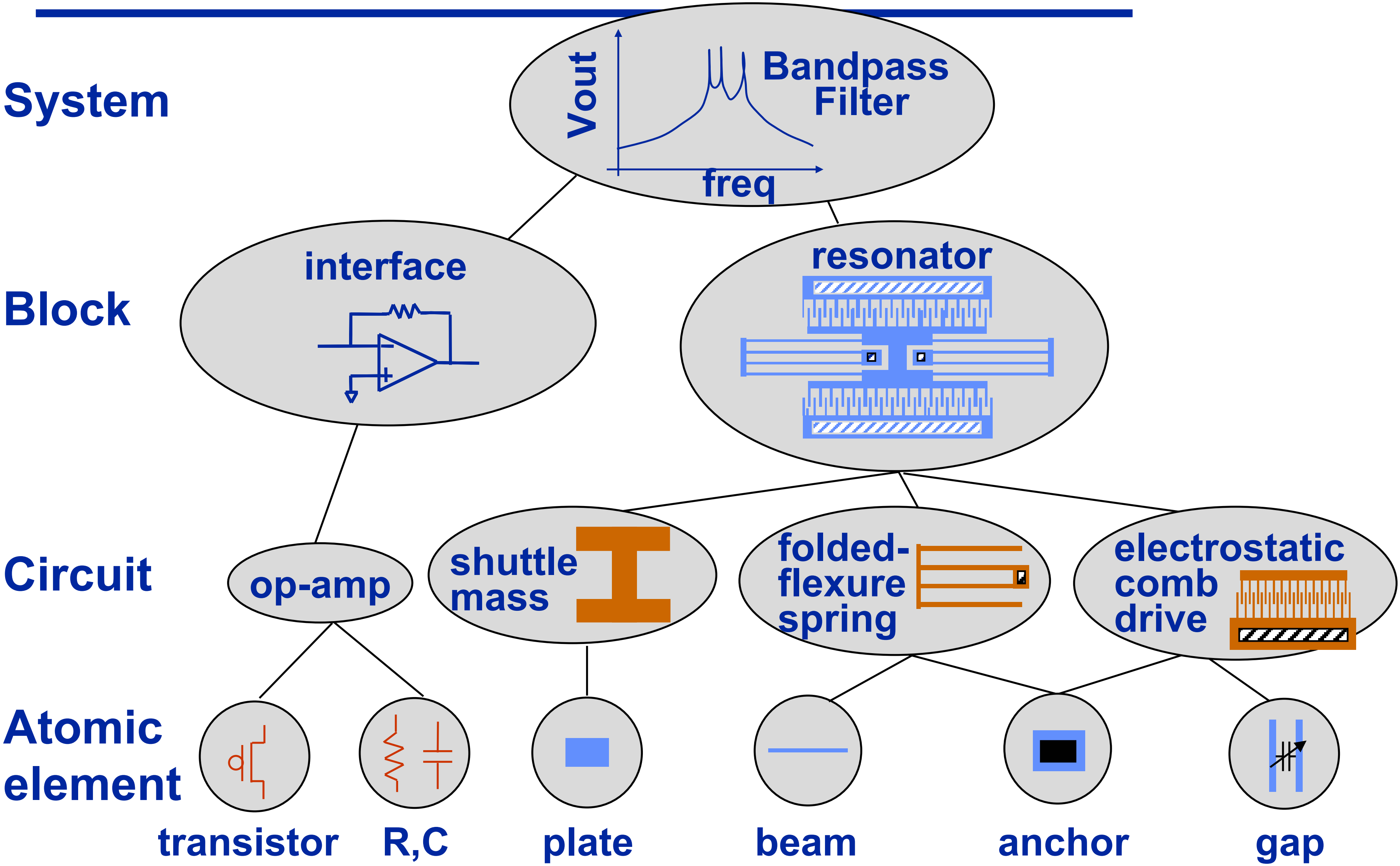
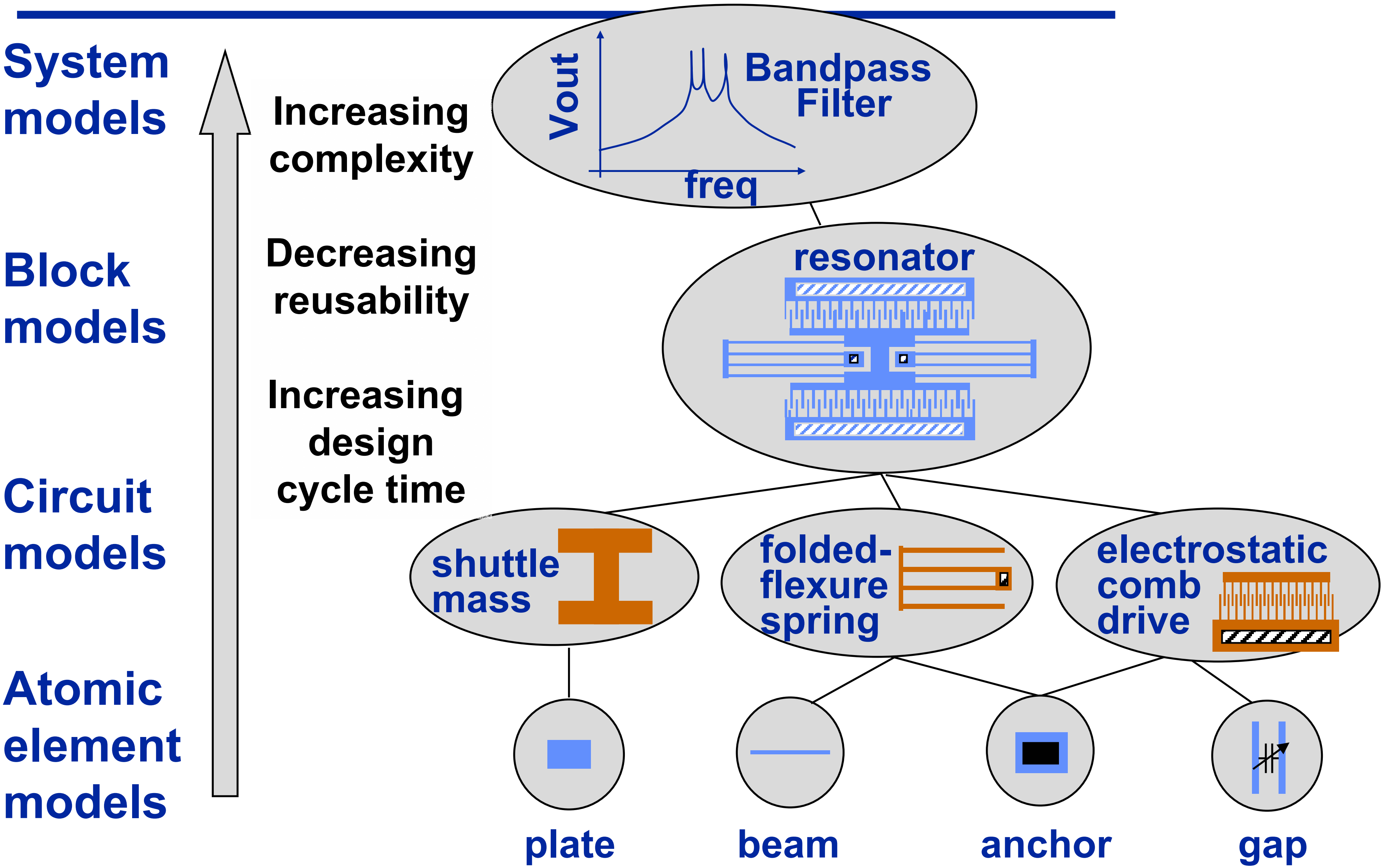
MEMS Design Hierarchy



Ref: G.K. Fedder & Q. Jing, IEEE TCASII, Oct. 1999

MEMS Behavioral Model Hierarchy



Ref: *G.K. Fedder & Q. Jing, IEEE TCASII, Oct. 1999*

Model Degrees of Freedom

- 1-D, 1 DOF
  - Simplest to macromodel
  - mass-spring-damper system
- Semi-analytic 2-D or 3-D
  - Assume lumped masses
  - 3 DOF (2-D) or 6 DOF (3-D) per mass
- Lowest N modes
  - Extracted from numerical analysis
  - True modal decomposition requires self-consistent electromechanical simulation
- DOF through structural modeling

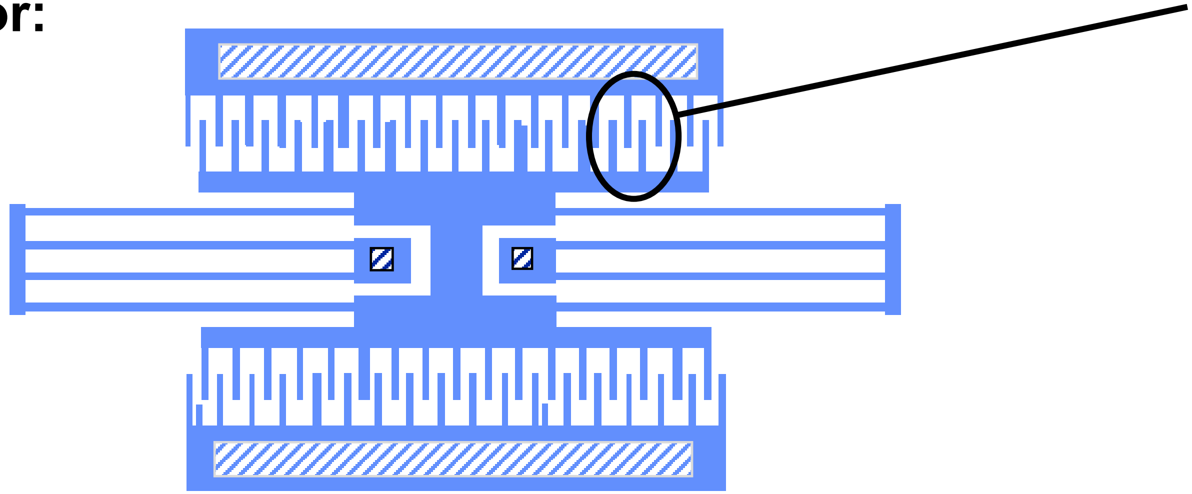
Danger of Degrees of Freedom

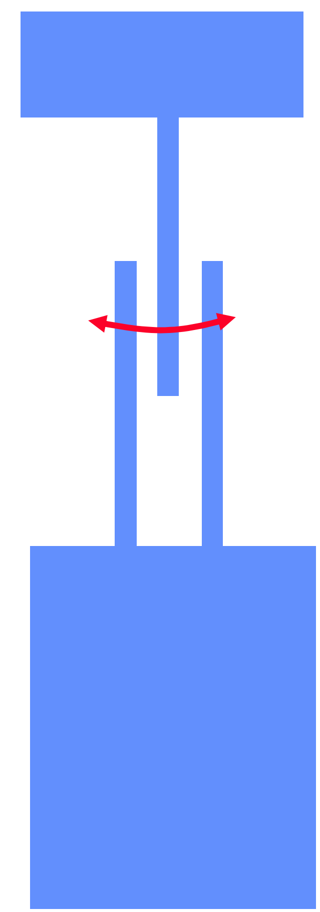
- Choice of DOFs is dependent on
  - stimulus location
  - stimulus frequency
- Recommendation in MEMS community:
  - Always model to the lowest level possible (i.e. tolerable)

comb fingers
can resonate!

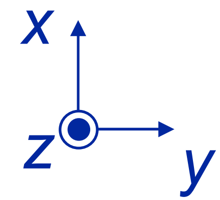
w = 1 μm
L = 50 μm
f_r = 550 kHz

Example folded-flexure resonator:

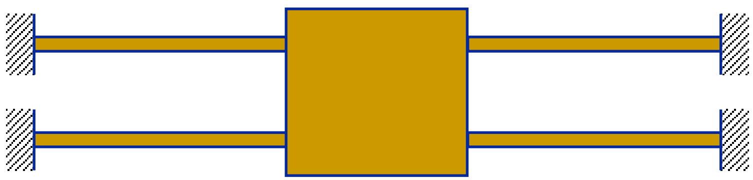




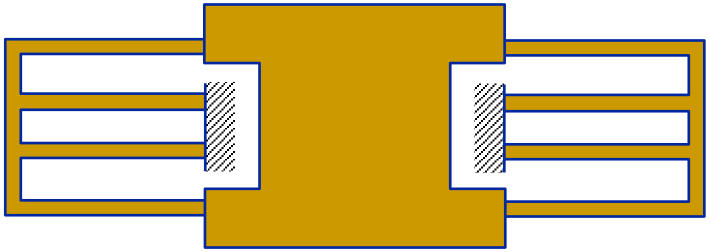
Selected MEMS Flexures



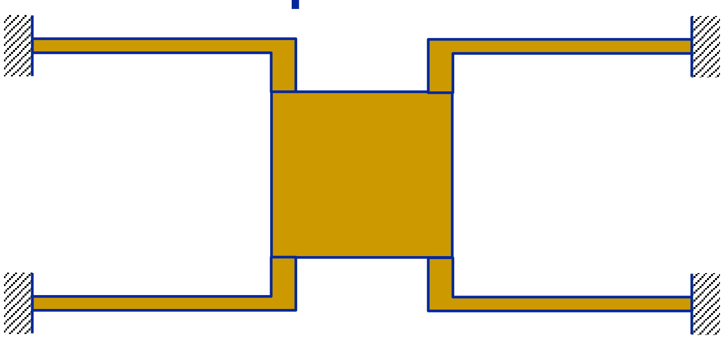
- **Fixed-fixed flexure**
  - stiff in y
  - nonlinear stiffening



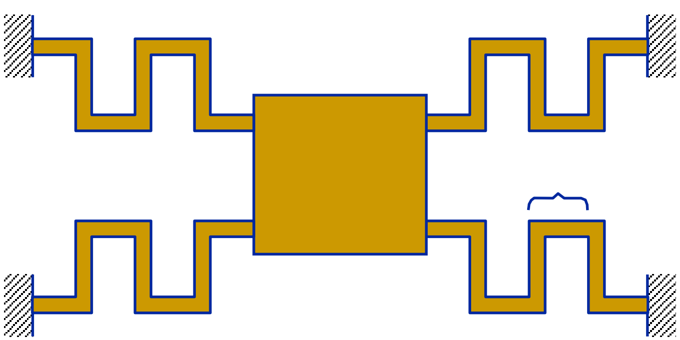
- **Folded flexure**
  - more linear
  - area efficient



