Symbolic Model Order Reduction (SMOR)

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Outline of Presentation

- Motivation for Symbolic MOR
- Review of PRIMA
- Isolation method
- Single frequency point method
- Multiple frequency point method
- Conclusion

Motivation for SMOR

- Numerical Model Order Reduction Methods (AWE,PACT, PRIMA, PVL, NORM etc) are in general efficient and accurate
- But the reduced model **lacks flexibility**

Motivation For SMOR

- **Sweep analysis** to find optimal design values
- **Process variation analysis** on interconnect (width, length, thickness, conductivity etc)
- **Repeating analysis** on similarly structured circuits with different element values

……

Motivation For SMOR

- Symbolic MOR is proposed to incorporate symbols in the Reduced Model
- With symbolic reduced model, we could handle those analyses more efficiently
- Symbolic model provides Flexibility compared to Numerical Reduced Model

Motivation for SMOR

Numerical MOR

One time reduction

SMOR

Sweep analysis

Variation analysis

Repeating analysis on similar circuits

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PRIMA review

$$C\dot{x} + Gx = b\mathbf{u} \qquad \Rightarrow \qquad C_r\dot{z} + G_r z = b_r\mathbf{u}$$

$$y = l^T x \qquad \qquad y = l_r^T z$$

By Congruence Transformation

$$C_r = V^T C V \quad G_r = V^T G V \quad b_r = V^T b \quad l_r = V^T l$$

$$V = \text{Krylov}\{G^{-1}C, G^{-1}b\}$$

PRIMA review cont’

SMOR is based on PRIMA for **stability** and **simplicity** of reduced model

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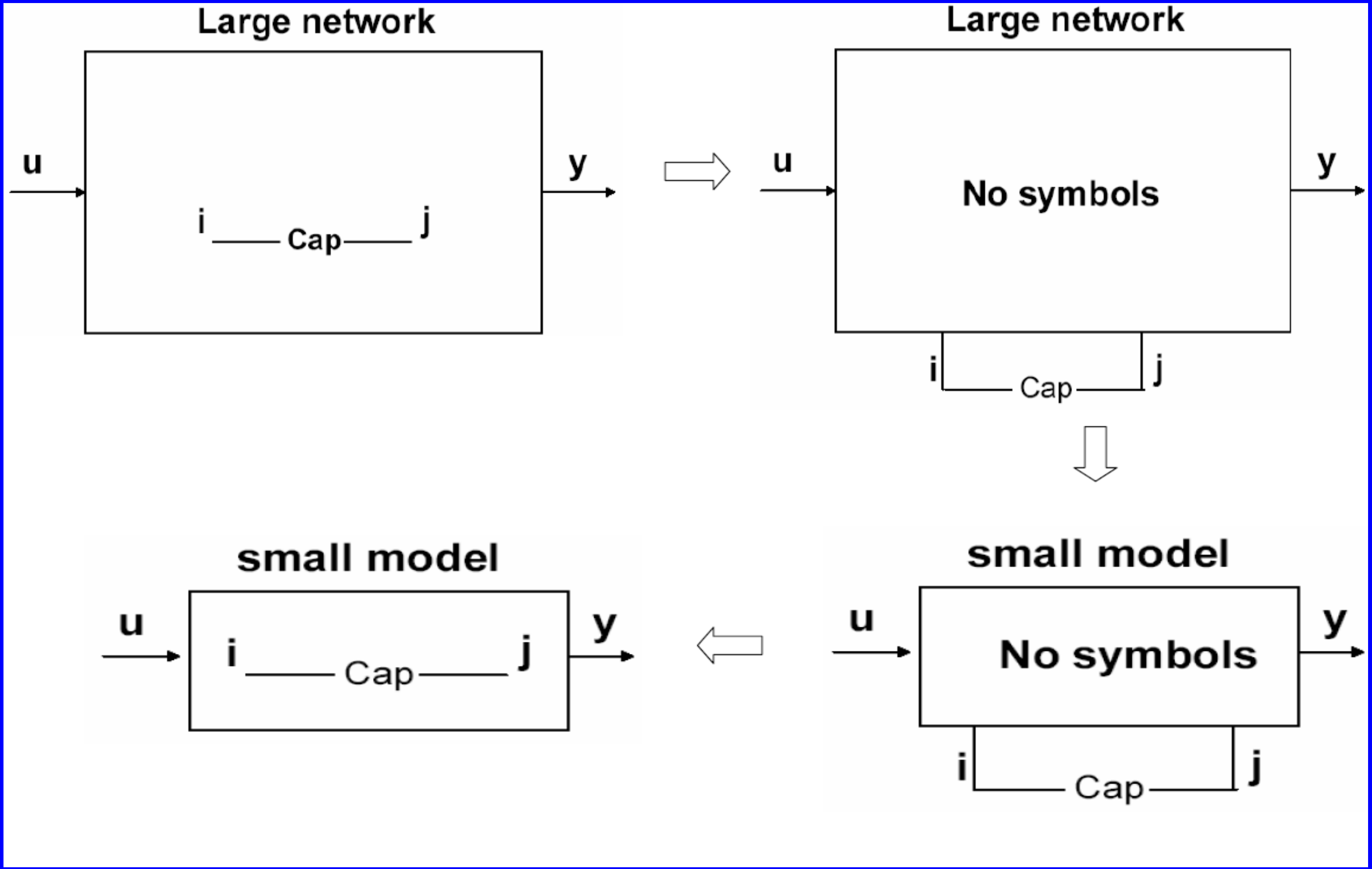
Symbol isolation method

