Modeling Nonlinear Communication ICs Using a Multivariate Formulation

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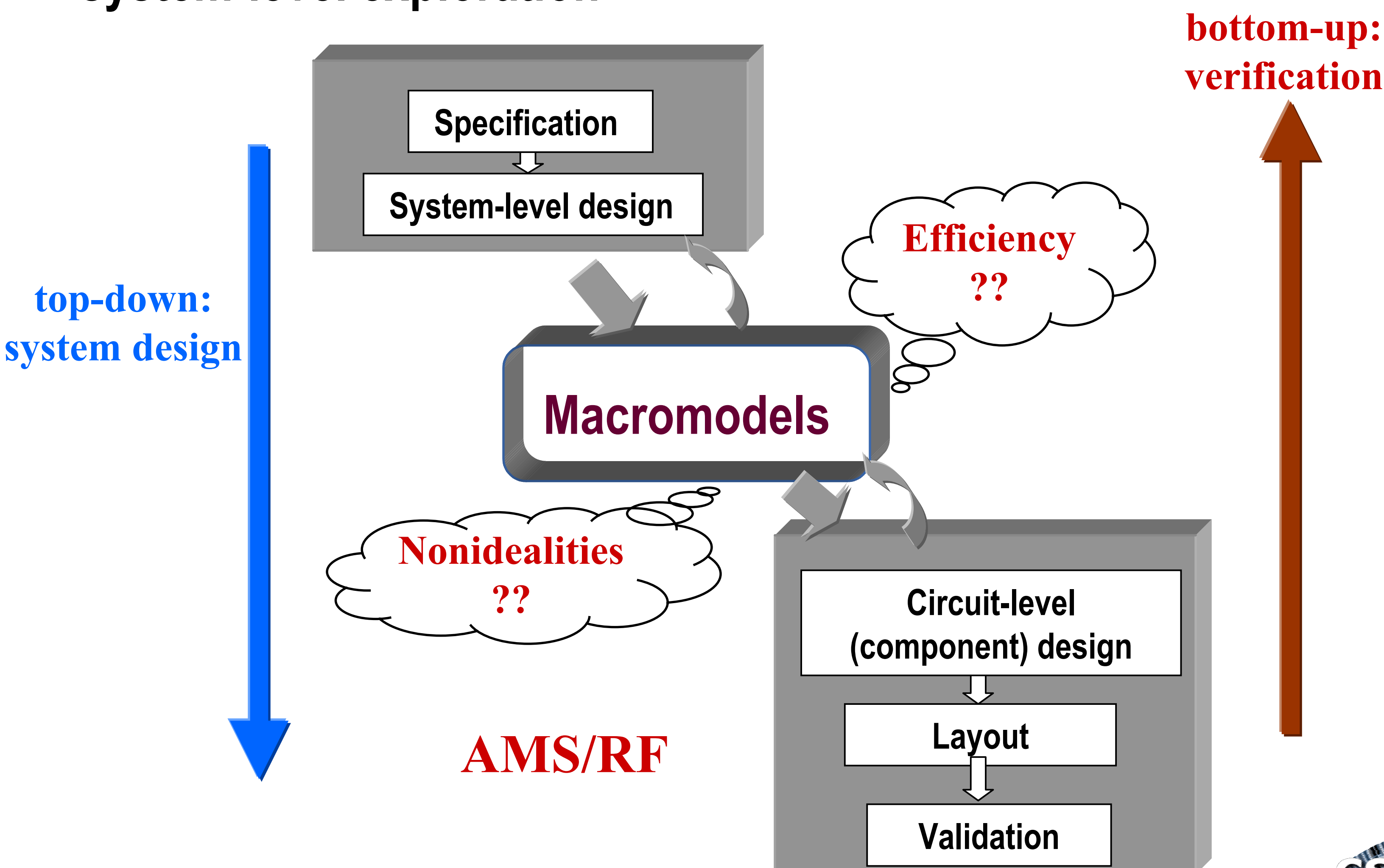
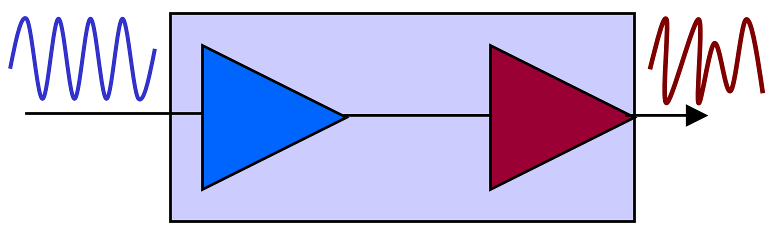
Motivation

- Macromodeling is a key for whole-system verification and system-level exploration



Motivation

- Nonlinear distortion is NOT negligible for analog & RF
  - Dynamic range, interference immunity, SNDR, THD, IM3 …
- Detailed transistor-level nonlinear analyses can lead to lengthy simulation time
  - Distortion effects must be characterized on a block/subsystem basis and reflected accurately and efficiently at system-level
- A systematic *nonlinear* macromodeling strategy is required – usually a very hard problem!