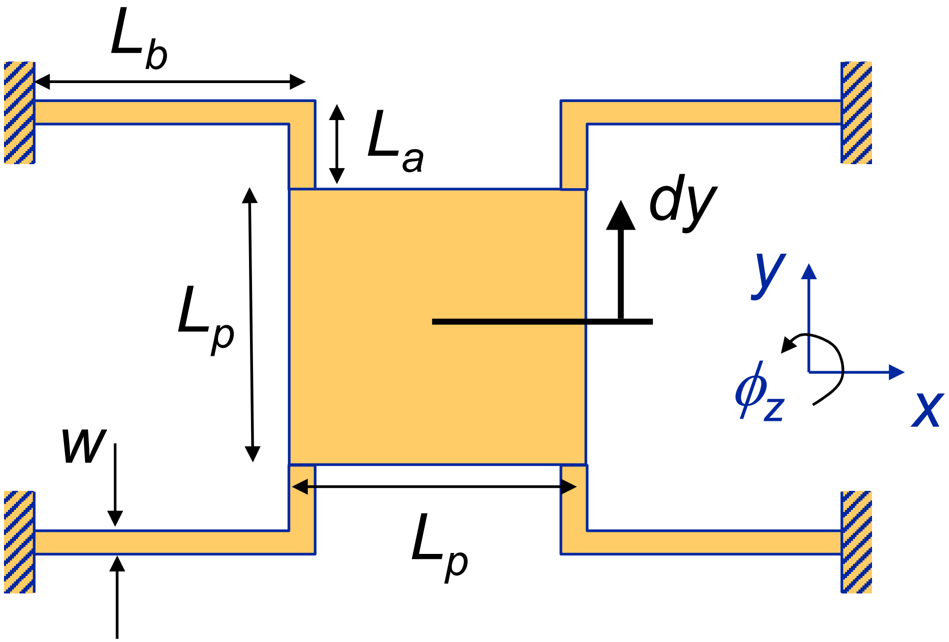
- **Crab-leg flexure**
  - less stiff in y
  - more linear
  - uses up area

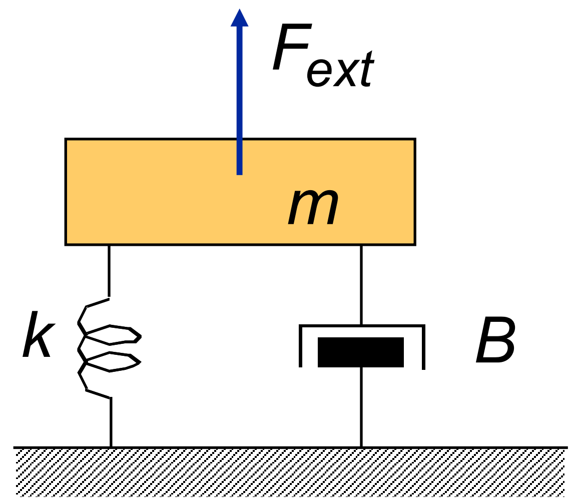


- **Serpentine flexure**
  - set stiffness in x, y
  - linear
  - area efficient



1-D Crab-Leg Resonator





Equation of motion:

$$F_{ext} = m\ddot{y} + B\dot{y} + ky$$

$$m \approx \rho h\left(L_p^2 + 4wL_a\right) \qquad B = \frac{\mu_{air} L_p^2}{d_{eff}} \qquad k = Eh\left(\frac{w}{L_b}\right)^3 \frac{(4L_b + L_a)}{(L_b + L_a)}$$

1-D Parameterized Behavioral Model

```
module crableg1D (dy);
inout dy;
kinematic_translational dy;
velocity vy;
parameter real La = 1e-5;
parameter real Lb = 1e-4;
real density = 2330, E = 165e9, viscosity = 1.79e-5, deff = 2e-6;
real h = 2e-6, w = 2e-6, Lp = 1e5;
real m, b, k;
analog begin
   m = density*h*(Lp^2+4*w*La);
   b = viscosity*Lp^2/deff;
   k = E*h*(w/Lb)^3*(4*Lb+La)/(Lb+La);
   Vel(vy) <+ ddt(Disp(dy));
   F(dy) <+ m*ddt(Vel(vy)) + b*Vel(vy) + k*Disp(dy);
   end
endmodule
```

$$m \approx \rho h\left(L_p^2 + 4wL_a\right)$$

$$B = \frac{\mu_{air} L_p^2}{d_{eff}}$$

$$k = Eh\left(\frac{w}{L_b}\right)^3 \frac{\left(4L_b + L_a\right)}{\left(L_b + L_a\right)}$$

$$F_{ext} = m\ddot{y} + B\dot{y} + ky$$

1-D Parameterized Behavioral Model

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inout dy;
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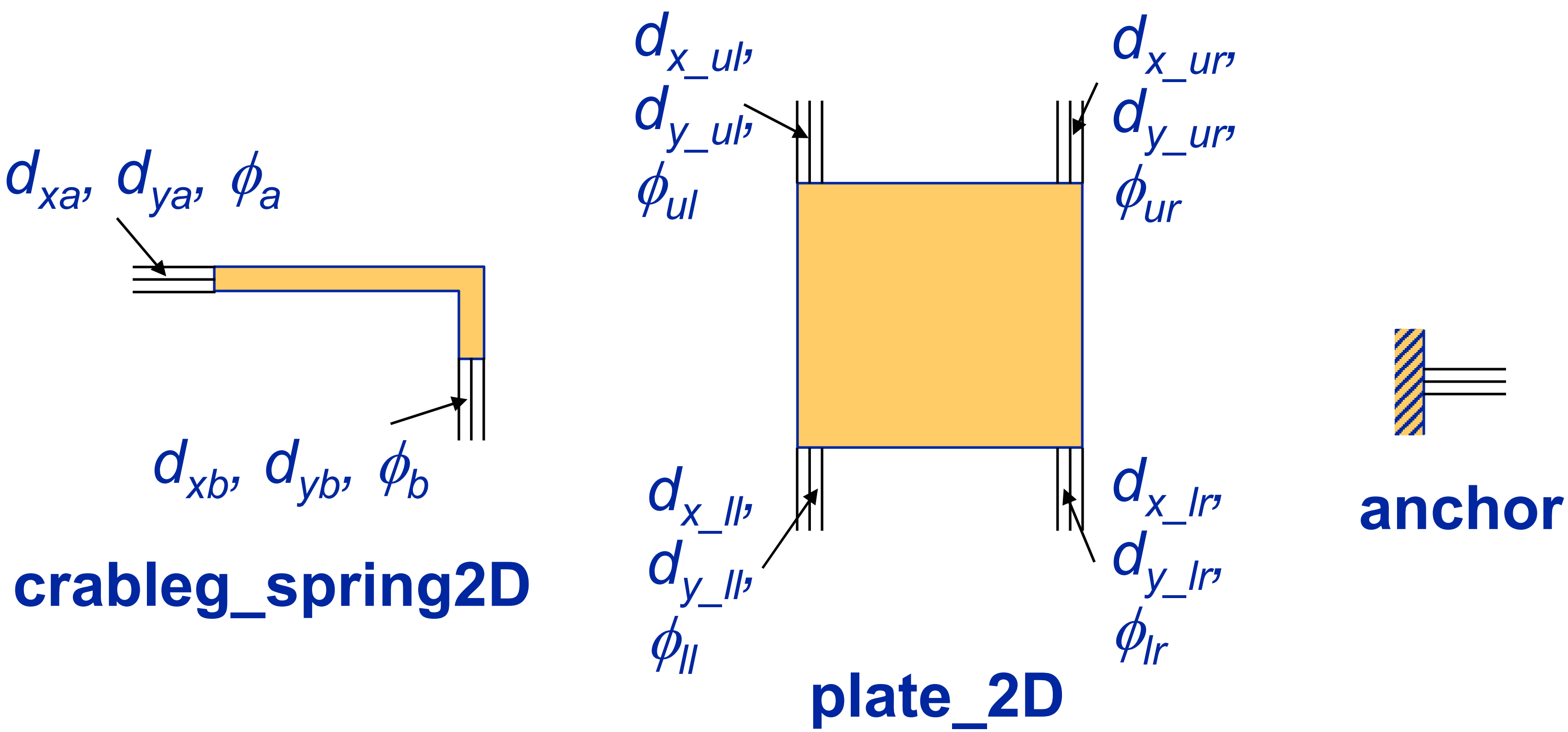
velocity vy;
parameter real La = 1e-5;
parameter real Lb = 1e-4;
real density = 2330, E = 165e9, viscosity = 1.79e-5, deff = 2e-6;
real h = 2e-6, w = 2e-6, Lp = 1e5;
real m, b, k;
analog begin
    m = density*h*(Lp^2+4*w*La);
    b = viscosity*Lp^2/deff;
    k = E*h*(w/Lb)^3*(4*Lb+La)/(Lb+La);
    Vel(vy) <+ ddt(Disp(dy));
    F(dy) <+ m*ddt(Vel(vy)) + b*Vel(vy) + k*Disp(dy);
    end
endmodule
```

velocity state needed

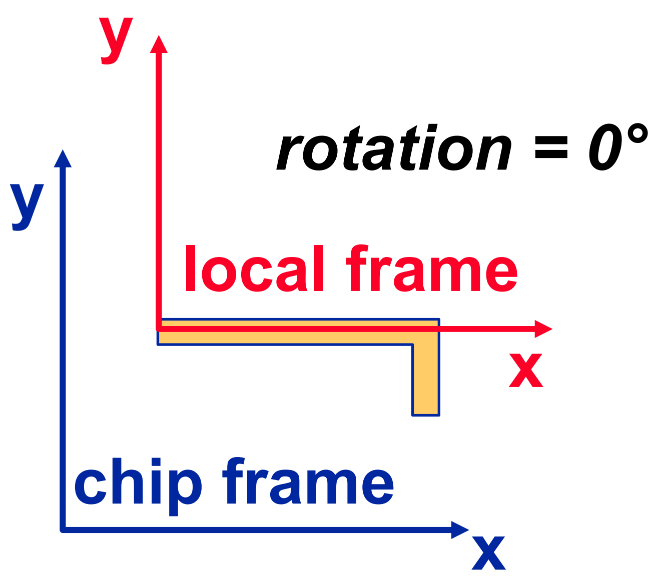
acceleration state unnecessary

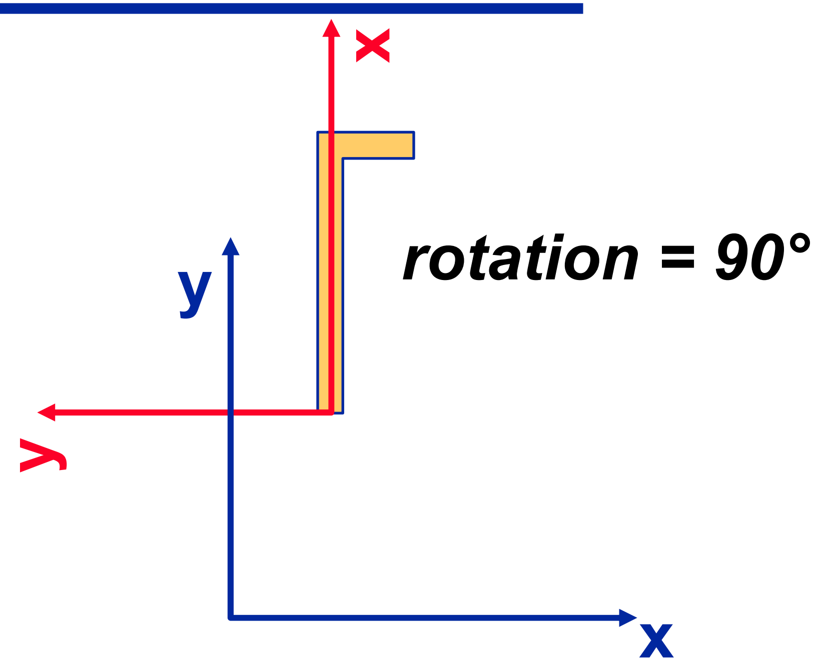
Functional-Level 2-D Models

- Divide into smaller functional units
- Easier to analytically model each unit
- Greater DOFs

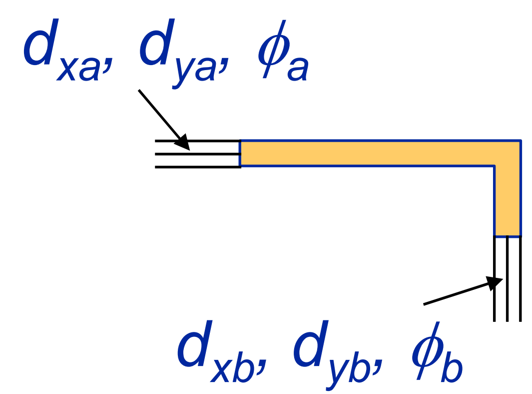


Chip and Local Frames of Reference



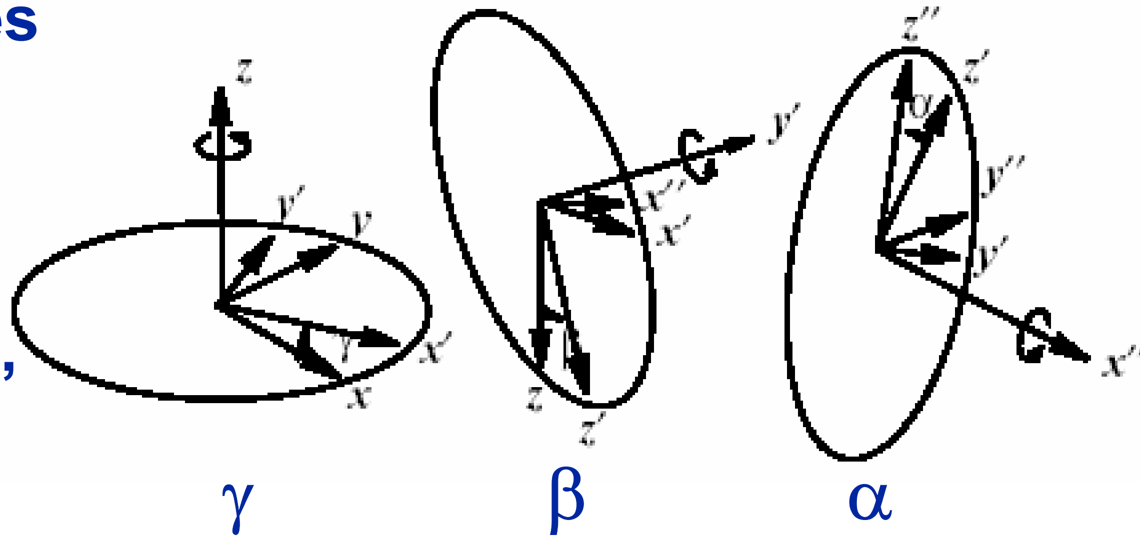


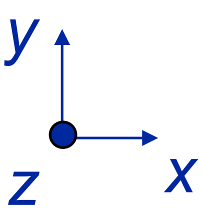
- External ports refer to chip frame
  - Direction of port is invariant with rotation
- Internal calculations refer to local frame
  - Must transform port inout variables



Functional Level 2-D Schematic

- Euler rotation angles specify layout position
- γ about z-axis, then β about y’-axis, last α about x’’-axis





$\gamma = 0°$

$\beta = 0°$

$\alpha = 0°$

$\gamma = 90°$

$\beta = 0°$

$\alpha = 0°$

$\gamma = 0°$

$\beta = 0°$

$\alpha = 180°$

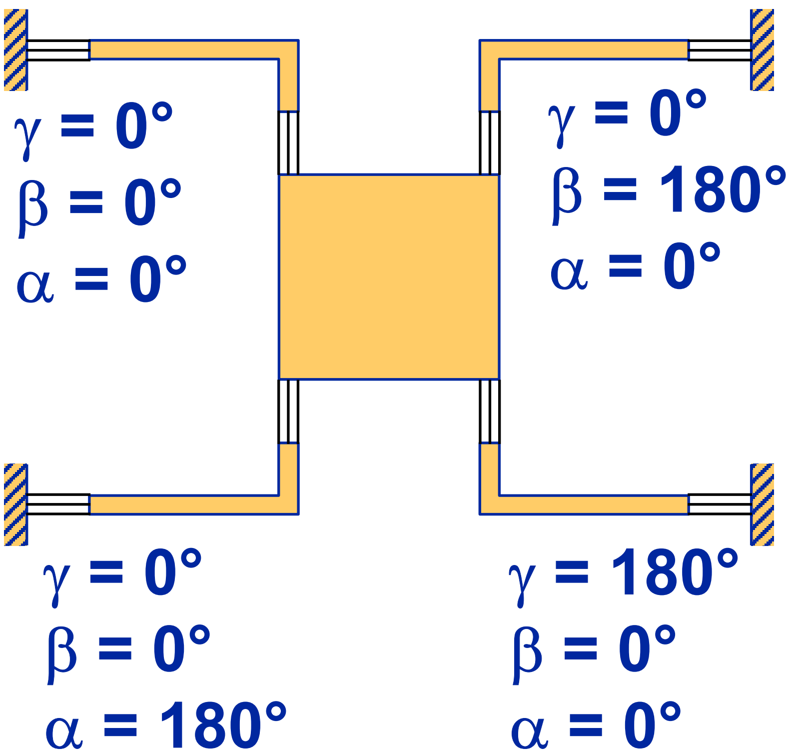
$\gamma = 0°$

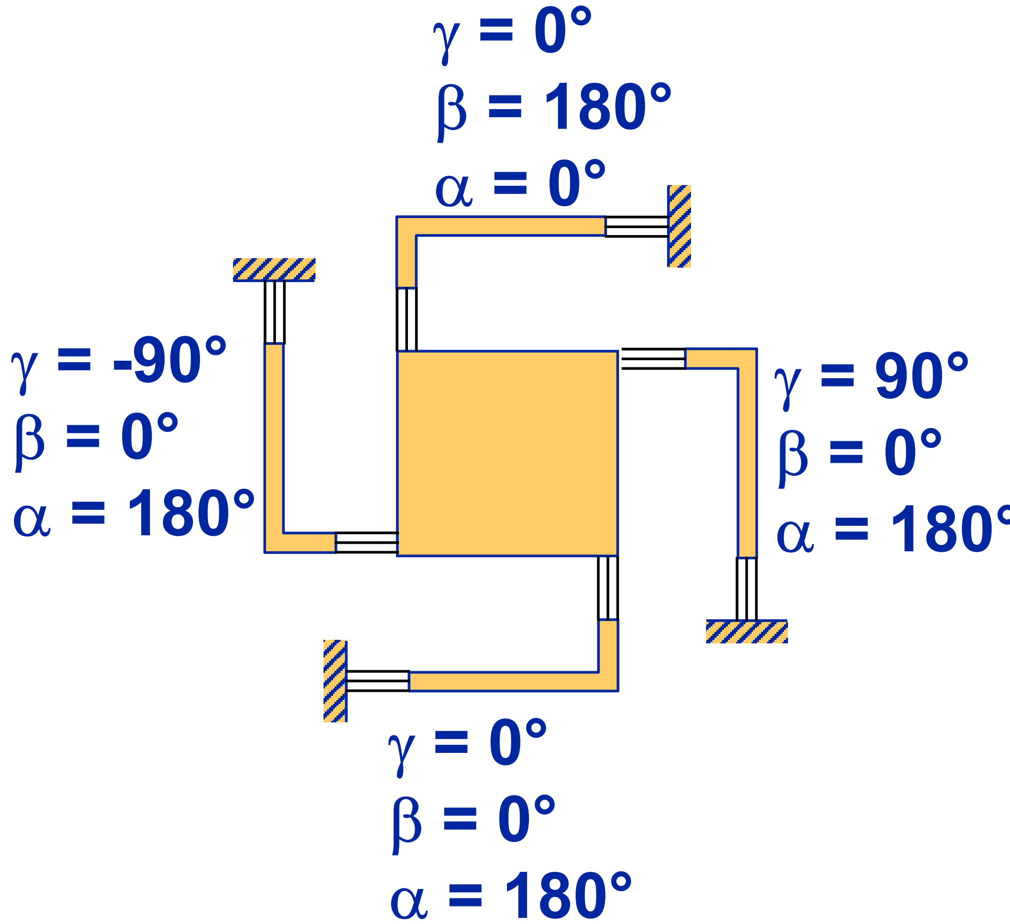
$\beta = 180°$

$\alpha = 0°$

Functional Level 2-D Schematic

- Can build any combination of crab-leg and plate





2-D Parameterized Behavioral Crab-Leg Model