- Applicable for sweep analysis
- Simple and Easy to implement

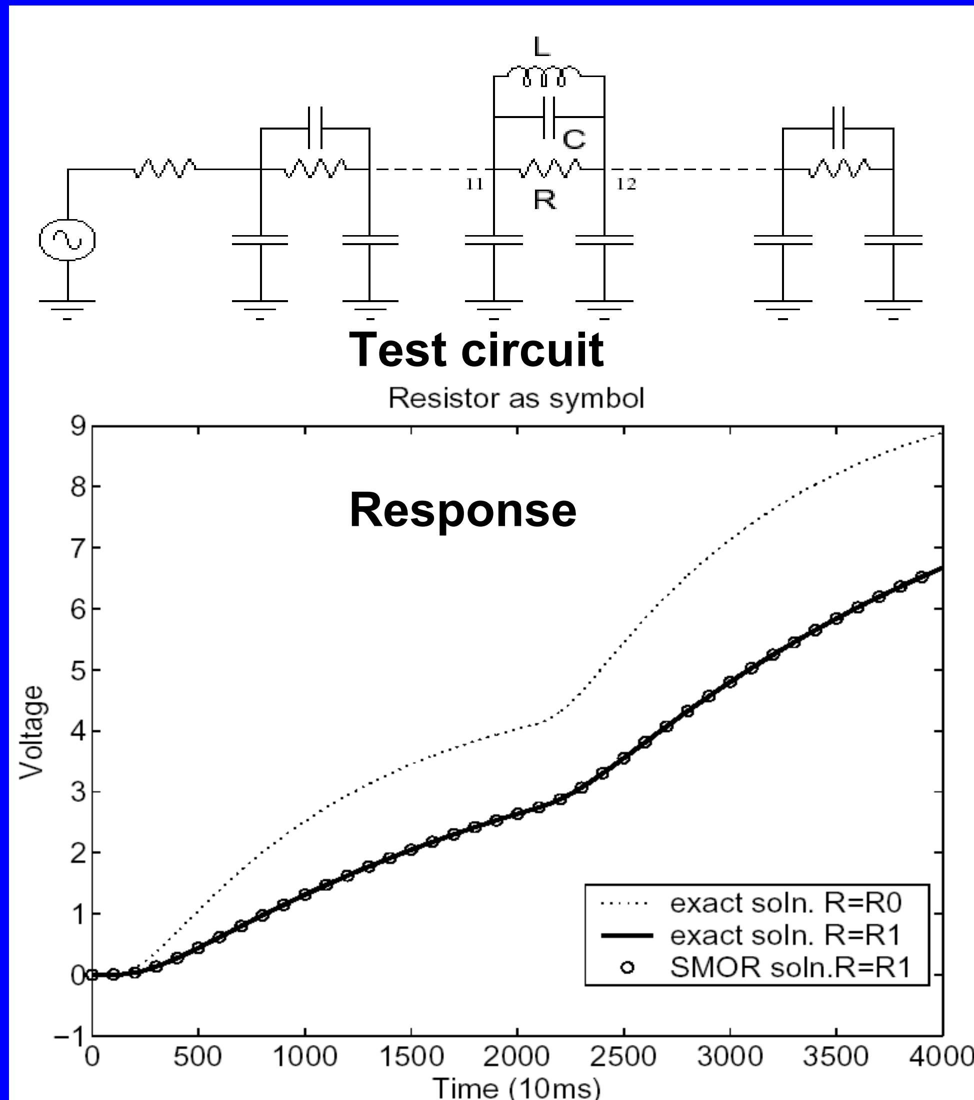
Symbol Isolation Method

- Each symbolized element is isolated from the circuit
- The interaction between each symbolized element and the remaining circuit is treated as one port
- Reduction is done on the remaining circuit
- Each symbolized element will be merged back into reduced model