Outline

- Motivation
- Prior work
- Multivariate formulation
- Model order reduction
- Results
- Conclusions

Prior Work

- Black-box based approaches
  - Neural nets, data mining, describing functions, support vector machines etc.
    [Liu et al DAC02] [Root et al DAC03][Bernardinis DAC03]
  - Can use measurement data

- White-box/equation based approaches
  - Based an explicit description of the nonlinear systems
    - Nonlinear ODEs:
      - piecewise-linear, Volterra, piecewise-polynomial representations
  - Resolve to model order reduction (MOR) or pruning for model generation

Prior Work

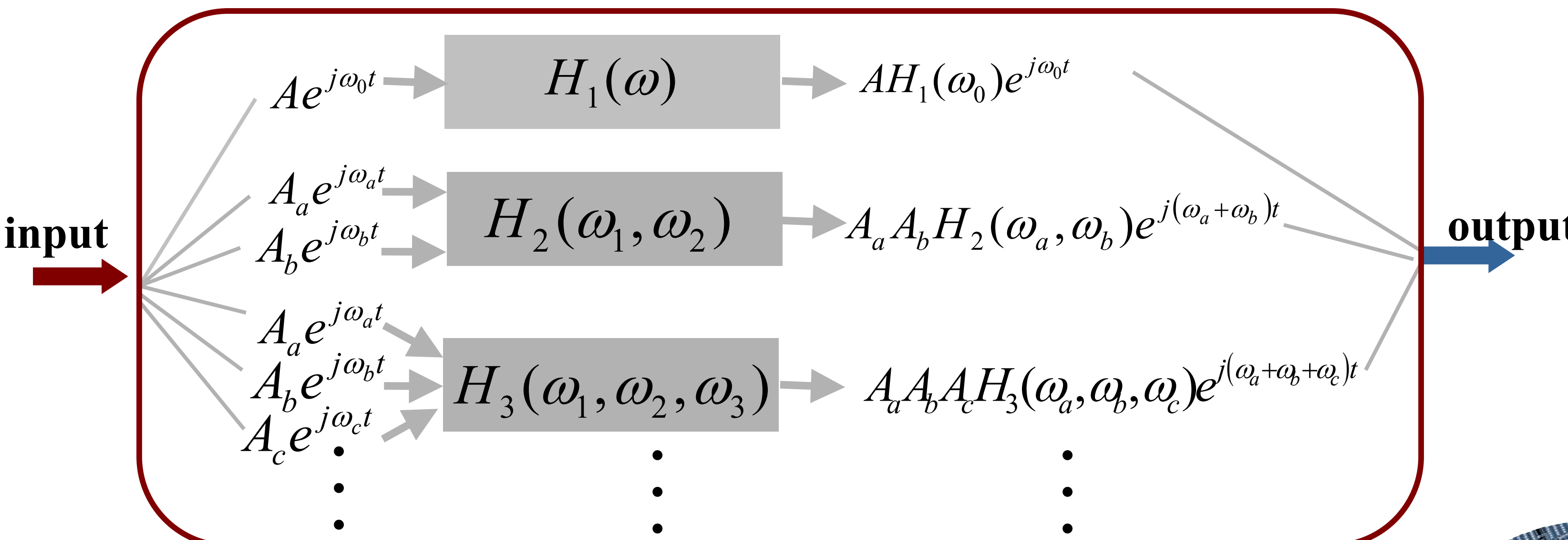
- Samples of recent equation-based works
  - Volterra & symbolic model generation
    - [Wambacq et al Kluwer98, DATE00]
  - Volterra & model order reduction
    - [Roychowdhury TCAS99], [Phillips CICC00, DAC00]
      [Li/Pileggi DAC03], [Li et al, ICCAD03]
  - Piecewise-linear & model order reduction
    - [Rewienski et al ICCAD01], [Vasilyev et al DAC03]
  - Piecewise-polynomial (Volterra) & model order reduction
    - [Dong/Roychowdhury DAC03]

Volterra Series

- A commonly used system description for weakly nonlinear systems

$$x(t) = \sum_{n=1}^{\infty} x_n(t)$$

$$x_n(t) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_n(\tau_1, \cdots, \tau_n) u(t-\tau_1) \cdots u(t-\tau_n) d\tau_1 \cdots d\tau_n$$