```
module crableg_spring2D (dxa, dya, phi_za, dxb, dyb, phi_zb);
inout dxa, dya, phi_za, dxb, dyb, phi_zb;
kinematic_translational dxa, dya, dxb, dyb;
kinematic_rotational phi_za, phi_zb;
   ...
parameter real alpha = 0.;
parameter real beta = 0.;
parameter real gamma = 0.;
parameter real La = 1e-5;
parameter real Lb = 1e-4;
analog begin
   ...
cos_a = cos(alpha*`M_PI/180.);
cos_b = cos(beta*`M_PI/180.);
cos_g = cos(gamma*`M_PI/180.);
   ...
dx_l = cos_b*cos_g*(Disp(dxb)-Disp(dxa)) + cos_b*sin_g*(Disp(dyb)-Disp(dya));
dy_l = -cos_a*sin_g*(Disp(dxb)-Disp(dxa)) + cos_a*cos_g*(Disp(dyb)-Disp(dya));
dphi = Phi(phi_zb) - Phi(phi_za);
```

Convert to radians

Transform displacements from chip frame to local frame

2-D Parameterized Behavioral Crab-Leg Model

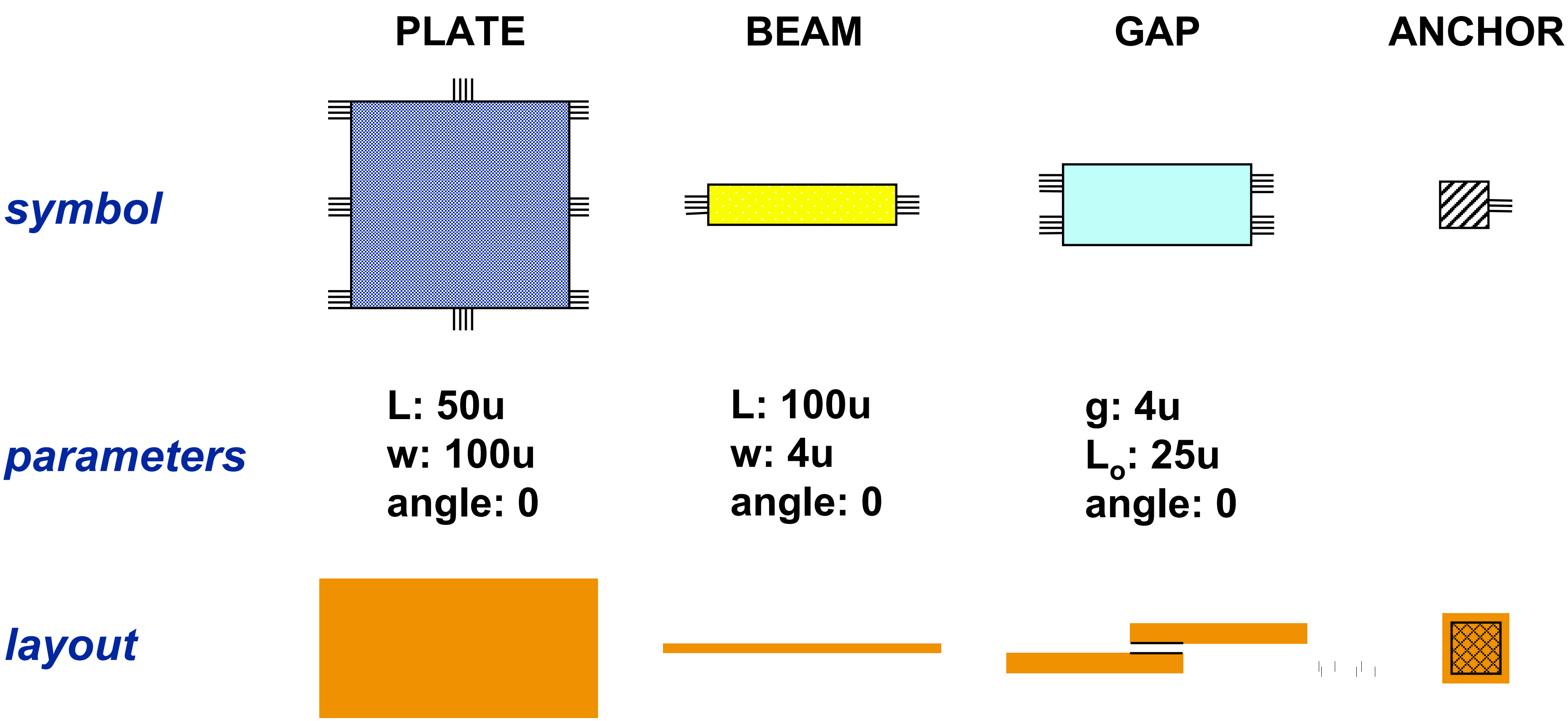
```
//Calculate spring forces and moments
Fkx_l = 3/(La+Lb)*((4*La+Lb)/La^3*dx_l-3/La/Lb*dy_l-(2*La+Lb)/La^2*dphi);
Fky_l = 3/(La+Lb)*(-3/La/Lb*dx_l+(La+4*Lb)Lb^3*dy_l+3/b*dphi);
Mkz_b = 1/(La+Lb)*(-3*(2*La+Lb)/La^2*dx_l+3/Lb*dy_l+(4*La+3*Lb)/La*dphi);
   ...
Fx_bl = Fmx_bl+Fbx_bl+Fkx_l; // Fm are inertial forces
Fy_bl = Fmy_bl+Fby_bl+Fky_l; // Fb are damping forces
Fx_al = Fmx_al+Fbx_al-Fkx_l;
Fy_al = Fmy_al+Fby_al-Fky_l;