Symbol Isolation Method cont'



test case


L
C
R
11
12
Test circuit
Resistor as symbol
Response
Voltage
9
8
7
6
5
4
3
2
1
0
−1
0
500
1000
1500
2000
2500
3000
3500
4000
Time (10ms)
exact soln. R=R0
exact soln. R=R1
SMOR soln.R=R1

Single frequency point method

- Single frequency point method handles symbols for more general case than symbol isolation method
- Based on PRIMA algorithm; focus is on the construction of transformation matrix V
- Constructs V without orthonormalization

  Or with pseudo orthonormalization
- Perform congruence transformation

Single Frequency Method

$$\mathrm{H(s)} = l^{\mathrm{T}} (\mathbf{G} + s\mathbf{C})^{-1} b \qquad (\mathrm{H(s)\ is\ transfer\ function})$$

$$= l^{\mathrm{T}} \left\{ \mathbf{G}^{-1} b - s\mathbf{G}^{-1}\mathbf{C}\mathbf{G}^{-1} b + s^2 \left(\mathbf{G}^{-1}\mathbf{C}\right)^2 \mathbf{G}^{-1} b + \ldots + s^{\mathrm{q}} \left(-\mathbf{G}^{-1}\mathbf{C}\right)^{\mathrm{q}} \mathbf{G}^{-1} b + .. \right\}$$

$$\mathbf{V} = colsp\left\{ \mathbf{G}^{-1} b, \mathbf{G}^{-1}\mathbf{C}\mathbf{G}^{-1} b, \left(\mathbf{G}^{-1}\mathbf{C}\right)^2 \mathbf{G}^{-1} b, \ldots, \left(\mathbf{G}^{-1}\mathbf{C}\right)^{\mathrm{q}} \mathbf{G}^{-1} b \right\}$$

V is constructed without orthonormalization

different from numerical reduction method such as PRIMA

Single Frequency Method

1). Symbolically inverse **G**

$$\mathbf{G}^{-1} = \text{inverse}(\mathbf{G})$$

2). Do matrix - vector multiplication

$$v_1 = \mathbf{G}^{-1} b$$

3). For k = 2 to q (q is the size of reduced model)

$$v_k = \mathbf{G}^{-1} \mathbf{C} v_{k-1}$$

4). For k = 1 to q, construct transformation matrix **V**

$$\mathbf{V}(:,k) = v_k$$

Problems

1. Condition number of V
2. Symbolic inversion of G
3. Model Accuracy

Pseudo orthonormalization

- Assume each symbol only varies moderately from its nominal value
- Use rotation matrix from numerical orthonormalization to rotate symbolically constructed matrix V