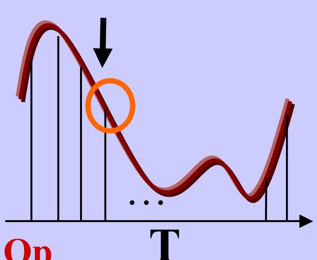
Time-Varying Volterra Series

- Need to perform weakly nonlinear distortion analysis over a varying operating condition for time-varying systems

- Prior works have focused on nonlinear model order reduction based periodically time-varying Volterra series
  - E.g. mixers and SC circuits

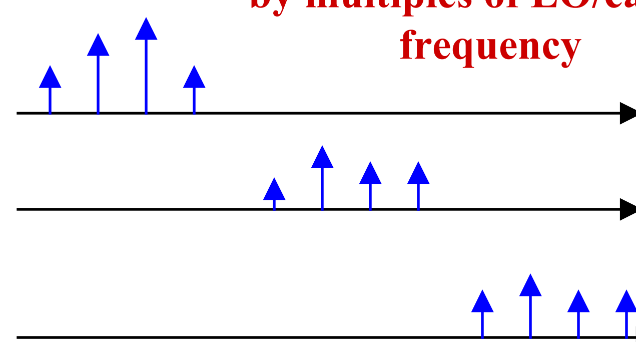
$$H_3(t,\omega_1,\omega_2,\omega_3) = \sum_k H_{3,k}(\omega_1,\omega_2,\omega_3)e^{jk\omega_0 t}$$

$$i(t) = a_{1,t}v(t) + a_{2,t}v^2(t) + a_{3,t}v^3(t)$$



Periodic Op

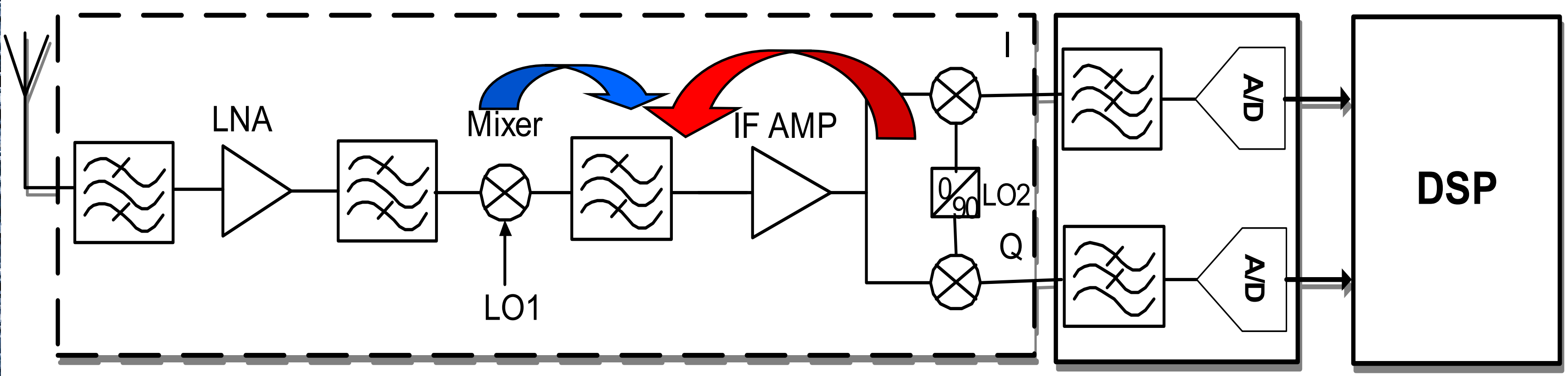
Distorted responses shifted by multiples of LO/carrier frequency



Proposed Work

- Multi-rate time variations are commonly present in communication ICs
  - Introduced by multiple LO's, sampling clocks of switched-capacitor channel select filters etc.

- Weakly nonlinear distortions are coupled with multi-rate time variations



Problem Formulation

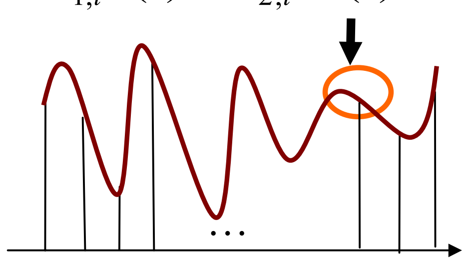
- Based upon a quasi-periodically varying operating point
- Using quasi-periodic nonlinear transfer functions for modeling weakly distortions due to the small input of interest

$$G_1(t)H_1(t)+\frac{d}{dt}(C_1(t)H_1(t,s))+sC_1(t)H_1(t,s)=b$$ ⟵ **First order**

$$[G_1(t)+(s_1+s_2)C_1(t)]H_2(t,s_1,s_2)+\frac{d}{dt}[C_1(t)H_2(t,s_1,s_2)]$$ ⟵ **Second order**

$$=-[G_2(t)+(s_1+s_2)C_2(t)][\overline{H_1(t,s_1)\otimes H_1(t,s_2)}]-\frac{d}{dt}[C_2(t)[\overline{H_1(t,s_1)\otimes H_1(t,s_2)}]$$

$$i(t)=a_{1,t}v(t)+a_{2,t}v^2(t)+a_{3,t}v^3(t)$$



Quasi-periodic time-varying op

Quasi-periodic nonlinear transfer functions:

$$H_1(t,\omega)$$

$$H_2(t,\omega_1,\omega_2)$$

$$H_3(t,\omega_1,\omega_2,\omega_3)$$

... ...