F(dxb) <+ cos_b*cos_g*Fx_bl - cos_a*sin_g*Fy_bl;
F(dyb) <+ cos_b*sin_g*Fx_bl + cos_a*cos_g*Fy_bl;
F(dxa) <+ cos_b*cos_g*Fx_al - cos_a*sin_g*Fy_al;
F(dya) <+ cos_b*sin_g*Fx_al + cos_a*cos_g*Fy_al;
Tau(phi_zb) <+ Mm_b + Mb_b + Mkz_b;
Tau(phi_za) <+ Mm_a + Mb_a - Mkz_b - La*Fkx_l - Lb*Fky_l;
end
endmodule
```

Transform forces from local frame to chip frame

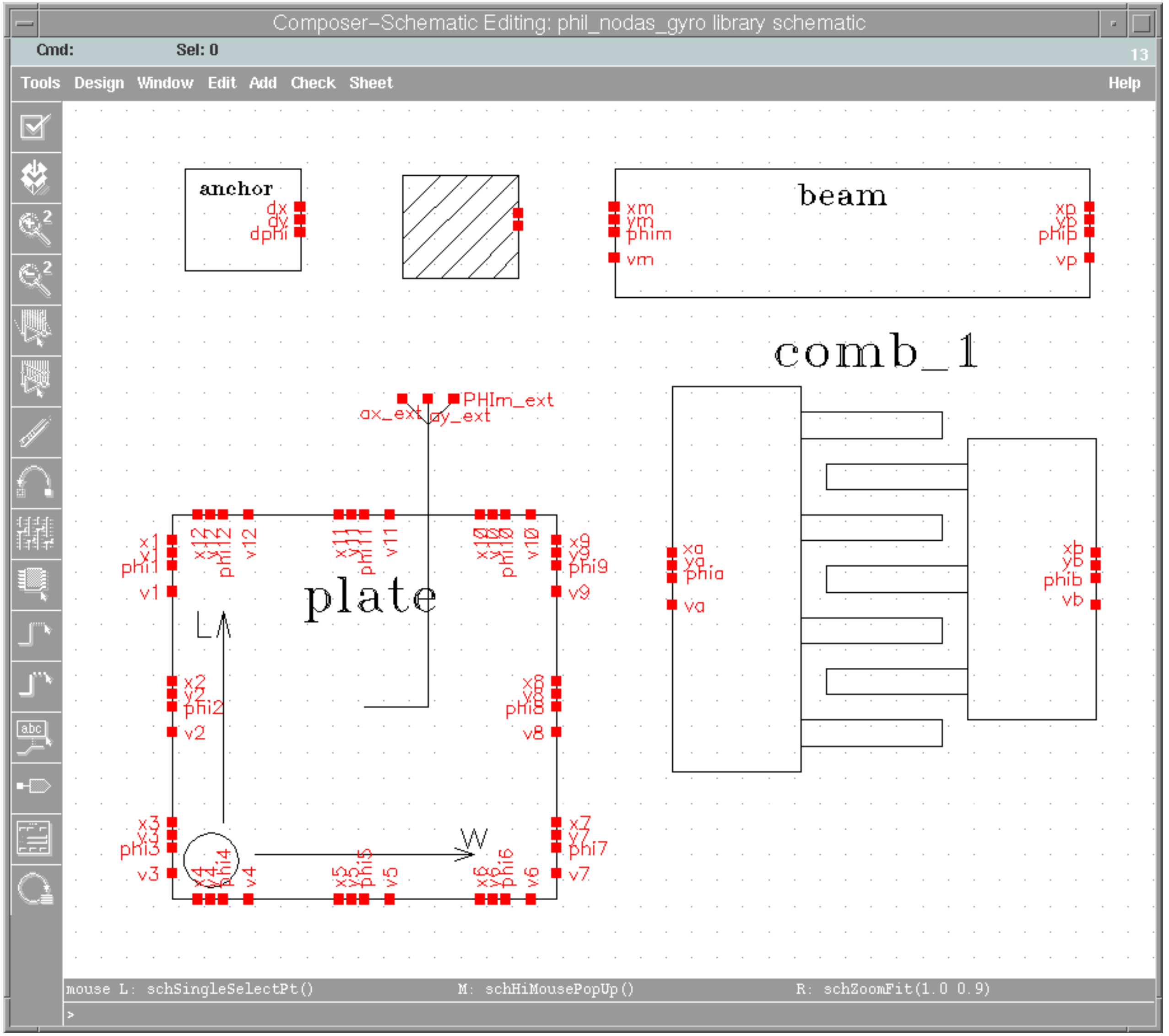
“Atomic” Elements

- Circuit representations of suspended MEMS can be partitioned into four basic lumped-parameter elements: plates, beams, gaps, and anchors



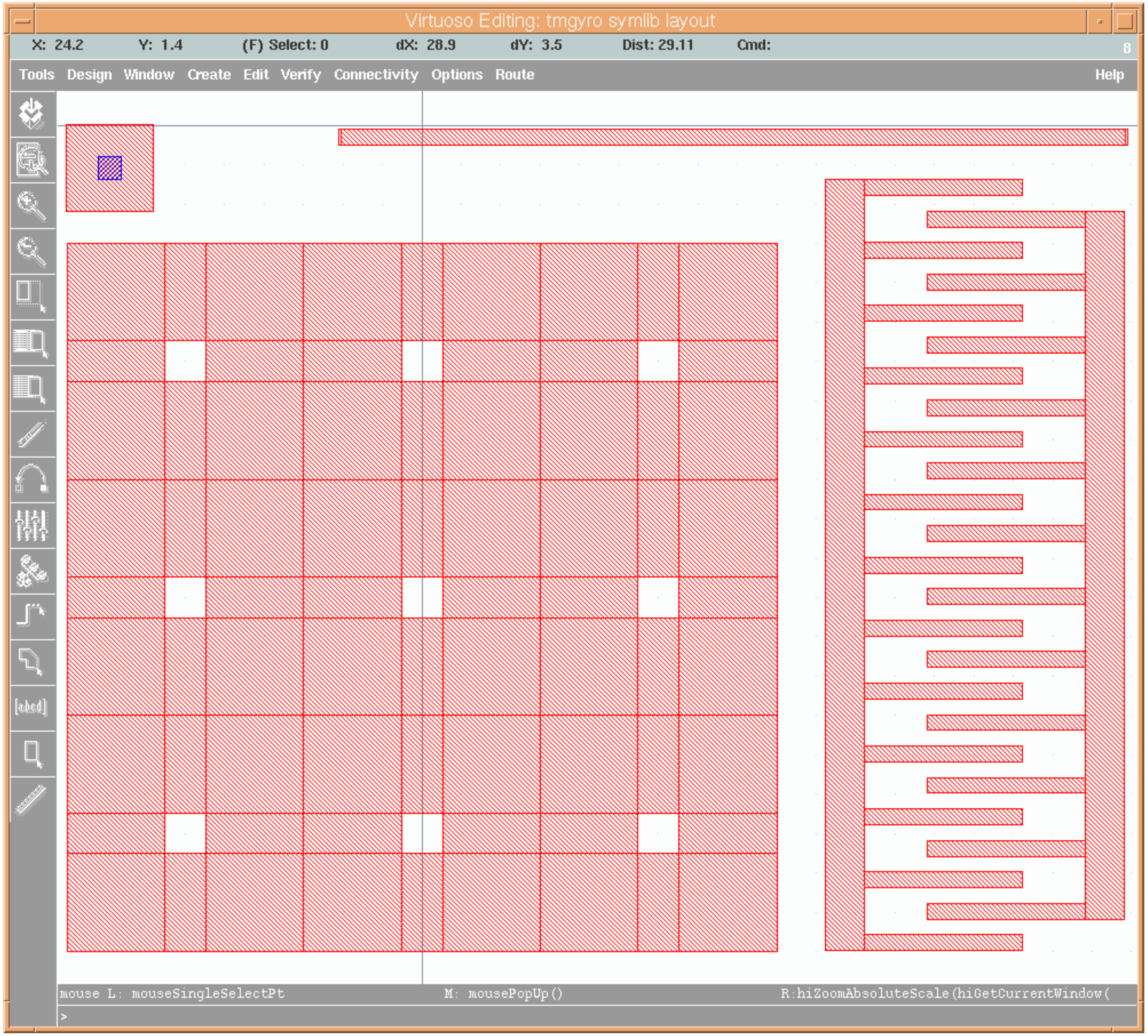
Multi-Level Design Reuse

- Elements (symbols and models) can be reused in new designs
- Low-level elements are:
  - Anchor
  - Beam
  - Plate
  - Gap
  - Comb



Layout Generation

- **Automated layout is hierarchically p-cell (parameterized cell) driven directly from elements**



Issues in MEMS Macromodeling

- Processes and Applications
- Design Views
- Natures and Disciplines
- Hierarchy
- Interoperability
- Beam Element Example
- Filter Example
- Language Extension

Port Interoperability

- Essential for composition
  - Atomic-level, functional-level schematic components

Three items must match at ports for meaningful interconnection:

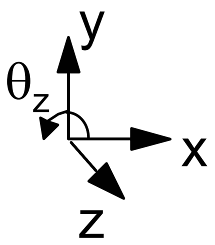
1. Disciplines must match
   - choose displacements, forces
2. Reference frames must match
   - choose chip frame of reference
3. Associated reference directions must match
   - ?

Associated Reference Directions

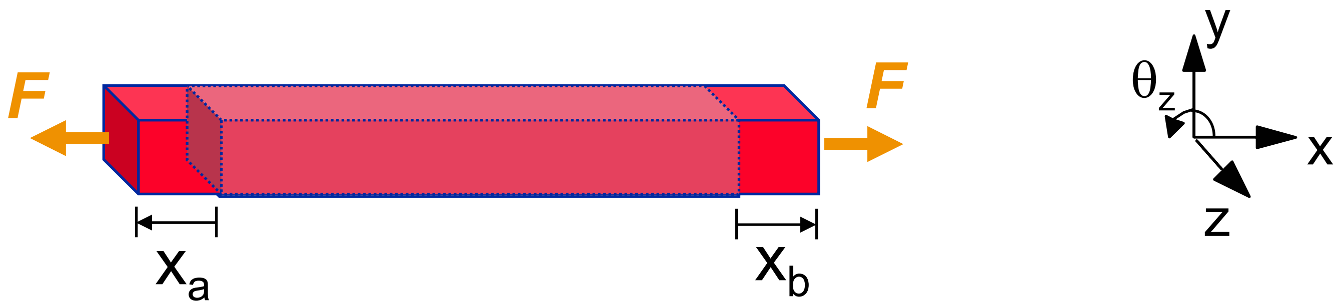
- Electrical:
  - positive valued flow → positive charge flow
  - positive valued potential → positive voltage
- What is convention for mechanical disciplines?
  - choice is arbitrary, but
  - interpretation can be frustrating

CMU Mechanical Nodal Conventions

- **Across variables (x, y, θ_z)**
  - **Positive valued displacements are in positive axial direction**
  - **Positive valued angles are counterclockwise around axis**
- **Through variables (Fx, Fy, Mz)**
  - **Positive force flowing into node acts in positive axial direction**
  - **Positive moment flowing into node acts counterclockwise around axis**



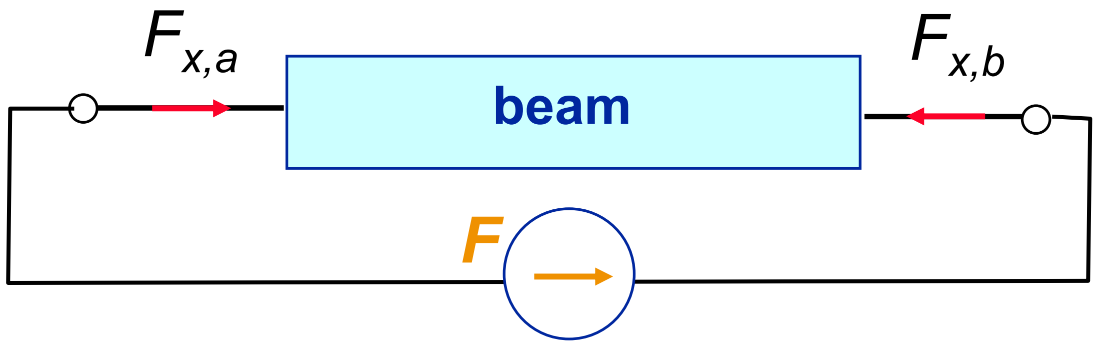
Beam in Tension



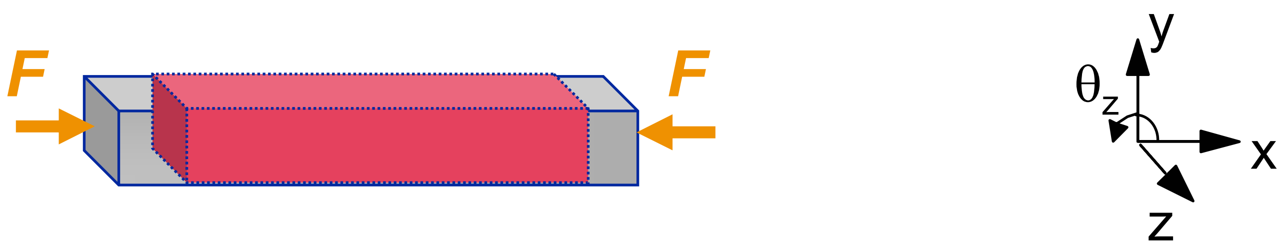
$F_{x,a}$ is negative; $F_{x,b}$ is positive

x_a is negative; x_b is positive

Equivalent schematic:

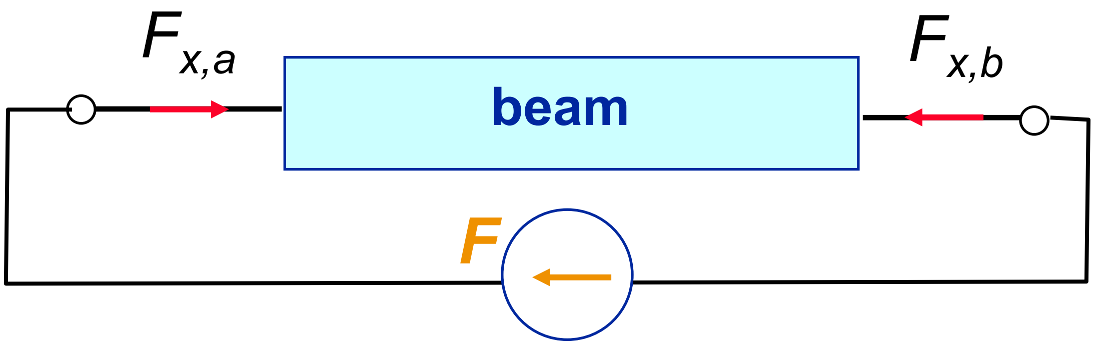


Beam in Compression

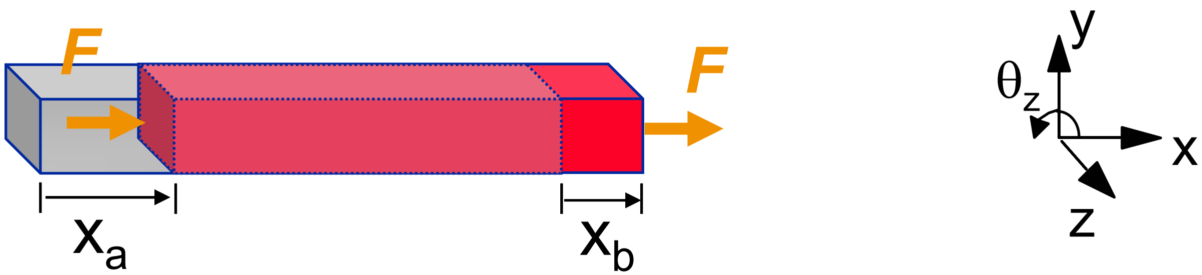


$F_{x,a}$ is negative; $F_{x,b}$ is positive

Equivalent schematic:



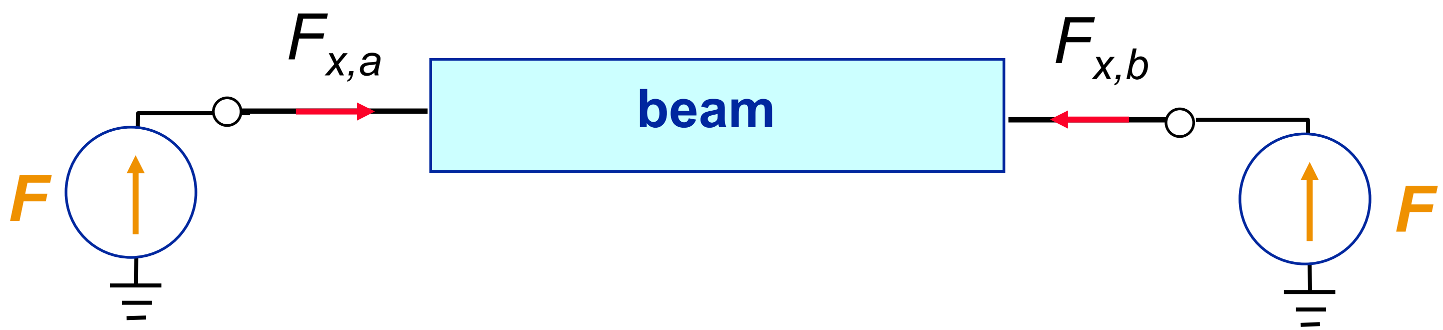
Moving Beam



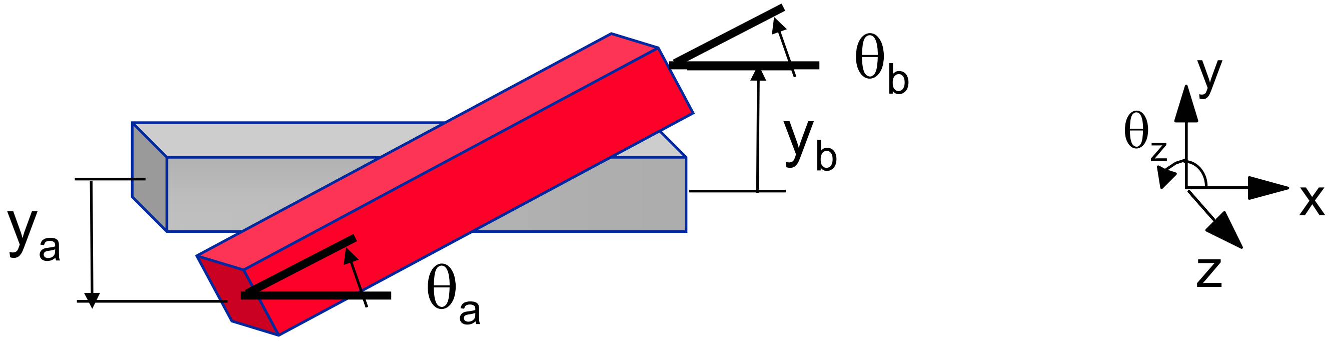
$F_{x,a}$ and $F_{x,b}$ are positive

x_a and x_b are positive

Equivalent schematic:



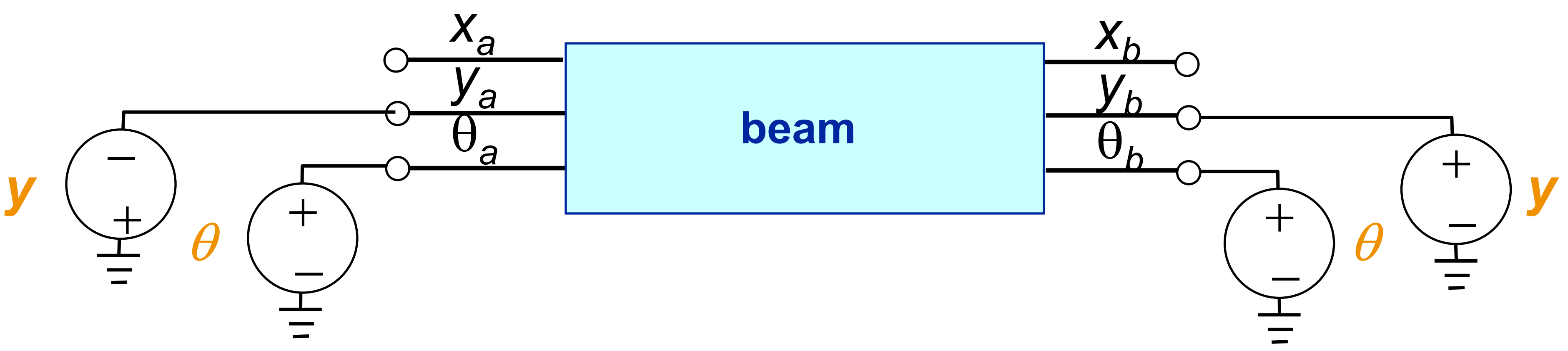
Rotated Beam



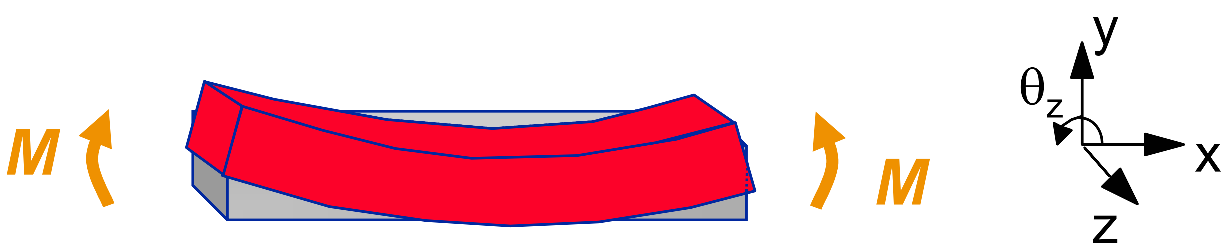
y_a is negative; y_b is positive

and both θ_a and θ_b are positive

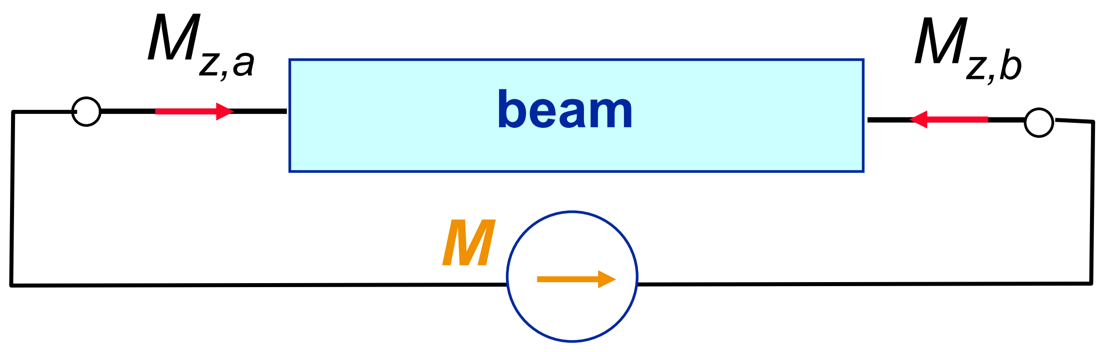
Equivalent schematic:



Pure Bending



Equivalent schematic:



Issues in MEMS Macromodeling

- Processes and Applications
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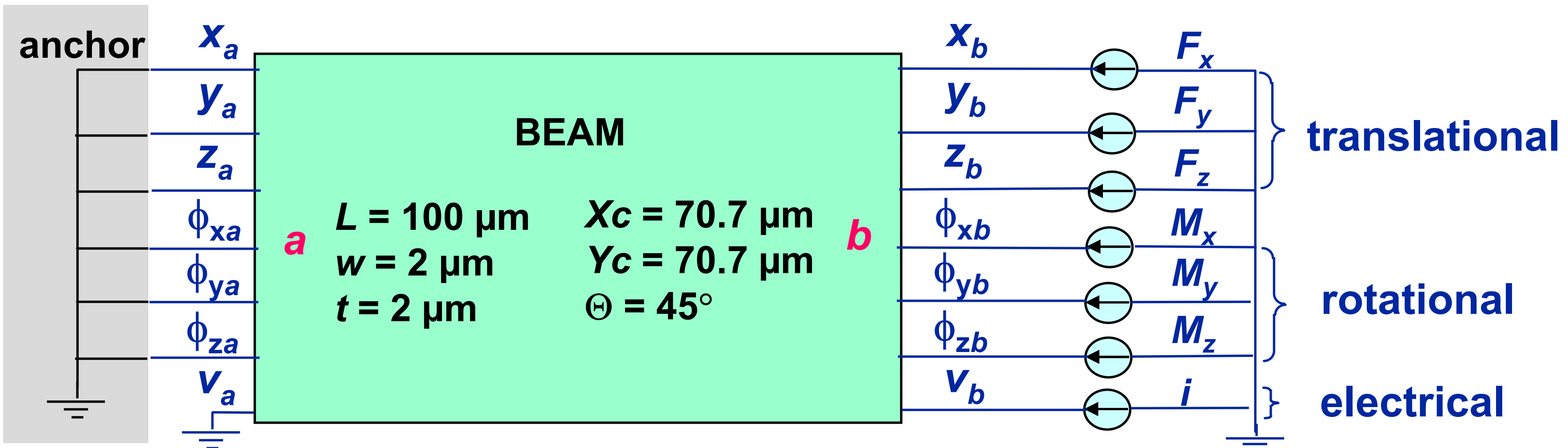
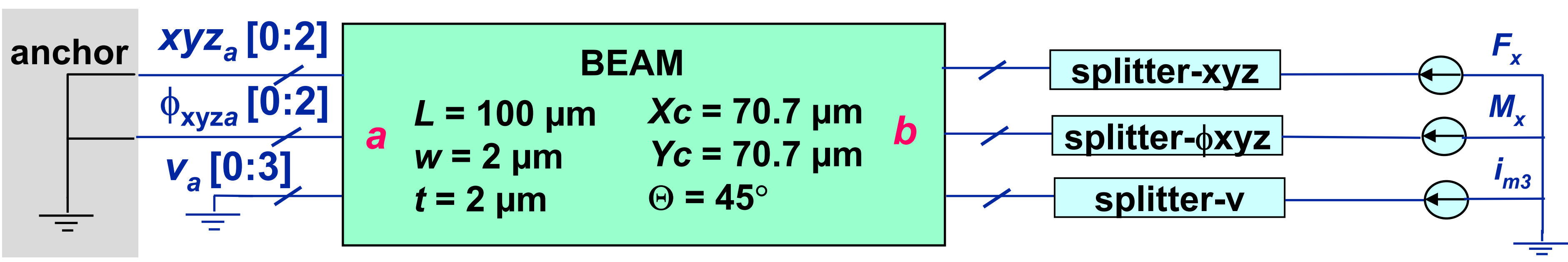
- Hierarchy
- Interoperability
- Beam Element Example
- Filter Example
- Language Extension

Bus Terminals

- Individual pins

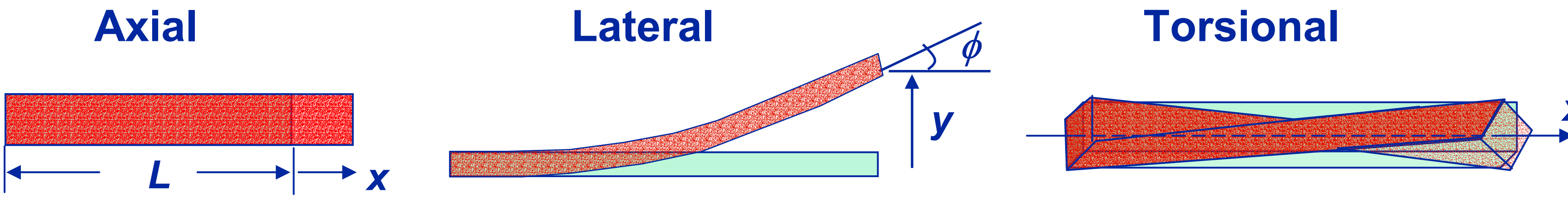


- Bus terminals

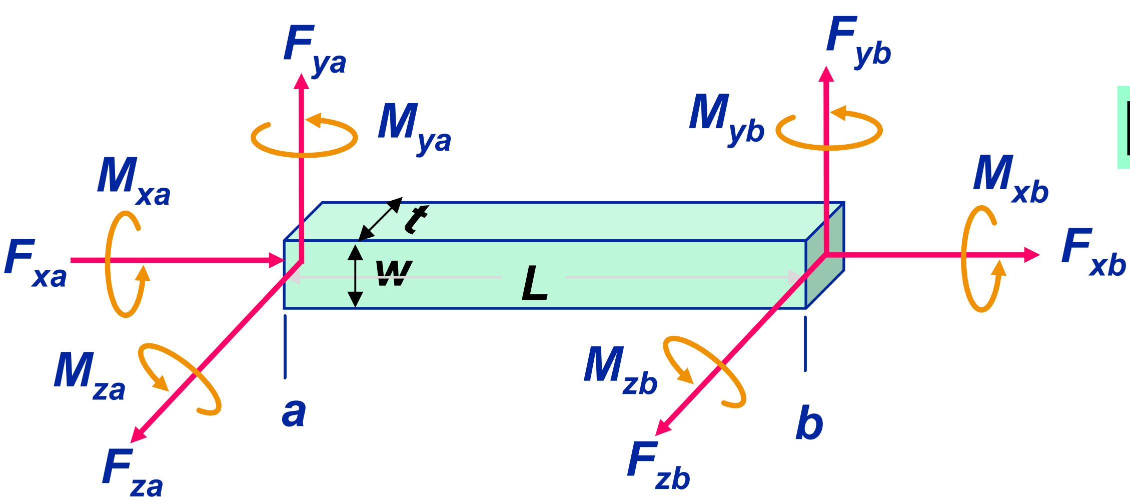


Linear Beam Model

- Captures linear beam mechanics, including



- 12-DOF symmetric composable model
- Model assumptions
  - Small displacements
  - Axial, lateral and torsional motions are independent



$$[F]=[m][\ddot{x}]+[B][\dot{x}]+[k][x]$$

mass matrix

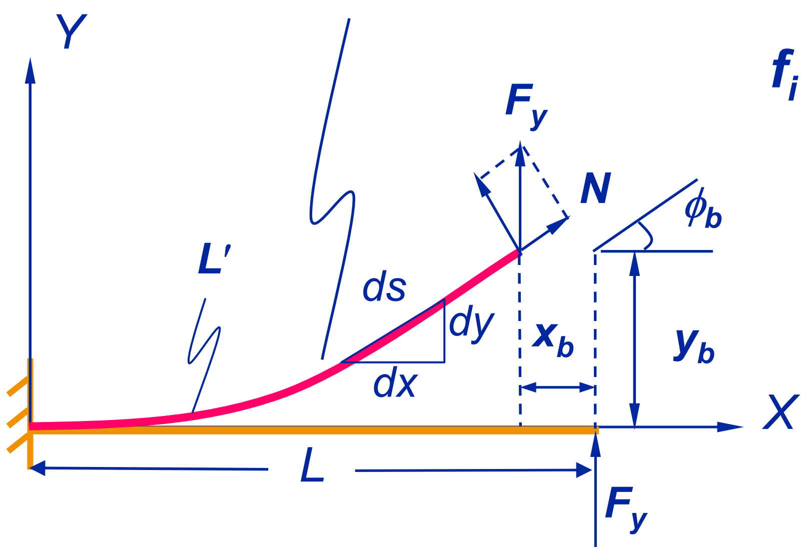
damping matrix

stiffness matrix

Nonlinear Beam Effect I: Large Geometric Deflection

- **Example: Cantilever beam**
- **Beam foreshortening, *x* and *y* are coupled**
- **Force projection into axial stress**
- **Not symmetric formulation**

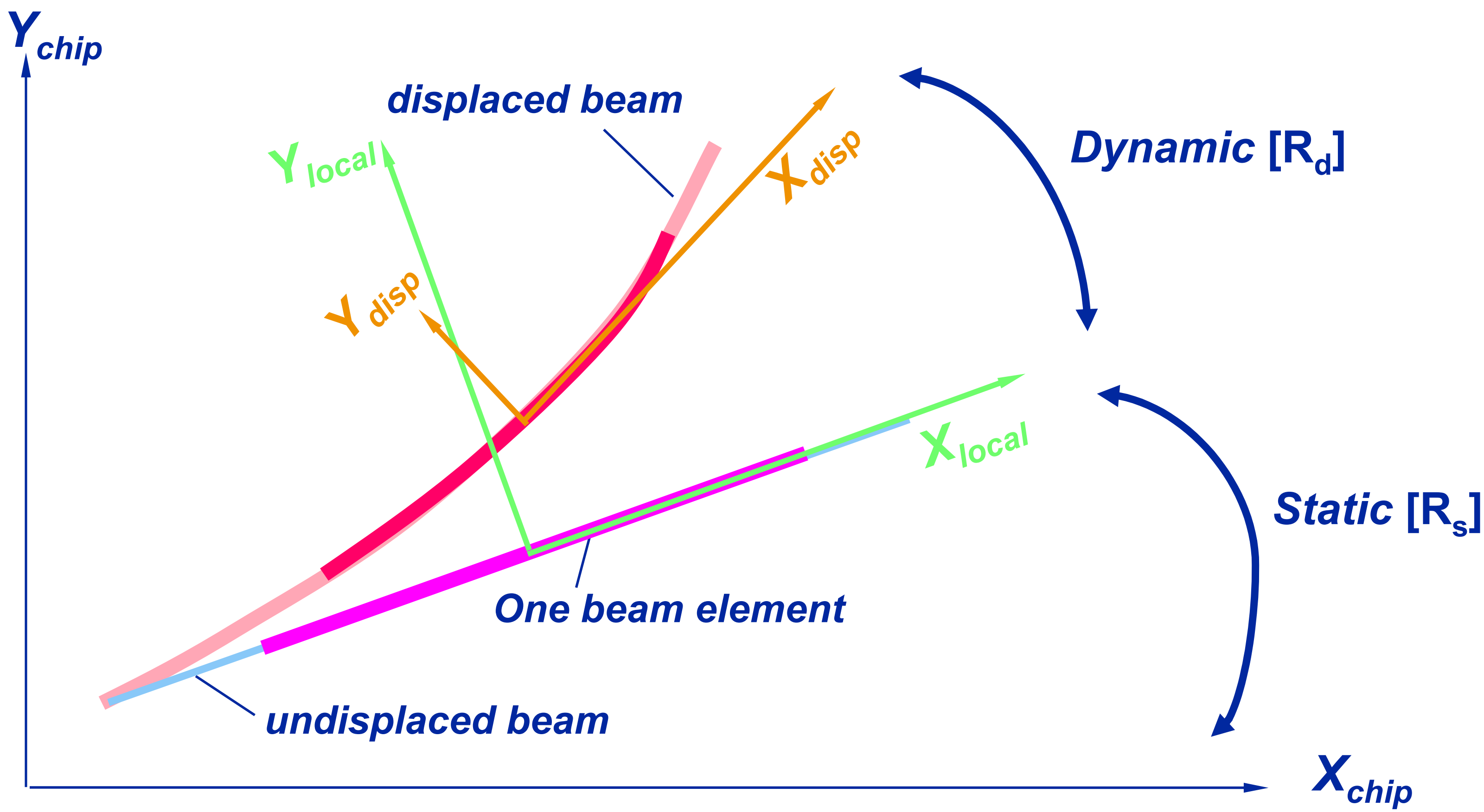
$$y(x) = f_1(x) y_a + f_2(x)\phi_a + f_3(x) y_b + f_4(x)\phi_b$$



f_i (x): cubic shape functions for small displacements

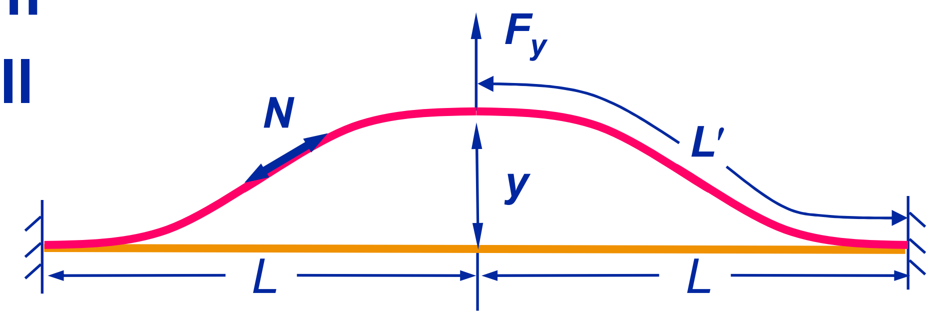
Coordinate Transformation

- Chip frame: specifies layout position
- Local frame: specific to each element
- Displaced frame: shape functions are applied

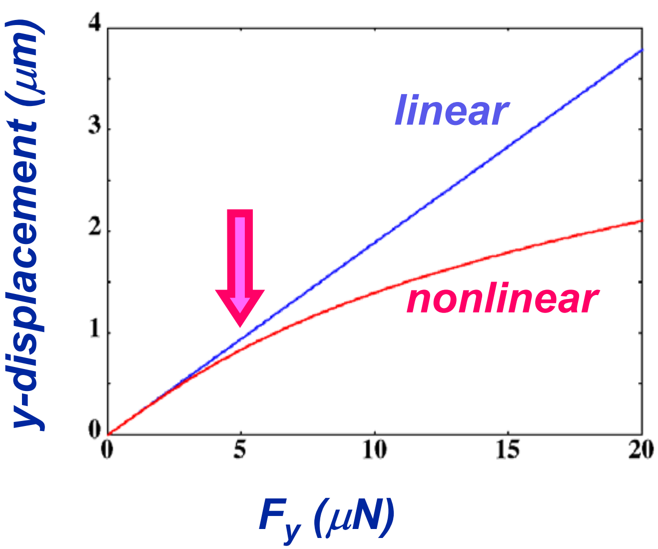


Nonlinear Beam Effect II: Large Axial Stress Stiffening

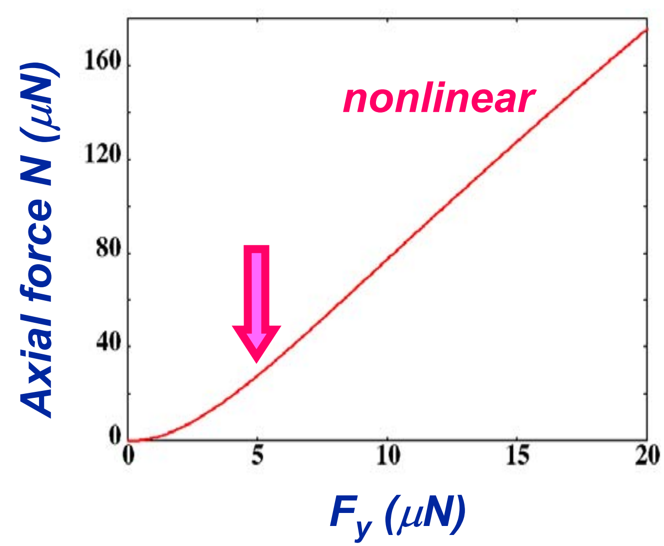
- Example: Fixed-fixed beam
- Nonlinearity starts at small displacement
- Effective beam length, *L′*
- Axial force, *N*



y-displacement



Axial force



Calculation of Axial Stiffening

- Use small angle shape function
- Refer to displaced frame
- Integrate shape to find displaced length

$$y(x) = f_1(x) y_a + f_2(x) \phi_a + f_3(x) y_b + f_4(x) \phi_b$$

$$L' = \int ds = \int_{x_a}^{L + x_b - x_a} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\delta L = L' - L$$

Geometric Stiffness Matrix

$$[F] = [K_0 + K_G(x)][x]$$

Linear stiffness matrix

$$\underbrace{N(x) * [K_{G0}]}_{} = \cancel{\frac{EA}{L}(x_a - x_b)}$$

Geometric stiffness matrix

$$N = \frac{EA}{L}\delta L$$

$$N(x) \quad [K_{G0}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & * & * \\ 0 & * & * & 0 & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & * & * \\ 0 & * & * & 0 & * & * \end{bmatrix}$$

Ref: Przemieniecki, Theory of Matrix Structural Analysis, 1968

Nonlinear Beam Code