Pseudo orthonormalization

1) Every symbol takes its nominal value

$$\mathbf{V}_{\text{symbolic}} \Rightarrow \mathbf{V}_{\text{nominal}}$$

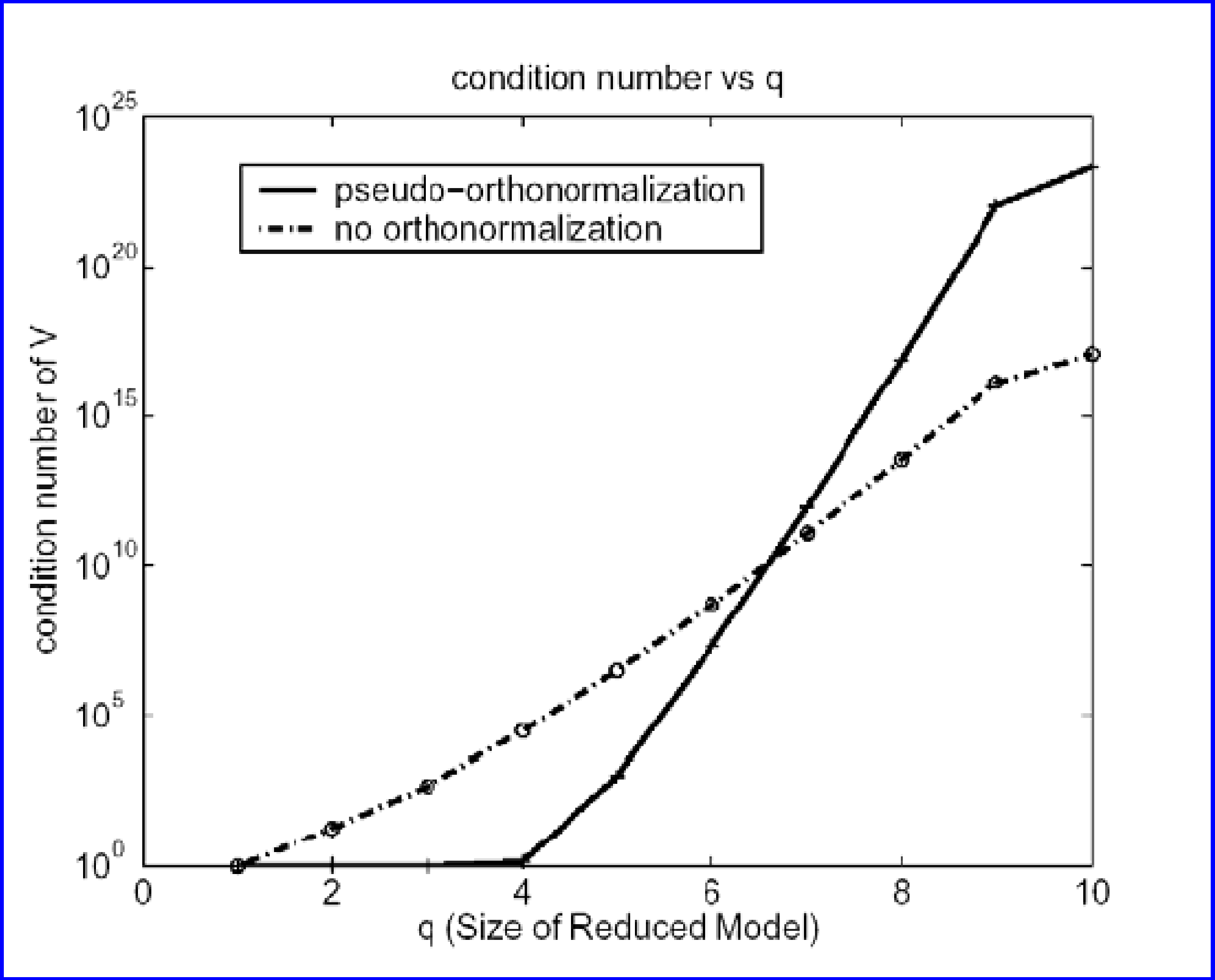
2) Find $\mathbf{R}_{\text{nominal}}$ to orthonormalize $\mathbf{V}_{\text{nominal}}$

$$\mathbf{V}_{\text{nominal}}\mathbf{R}_{\text{nominal}} \Rightarrow \mathbf{V}_{\text{orthnorm}}$$

3) Use $\mathbf{R}_{\text{nominal}}$ to pseudo orthonormalize $\mathbf{V}_{\text{symbolic}}$

$$\mathbf{V}_{\text{symbolic}}\mathbf{R}_{\text{nominal}} \Rightarrow \mathbf{V}_{\text{pseudo-orthnorm}}$$

Pseudo orthonormalization



Successive first order approximation

- **Symbolic inversion** of G is computationally very expensive operation*
- **Successive first order approximation** method will reduce the computation cost to a manageable level

Successive first order approximation

Suppose $\mathbf{G} = \mathbf{G}_0 + \Delta\mathbf{G}$

$\mathbf{G}_0$: nominal valued numerical matrix

$\Delta\mathbf{G}$: symbolic perturbation matrix

The perturbation is small (+/- 30% of nominal value)

By First Order Approximation

$$\mathbf{G}^{-1} \approx \mathbf{G}_0^{-1} - \mathbf{G}_0^{-1}\Delta\mathbf{G}\mathbf{G}_0^{-1}$$

Successive first order approximation

$C = C_0 + \Delta C, G = G_0 + \Delta G$

ΔC and ΔG are small perturbation matrices (symbolic)