Proposed Work

- **Multi-rate RF/analog blocks should be modeled compactly for whole system simulation**

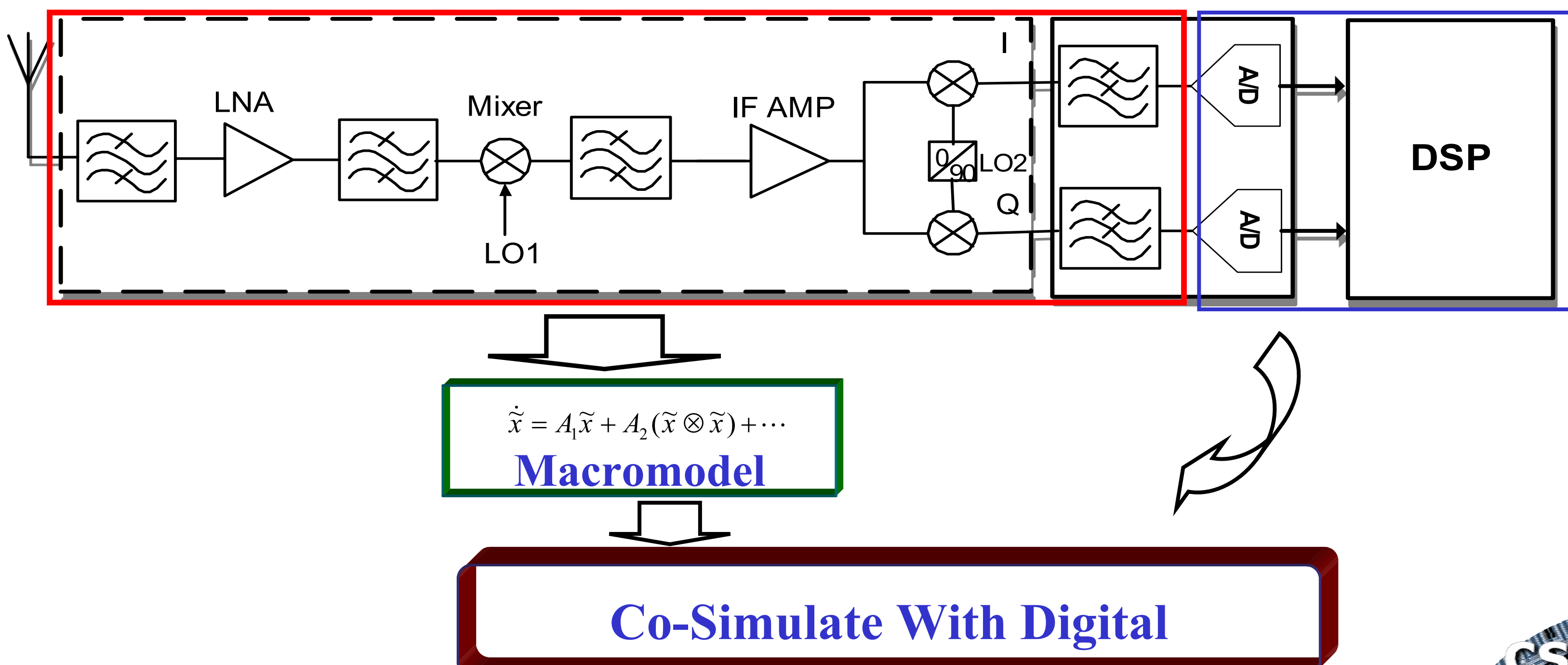
- **Divide-and-conquer**
  - **partition a multi-rate subsystem into chucks and model each chuck as periodically-varying (associated with a single LO/clock)**

- **Direct approach**
  - **Treat the multi-rate RF/analog block more generally as *quasi-periodic***
  - **Cons: Larger problem size**
  - **Benefits**
    - Allow a full consideration of interactions between different stages
      - Parasitic coupling, leakage
    - Potential increase of modeling efficiency – a single piece model for the complete multi-rate subsystem

Our Approach

- Previous works have focused on reduced-order modeling of periodically time-varying weakly nonlinear circuits (e.g. mixer)
  - Using a Volterra description

- We need to consider more general quasi-periodically time-varying systems to accommodate multi-rates