```
rphix = (l_phixa+l_phixb)/2.0;
rx = rphix/rphi;
new_l1 = rt*rx*rx + rc;
...
new_l_xb = new_l1*(l_xb+l) + new_m1*l_yb + new_n1*l_zb - l;
...
lp = l+new_l_xb-new_l_xa;
l_eff = (lp*(15*pow(l_new,6)*(2+pow(new_l_phiya,2)...
// calculate the axial force
F_axial = ea*(l_eff-l)/l;
// linear stiffness matrix [k0], with shear deformation
// variables for shear deformation
Asy = 2.0/3.0*area; //effective shear area
...
k0_1_1 = ea/l;
k0_2_2 = 12.0*E*Iz/pow(l,3);
...
k1_1_1 = 0;
k1_2_2 = F_axial*2.0*(6+10*Sy+5*pow(Sy,2))/(10*l*pow(1+Sy,2)); matrix
...
k_1_1 = k0_1_1 + k1_1_1;
k_2_2 = k0_2_2 + k1_2_2;
...
new_Fkxa = -F_axial;
new_Fkxb = F_axial;
new_Fkya = k_2_1*new_l_xa+k_2_2*new_l_ya+k_3_2*new_l_za...
...
Fkxa=inv_new_l1*new_Fkxa+inv_new_m1*new_Fkya+inv_new_n1*new_Fkza;
```

// dynamic rotation matrix from translated local frame to the displaced frame

// transform displacements into the displaced frame

// calculate effective beam length in the dispalced frame

// geometric nonlinear stiffness matrix [kG], with shear deformation

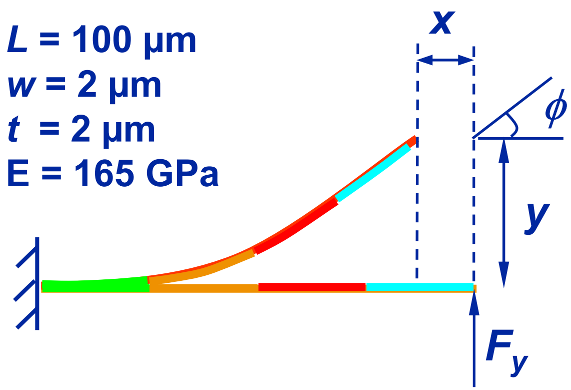
// calculate the spring forces in the displaced frame

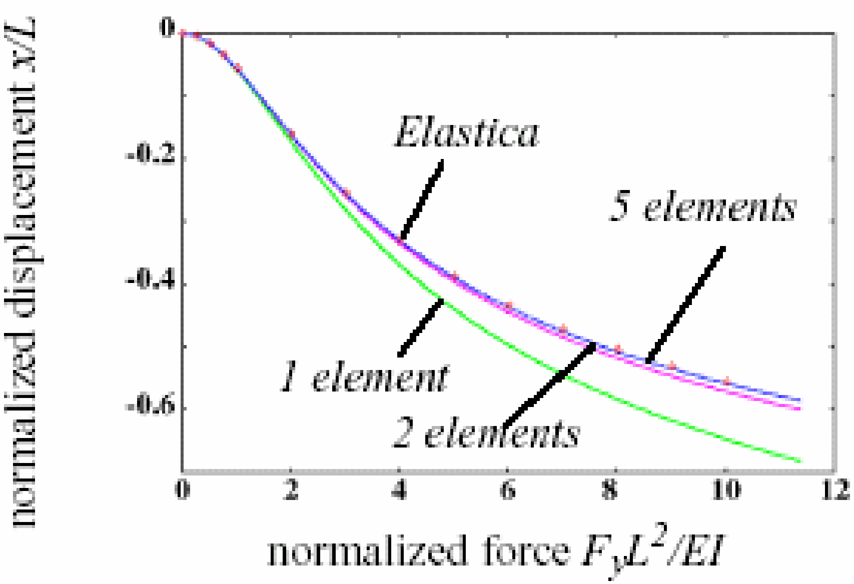
// transform spring forces from the displaced frame back to the local frame

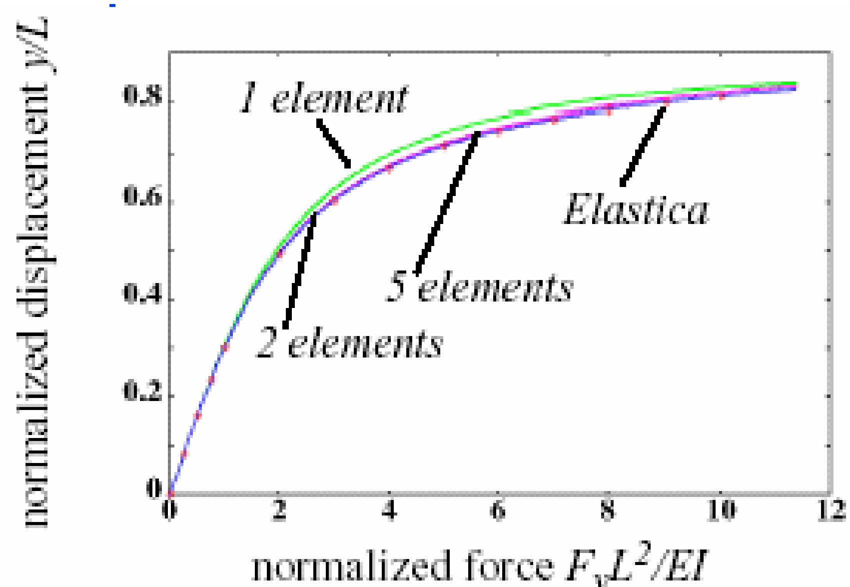
Nonlinear Beam Model: Verification I

- Static analysis of a cantilever beam
- Compared to Elastica

L = 100 µm
w = 2 µm
t = 2 µm
E = 165 GPa

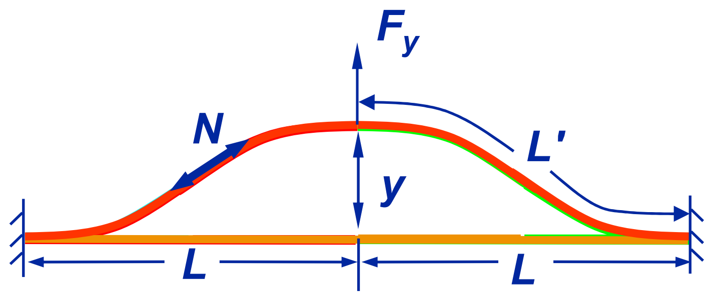




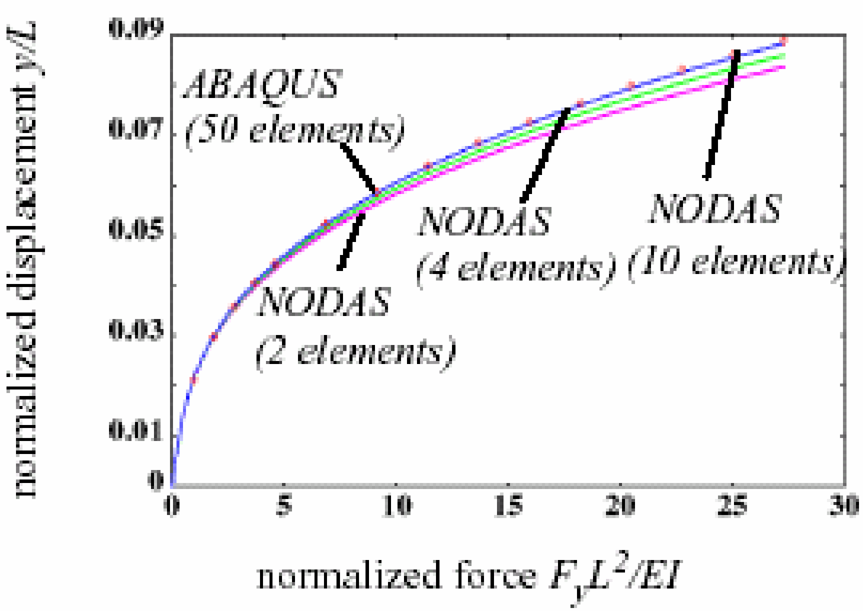


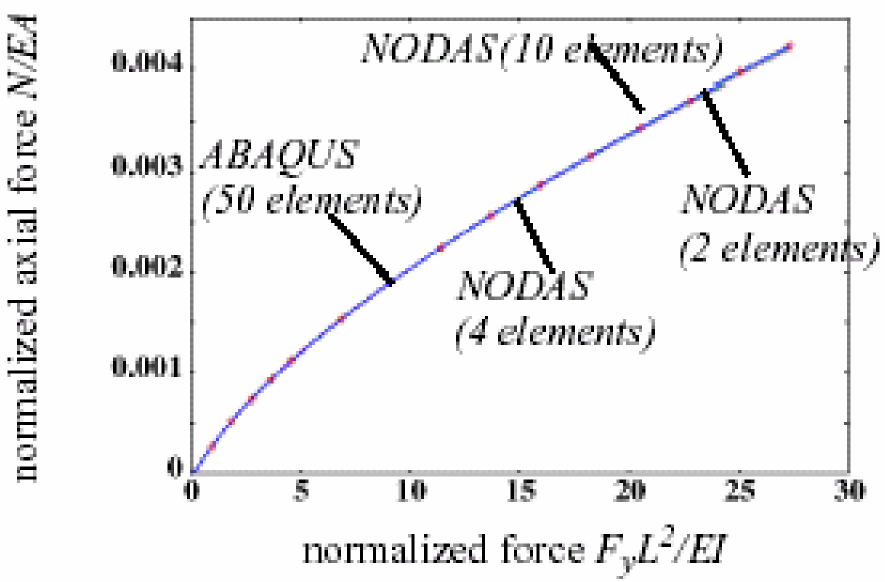
Nonlinear Beam Model: Verification II

- Static analysis of a fixed-fixed beam
- Compared to ABAQUS



L = 100 μm
w = 2 μm
t = 2 μm
E = 165 GPa

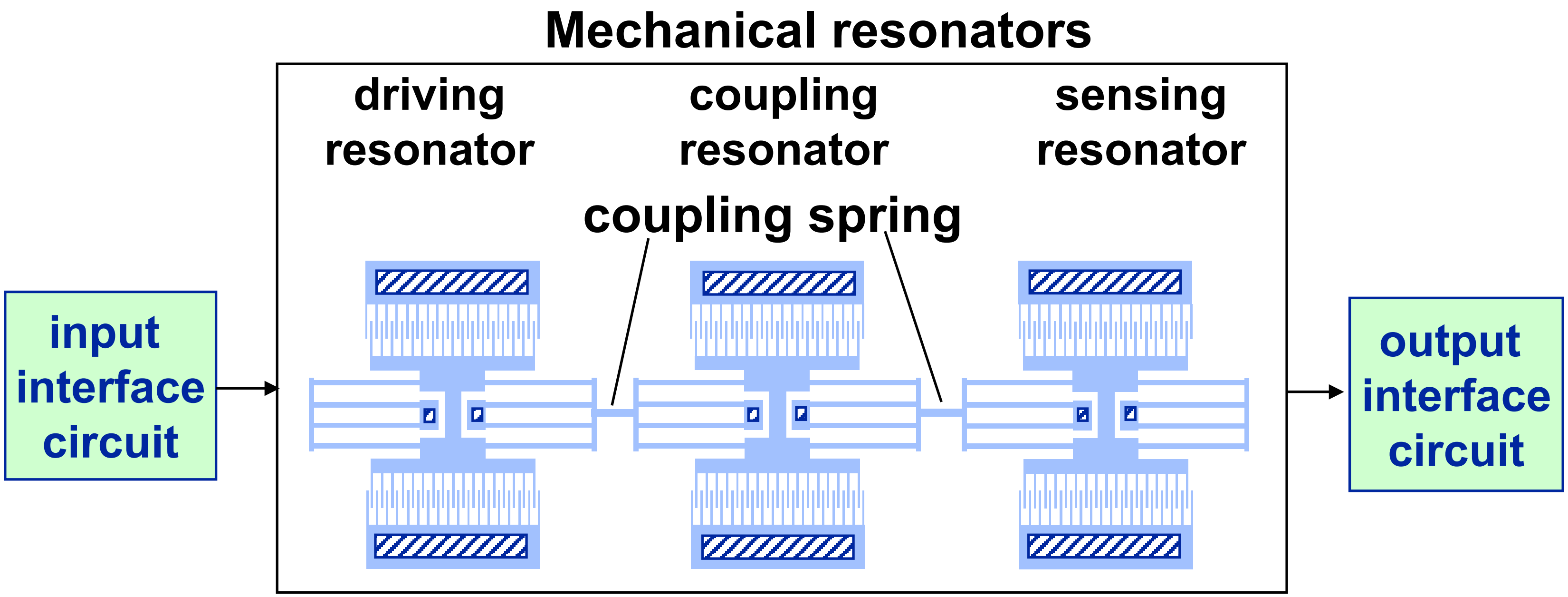




Issues in MEMS Macromodeling