Keep only first order

$\left(G + \Delta G\right)^{-1} b$ ← **First column vector of V**

$\approx G_0^{-1} b - G_0^{-1} \Delta G G_0^{-1} b$

$= \left\{G_0^{-1} b\right\} + \left\{\Delta G\right\}$

……

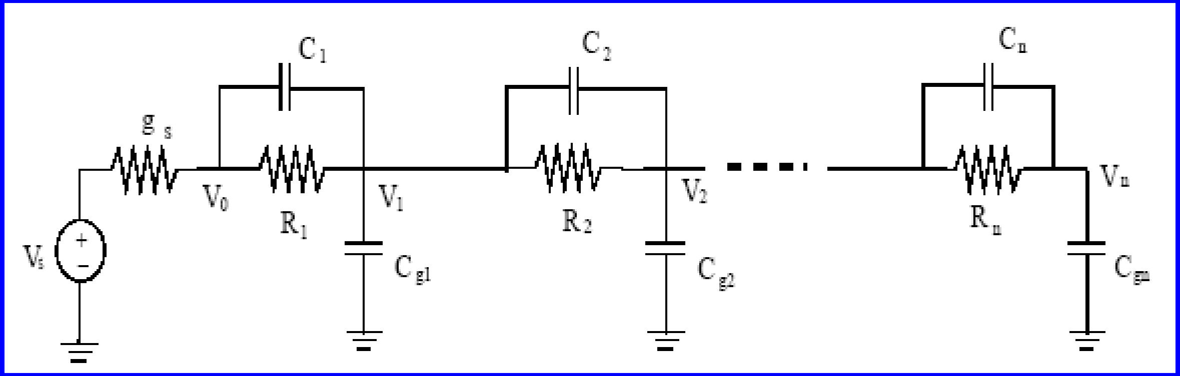
$\left(G_0 + \Delta G\right)^{-k} \left(C_0 + \Delta C\right)^{k} \left(G_0 + \Delta G\right)^{-1} b$

$\approx \left\{0\text{th order}\right\} + \left\{1st \text{ order of } \Delta G\right\} + \left\{1\text{st order of } \Delta C\right\}$

Successive first order approximation

- After V is constructed, perform Congruence Transformation as in PRIMA
- Keep all orders in Congruence Transformation, otherwise, model could diverge