Problem Formulation

- Model the weakly nonlinear distortions under a multi-rate varying operating condition

- Need to formulate proper quasi-periodic boundary conditions for nonlinear transfer functions
  - Formulating matrix-form system description for reduced-order modeling

- Can use a boundary condition formulation very similar to that used in steady-state analyses
  - Delay operator [Kundert et al JSCC89] [Feng et al DAC99]
  - MPDE approach [Roychowdhury TCAS01]

Multivariate Formulation

- Introduce one time-variable for each large time-variation
- Can simply a enforce periodic boundary condition for each time-variable
- Birate case

$$G_1(t)\ \ G_2(t)$$

$$C_1(t)\ \ C_2(t)$$

$$H_1(t,\omega)$$

$$H_2(t,\omega_1,\omega_2)$$

$$G_1(t)H_1(t) + \frac{d}{dt}(C_1(t)H_1(t,s))$$

$$+ sC_1(t)H_1(t,s) = b$$

Quasi-periodic

$$X(t) = \hat{X}(t,t)$$

$$\hat{G}_1(t_1,t_2)\ \ \hat{G}_2(t_1,t_2)$$

$$\hat{C}_1(t_1,t_2)\ \ \hat{C}_2(t_1,t_2)$$

$$\hat{H}_1(t_1,t_2,\omega)$$

$$\hat{H}_2(t_1,t_2,\omega_1,\omega_2)$$

$$\hat{G}_1(t)\hat{H}_1(t_1,t_2) + \frac{\partial}{\partial t_1}(\hat{C}_1(t_1,t_2)\hat{H}_1(t_1,t_2,s)) +$$

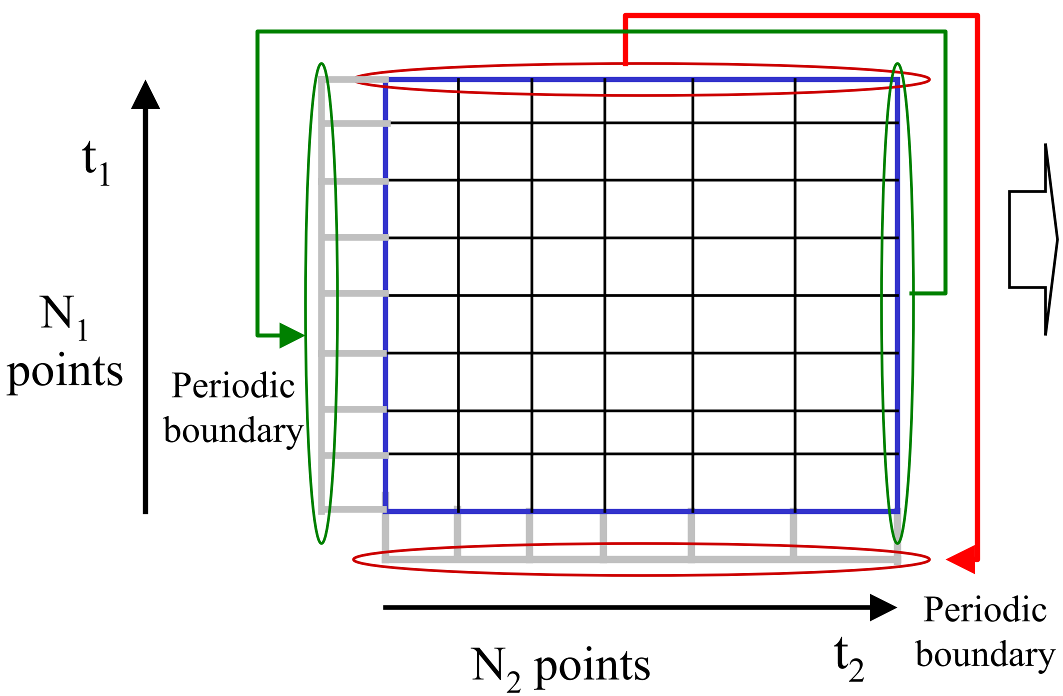
$$\frac{\partial}{\partial t_2}(\hat{C}_1(t_1,t_2)\hat{H}_1(t_1,t_2,s)) + s\hat{C}_1(t_1,t_2)\hat{H}_1(t_1,t_2,s) = b$$

Biperiodic:

$$\hat{X}(t_1+T_1, t_2+T_2) = \hat{X}(t_1,t_2)$$

Multivariate Formulation

- Use finite-difference discretizations lead to a set of linear algebraic equations for transfer functions
- Birate case
  - Discretize the PDEs (in terms of Volterra) on a 2D grid
  - Substitute in biperiodic boundary conditions