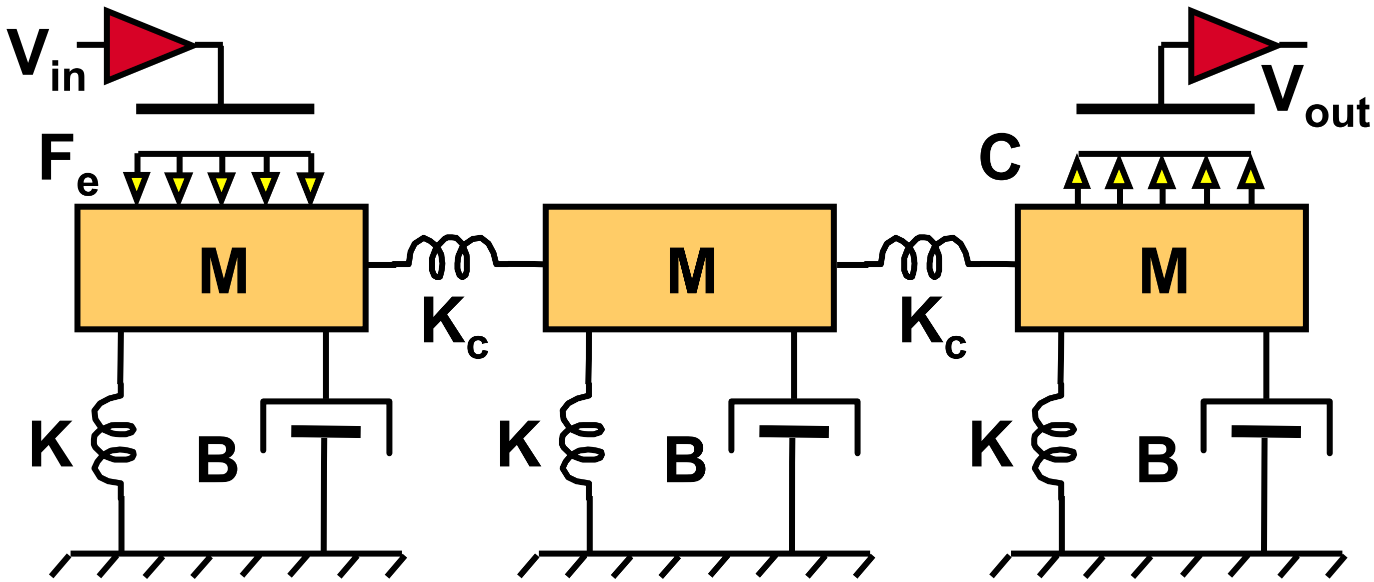
- Processes and Applications
- Design Views
- Natures and Disciplines
- Hierarchy
- Interoperability
- Beam Element Example
- Filter Example
- Language Extension

Design Example: Third order Bandpass Filter

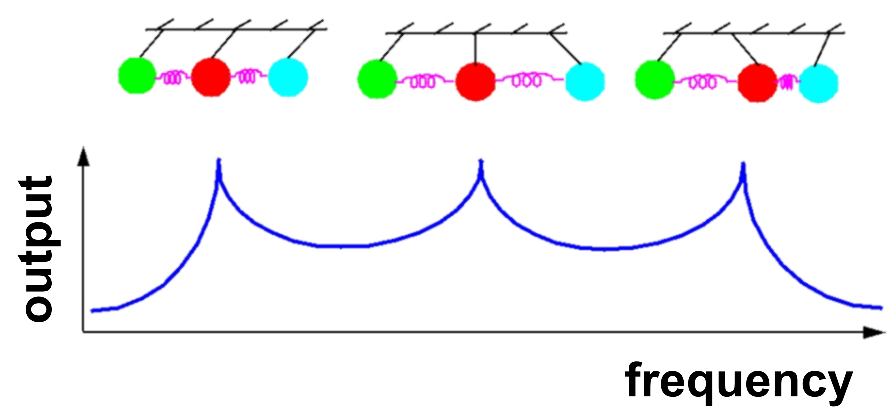


- Topology due to Wang & Nguyen (MEMS ’97)
- More complex than just a single resonator
  - Multi-domain interactions
  - Multiple functional devices
  - Coupling is crucial to the performance

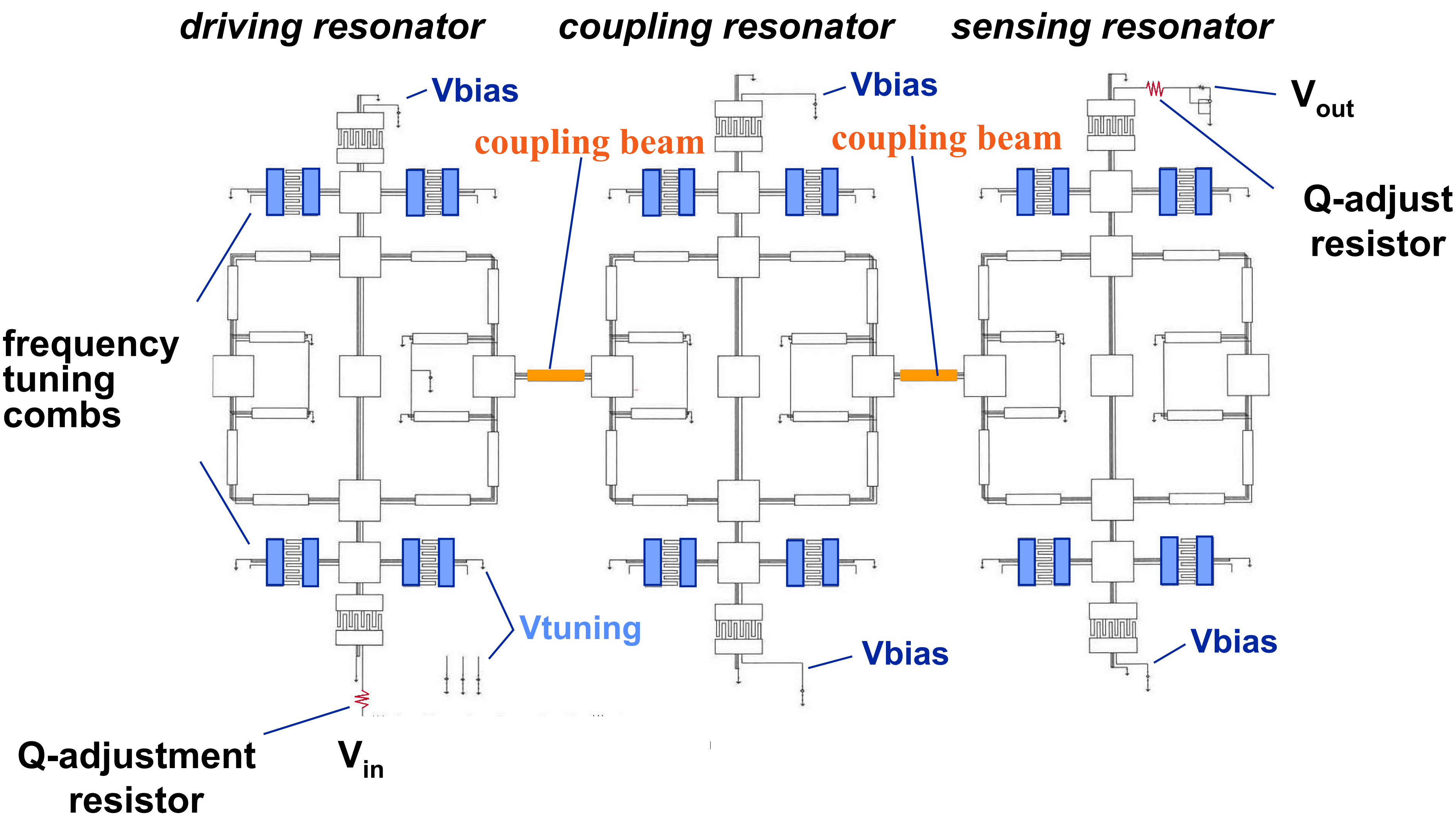
Design Example: Micromechanical Bandpass Filter



- Cascade of 3 coupled microresonators

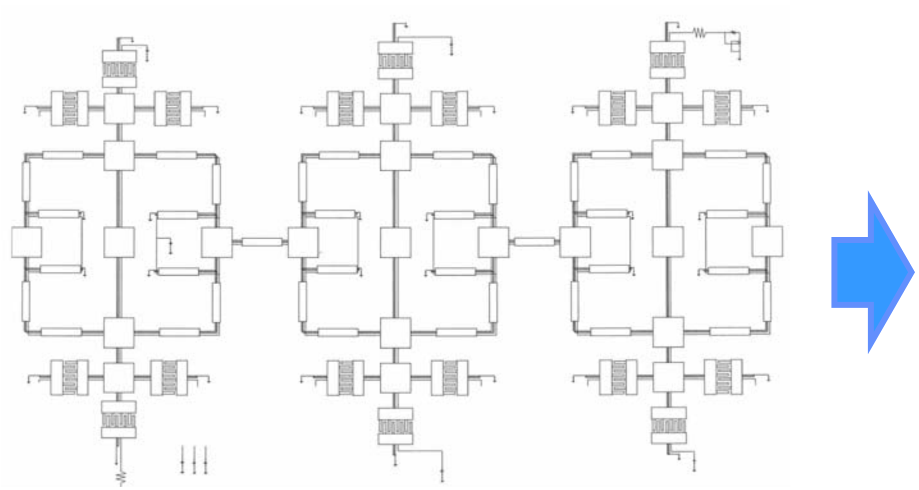


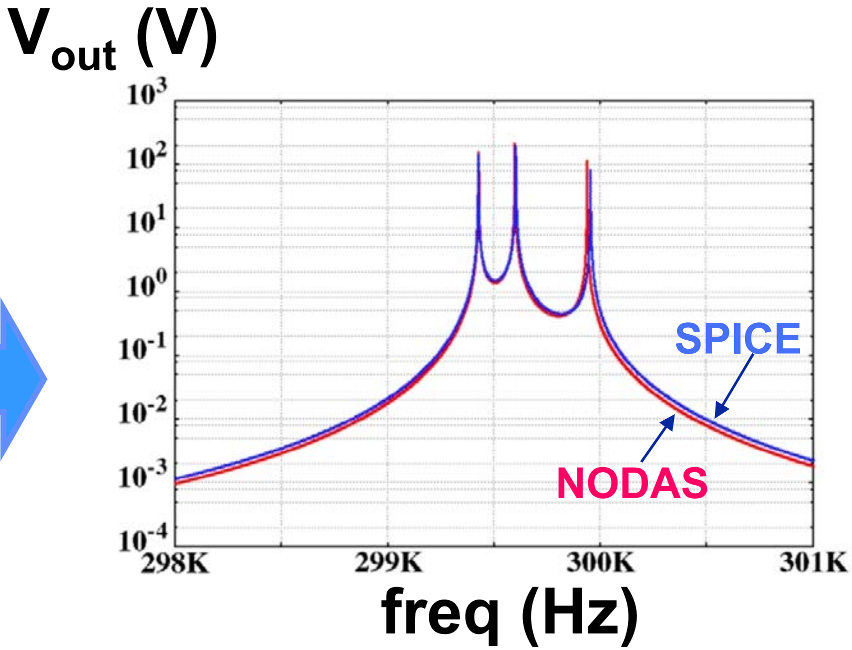
MEMS Bandpass Filter



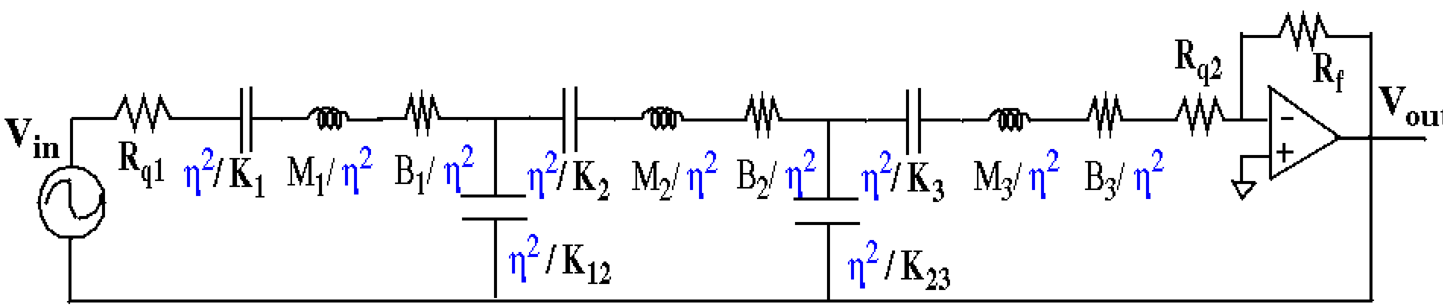
Bandpass Filter Verification

- **NODAS MEMS Circuit**



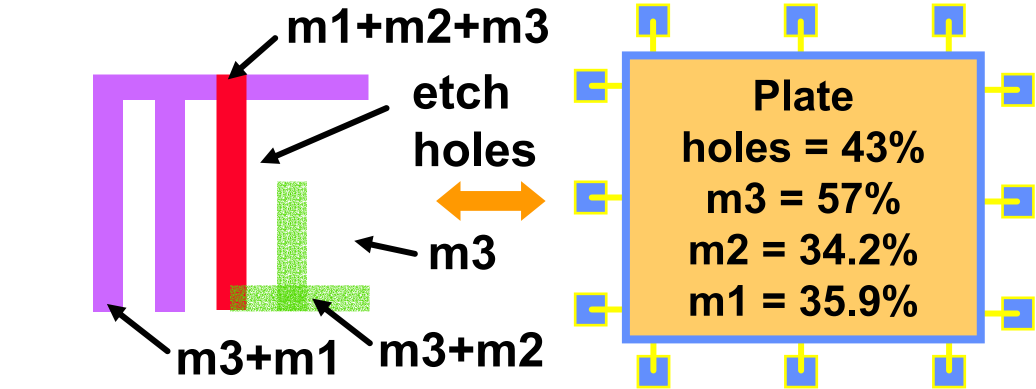


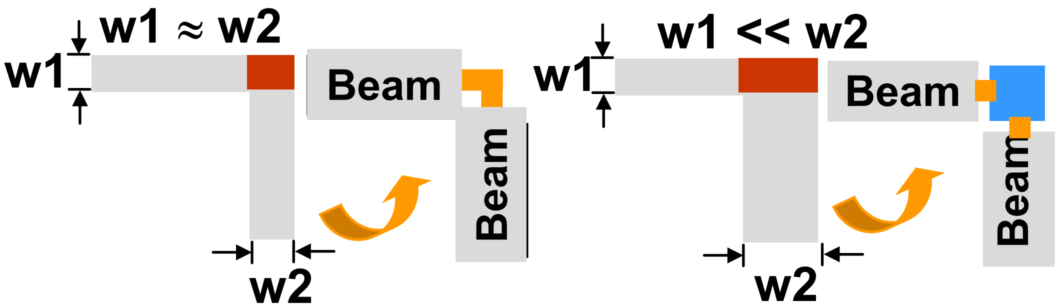
- **Equivalent Linear SPICE Circuit**

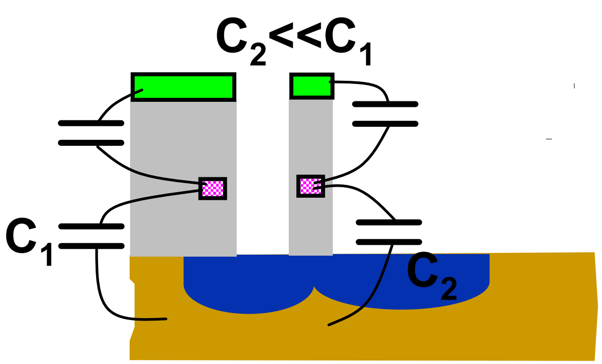


Ref: Q. Jing et al., IEEE MEMS 2000

Parasitics in CMOS MEMS



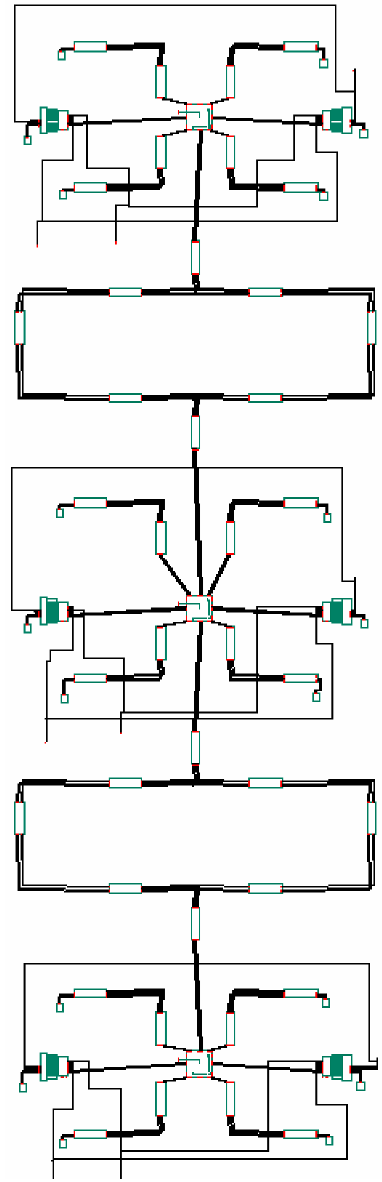


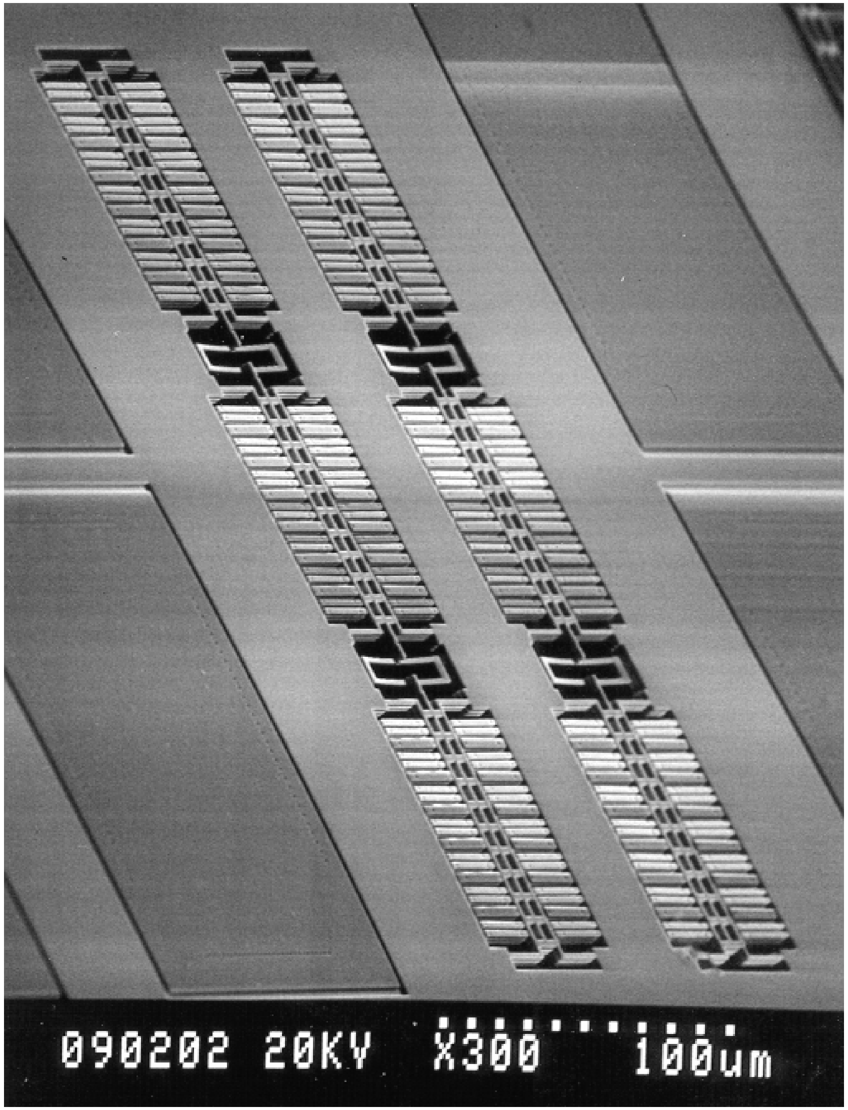


additional substrate gap in MEMS areas

- Mechanical parasitics
  - Parasitic mass
    - Holes
    - Routing metal
  - Parasitic joints
    - Varying beam widths
    - Similar length of beam and joints
- Electrical parasitics
  - Metal to substrate gap

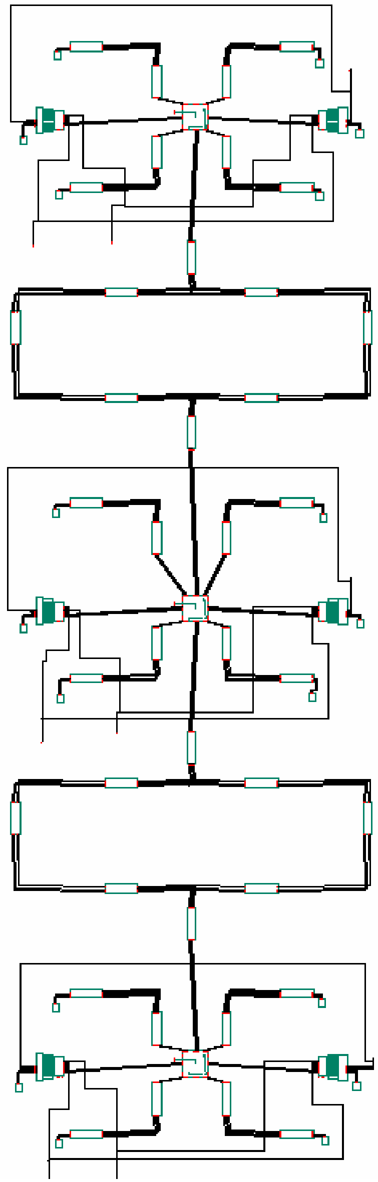
Verification by Layout Extraction

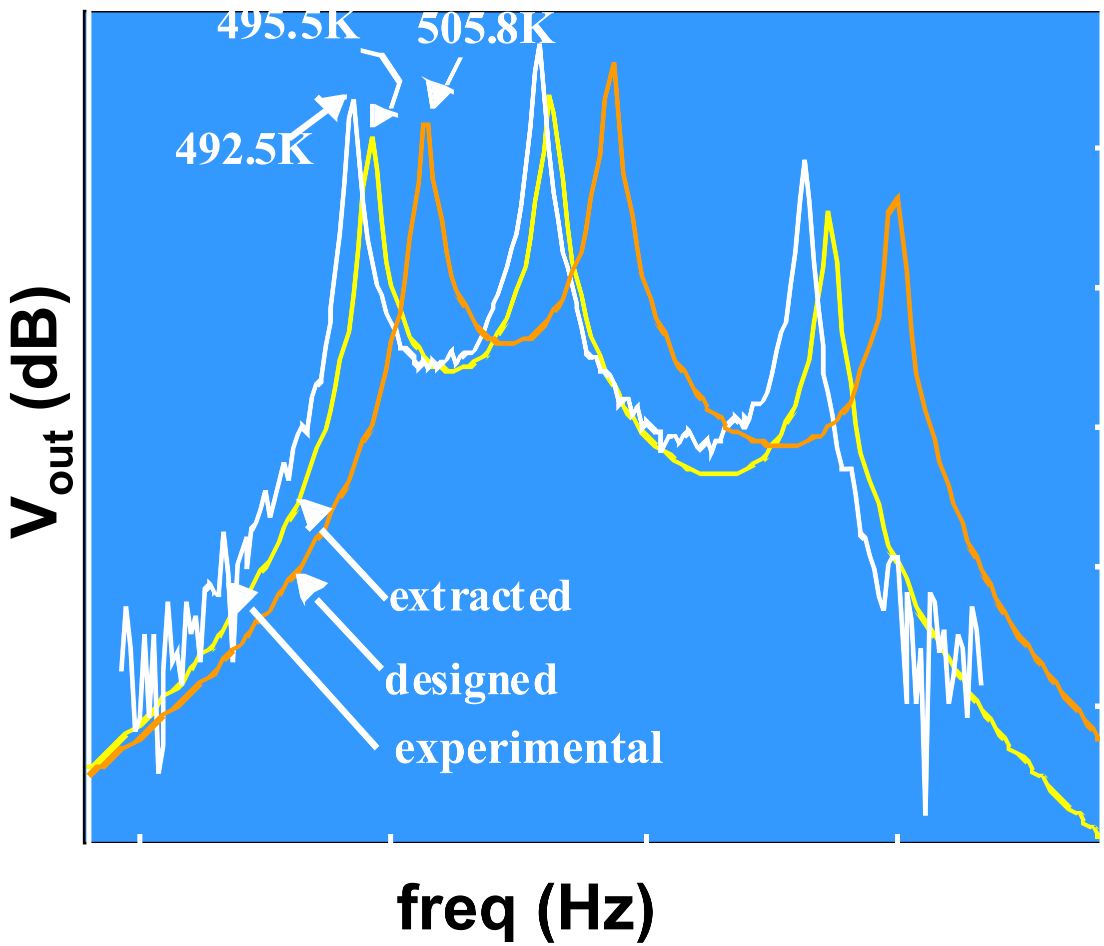




Baidya, Gupta & Mukherjee, IEEE JMEMS, Feb 2002

Verification by Layout Extraction





- Experimental results match extracted schematic

Baidya, Gupta & Mukherjee, IEEE JMEMS, Feb 2002

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Proposed Composite Discipline