Test case and experimental observations

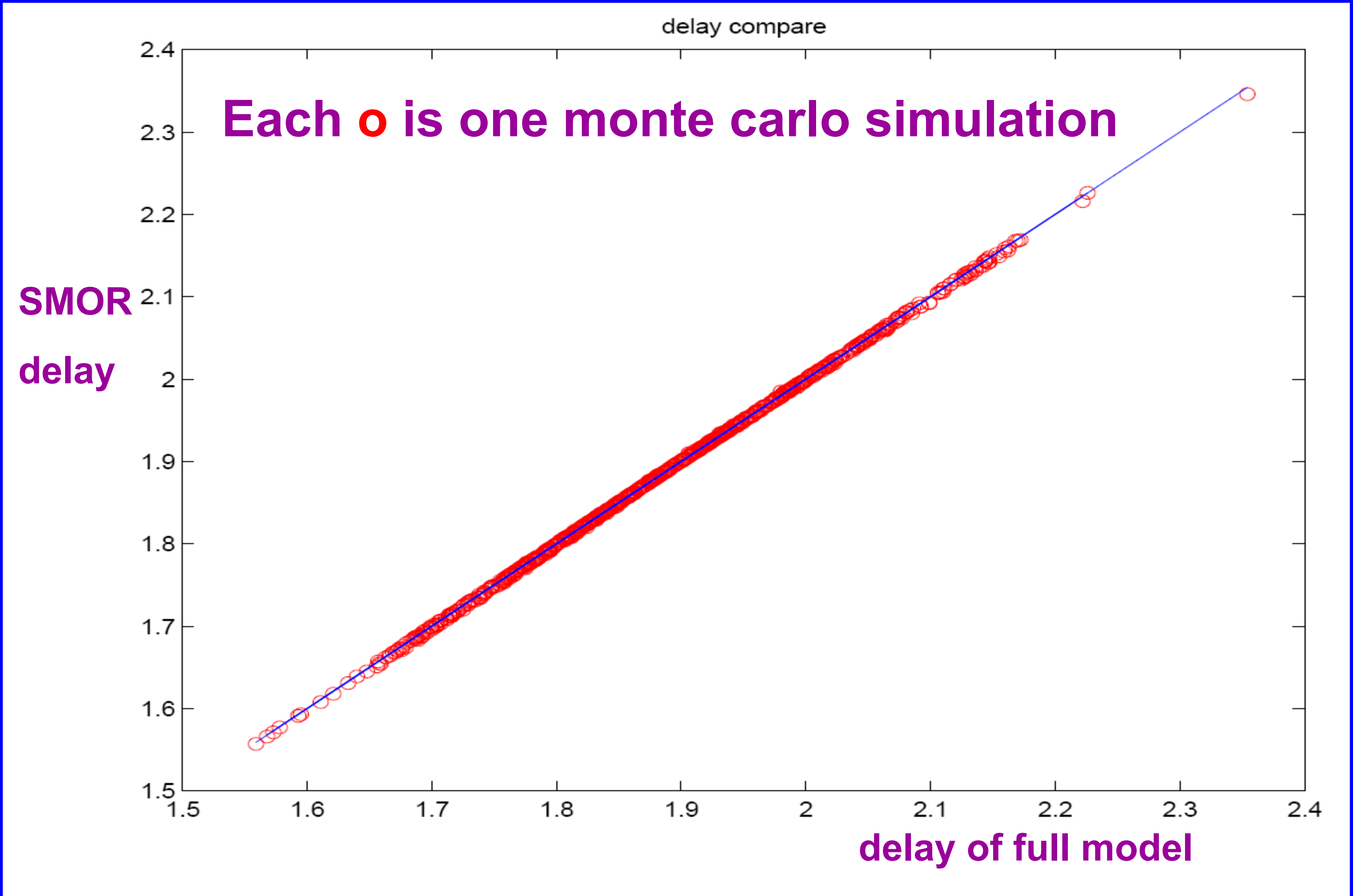


- RC ladder example to simulate the process variation(+/- 30%)
- 4 symbolic parameters are put into the circuit