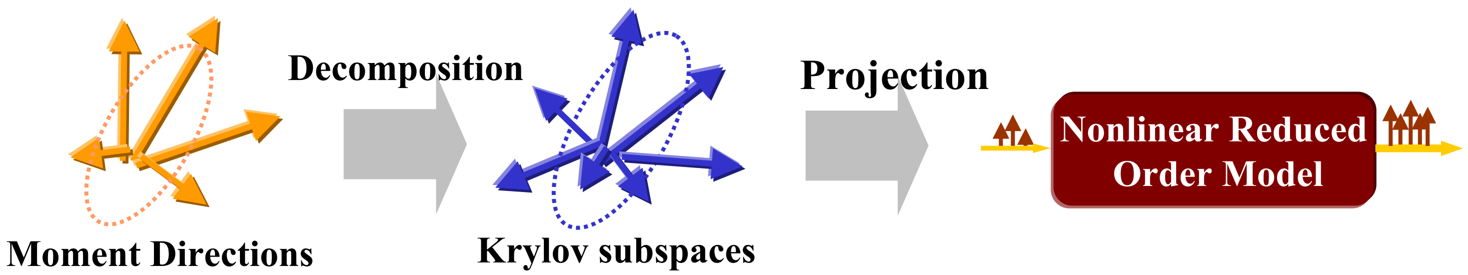
$$[\tilde{J}_1 + s\tilde{C}_1]\tilde{H}_1(s) = \tilde{b}$$

$$[\tilde{J}_1 + (s_1 + s_2)\tilde{C}_1]\tilde{H}_2(s_1, s_2) = -[\tilde{J}_2 + (s_1 + s_2)\tilde{C}_2]\tilde{H}_{11}$$

$$\vdots \qquad \vdots$$

Model Order Reduction

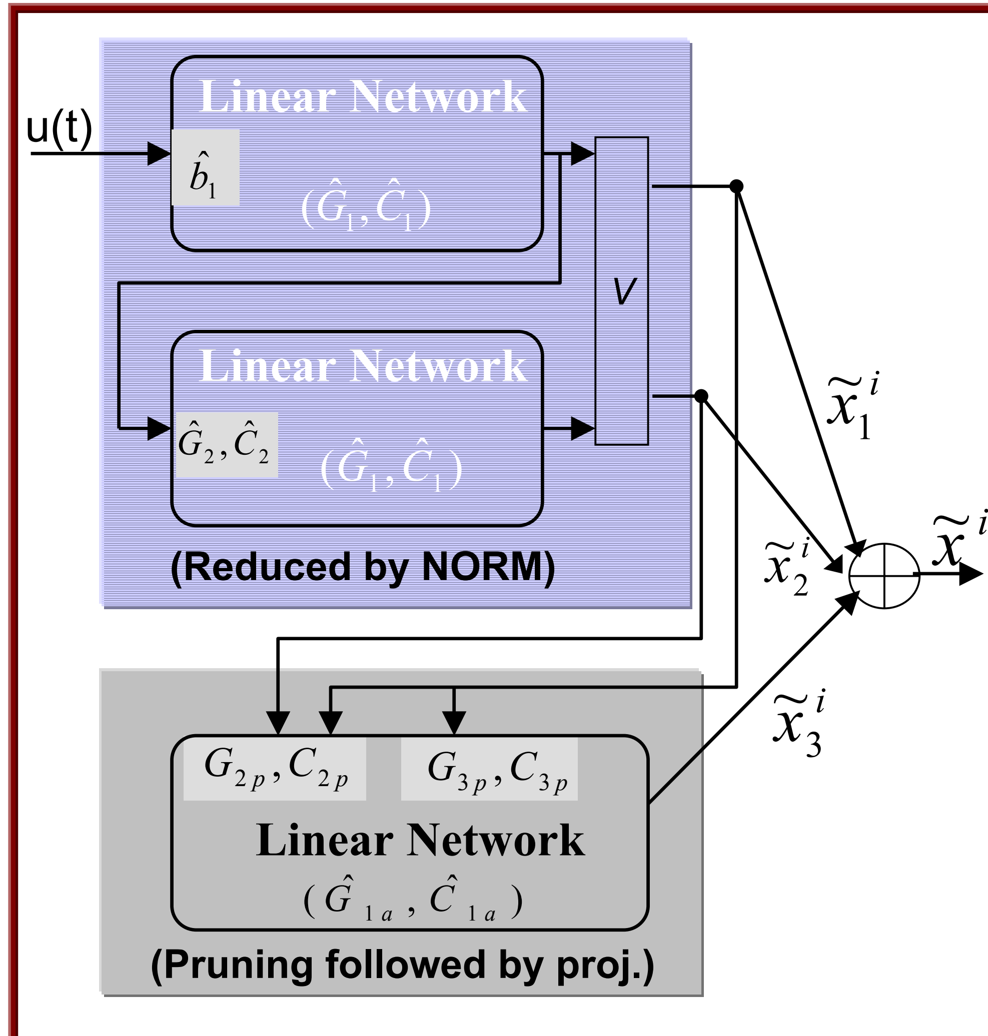
- Discretized multivariate transfer functions can be reduced by recently developed nonlinear model order reduction algorithm NORM [DAC 03]
  - Nonlinear transfer function moments are matched by projection-based nonlinear Padé approximations