```
multidiscipline mems2D
   discipline dx : kinematic_translational
         potential.access = "Dx";
         flow.access = "Fx";
   enddiscipline
   discipline dy : kinematic_translational
         potential.access = "Dy";
         flow.access = "Fy";
   enddiscipline
   discipline phi : kinematic_rotational
   enddiscipline
endmultidiscipline
```

definitions from base discipline

Proposed Composite Discipline

```
multidiscipline mems2D
  discipline dx : kinematic_translational
      potential.access = “Dx”;
      flow.access = “Fx”;
  enddiscipline
  discipline dy : kinematic_translational
      potential.access = “Dy”;
      flow.access = “Fy”;
  enddiscipline
  discipline phi : kinematic_rotational
  enddiscipline
endmultidiscipline
```

redefine
access
functions

Composite Discipline Use

- Desire equations that are concise, easy to read
- Syntax looks like standard disciplines
- Could use matrix notation…


```
inout a, b;
mems2D a, b;
      ...
real kxx, kyy, ktaa, ktab, ktbb, kxy, kxt, kyt;
analog begin
      ...
      Fx(a, b) <+ kxx*Dx(a,b) + kxy*Dy(a,b) + kxt*Phi(a,b);
      Fy(a, b) <+ kxy*Dx(a,b) + kyy*Dy(a,b) + kyt*Phi(a,b);
      Tau(a)   <+ kxt*Dx(a,b) + kyt*Dy(a,b) + ktaa*Phi(a) + ktab*Phi(b);
      Tau(b)   <+ kxt*Dx(a,b) + kyt*Dy(a,b) + ktab*Phi(a) + ktbb*Phi(b);
end
```

Conclusions

- MEMS behavioral modeling with A-HDLs is slowly being adopted
- Still far from the preferred starting point in design

- Accurate behavioral models exist for mechanics
- Near-term Work:
  - General electrostatic gap
  - Thermomechanics, electrothermal, piezo-effects
- Atomic-level models in other domains way behind

- Open standard cell library is needed
  - Need standard conventions adopted for interoperability
  - Provides templates for behavioral modeling
  - Promotes structured modeling

Acknowledgements

- NSF, DARPA and Pittsburgh Digital Greenhouse
- Tamal Mukherjee
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  - Jan Vandemeer
  - Mike Kranz
  - Qi Jing
  - Gilbert Wong