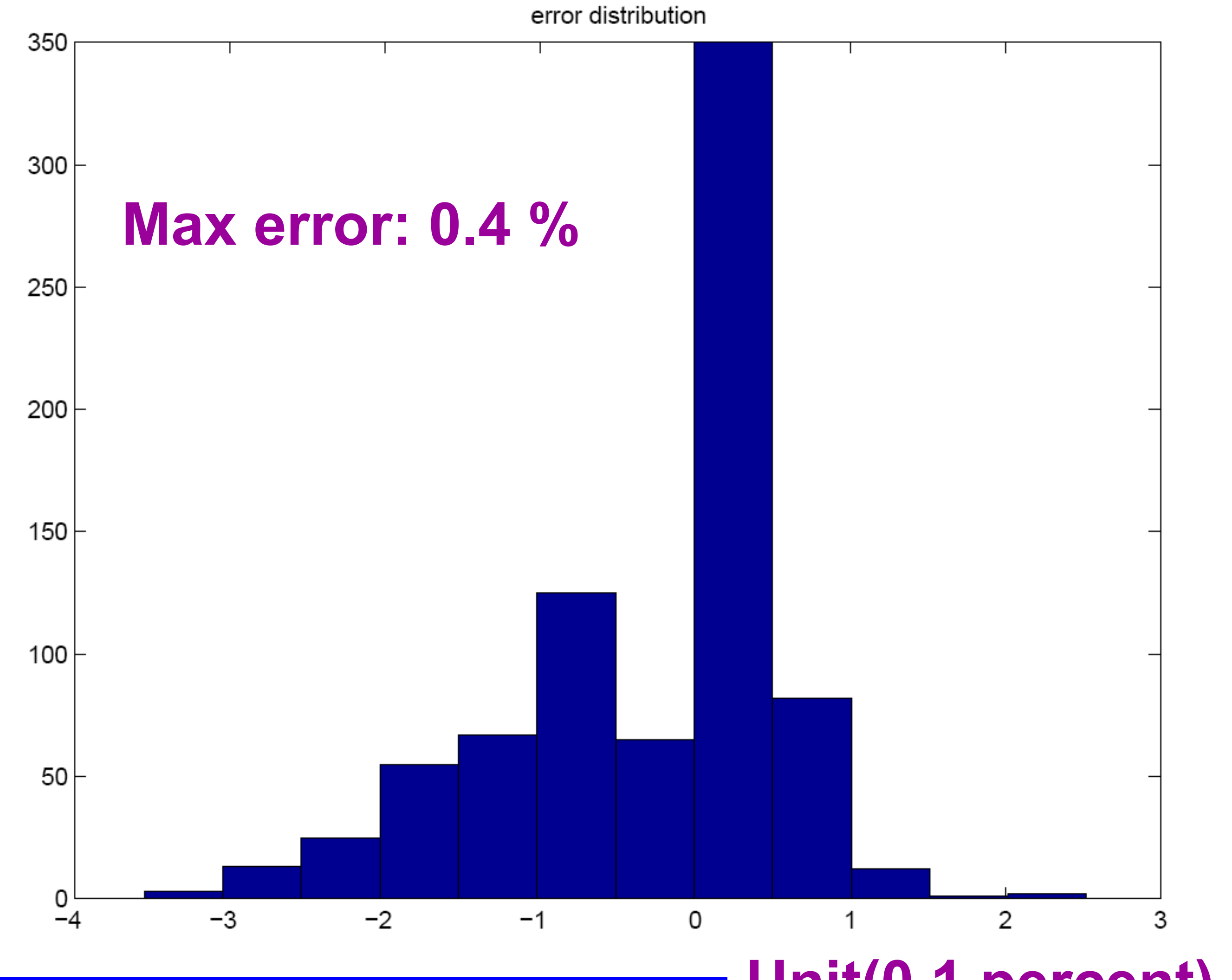
Test case and experimental observation

- Original size of circuit matrices: 100
- Reduced symbolic model size: 4
- Statistical Perturbation of 4 symbolic parameters: +/- 30%
- 50-50 Delay of step input response is measured

delay-delay plot



$$\frac{\text{Delay of reduced model} - \text{Delay of full model}}{\text{Delay of full model}}.$$



Unit(0.1 percent)

Multiple Frequency Point Method

- Using Multiple frequency points to improve model accuracy at higher frequency range
- In the same time, reduce the condition number of V during Symbolic MOR

Multiple Frequency Method

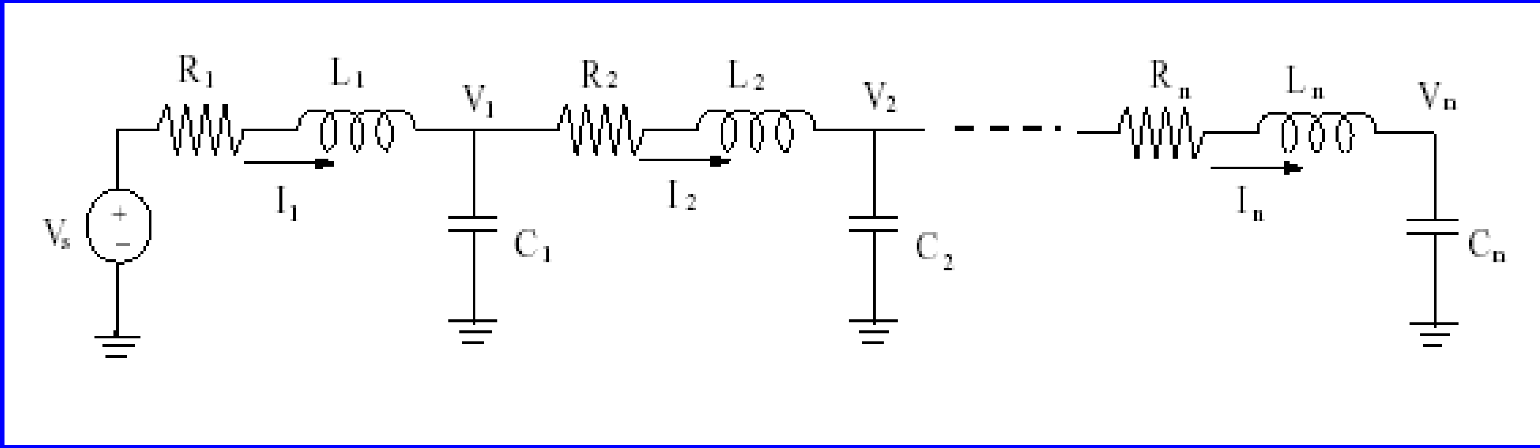
For each choice of σ, we generate a few vectors in the Krylov subspace:

$$\mathrm{V}_\sigma = \mathrm{Krylov}\left\{(C\sigma + G)^{-1} C, (C\sigma + G)^{-1} b\right\}$$

If we use multiple frequency points $\{\sigma_1, \sigma_2, \ldots, \sigma_\mathrm{n}\}$ transformation matrix V is given by grouping each subspace together:

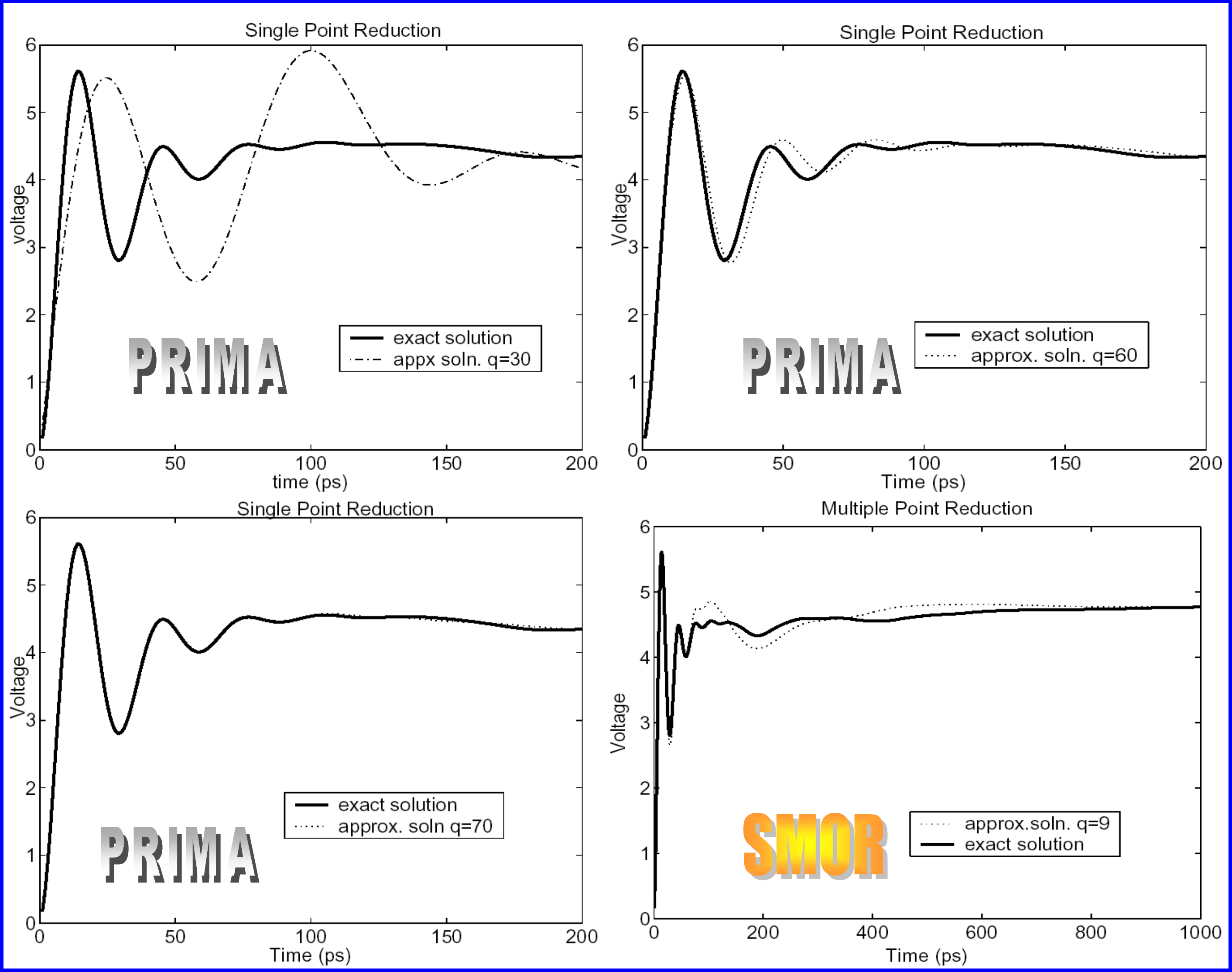
$$V = colsp\{V_{\sigma_1}, V_{\sigma_2}, \ldots, V_{\sigma_n}\}$$

Multiple Frequency method



Test circuit (300 blocks)

Multiple Frequency Method



Conclusion

- Isolation method is simplest, but applicable to only special cases
- Single frequency point method applies to low order reduction
- Multiple frequency point method improves the order of reduced model

Conclusion cont'

- Difficulties in reducing the condition number of symbolic model
- Symbolic inversion is expensive*
- Number of symbols is limited(a few)

SMOR need further research!