- But for high-Q circuits, a full projection approach becomes ineffective for modeling high-order distortions [Li et al, ICCAD03]
  - Many projection vectors are needed for modeling the full frequency range
  - Large model size

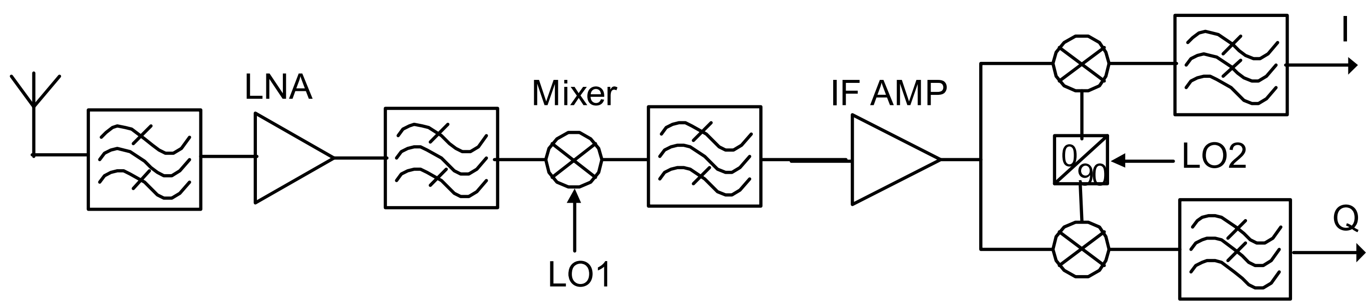
Reduced Hybrid Model

- Low-order responses are projected onto the reduced coordinates and then mapped back.
- High-order response is evaluated in the original coordinates based on sparsified matrices and the propagation through reduced adjoint network.



Results

- A heterodyne receiver
  - flo1 = 880MHz, flo2 = 70MHz
  - Quasi-periodic operating condition is computed by 2-tone steady-state analysis
  - The system is modeled using multivariate Volterra series using 40,000 unknowns
  - The extracted full model is then reduced by a combination of projection-based reductions



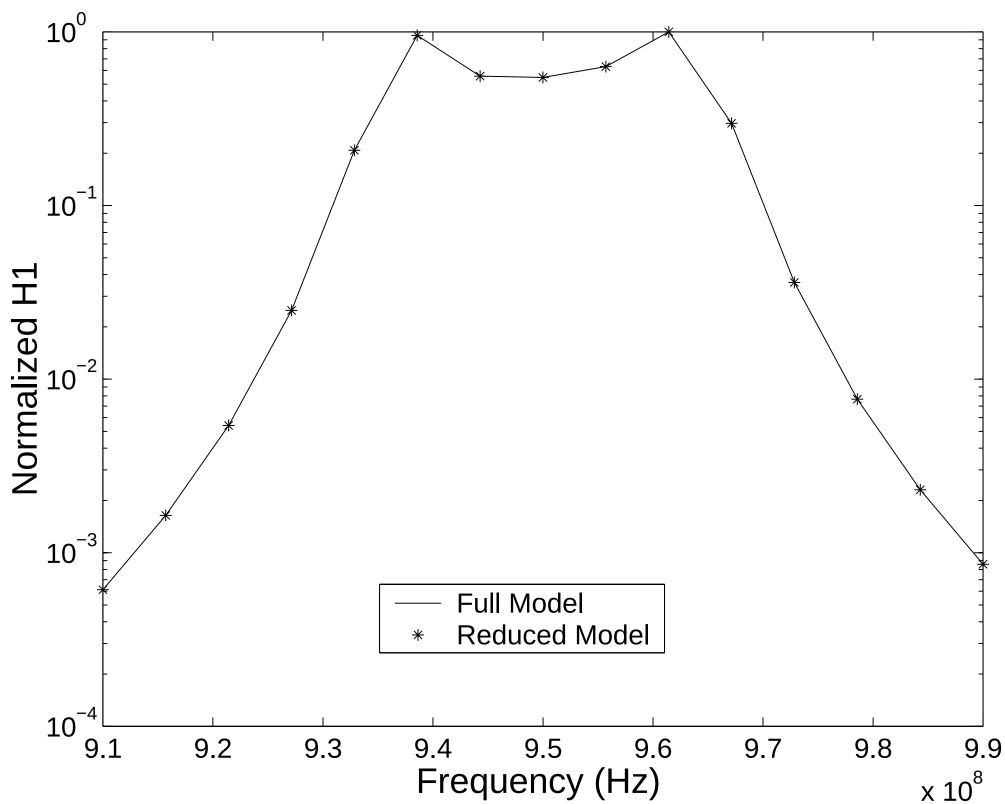
Results

- Model structure
  - First and second order nonlinear responses
    - Approximated using a nonlinear reduced model of size 36 produced by multi-point NORM
  - Third order response
    - Approximated using the above model and a reduced adjoint model of size 20

- Achieved 13x runtime speedup using the reduced order model
  - Further model simplification is possible via a pruning processing in the original system coordinates

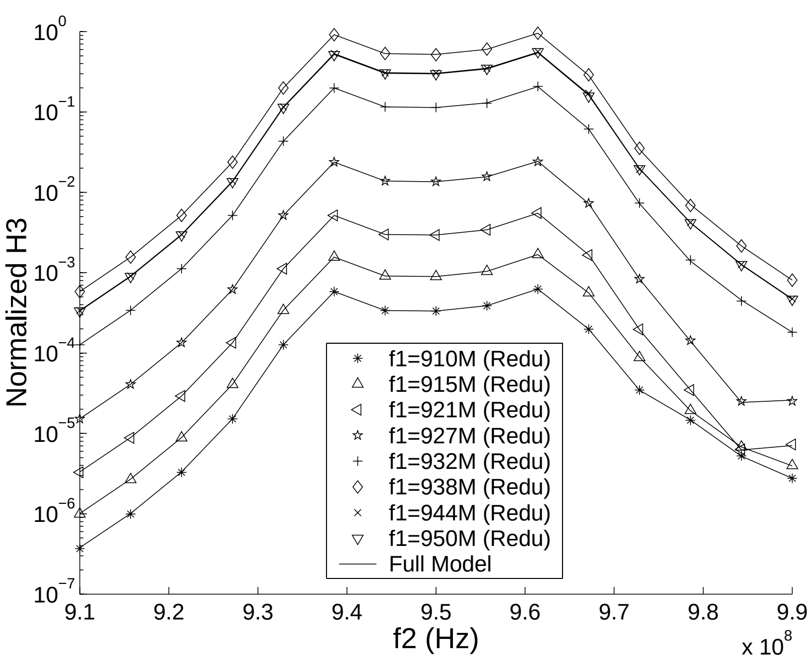
Results

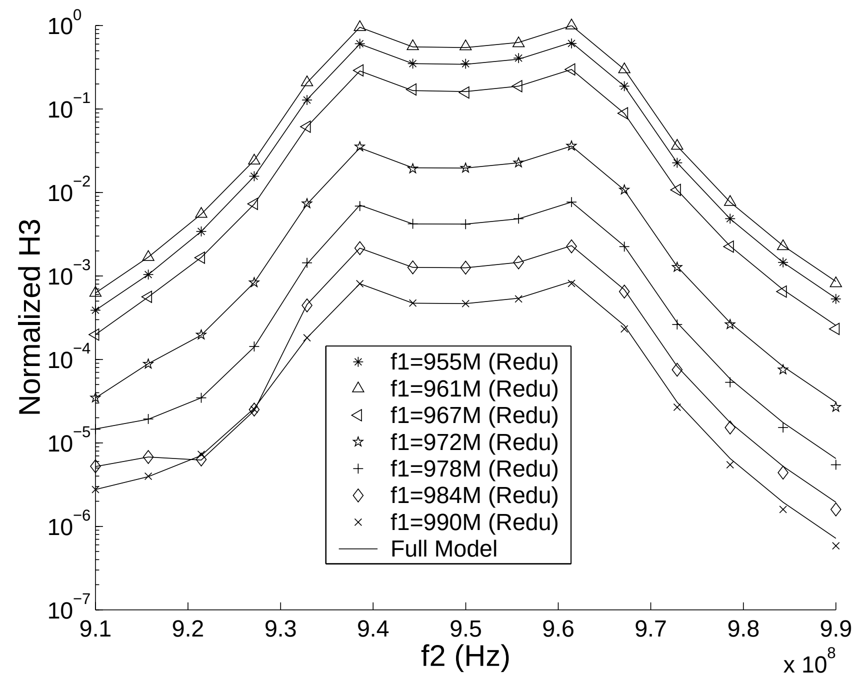
- One harmonic of the 1st order transfer function at the output
  - Specifying the linear conversion gain



Results

- One harmonic of the time-varying third order nonlinear transfer function at the output
  - Specifying the amount of IM3 translated to the base band





Conclusions

- A nonlinear model generation approach is presented for modeling multirate nonlinear communication ICs
- Nonlinear distortion effects are characterized using a multivariate formulation of Volterra nonlinear transfer functions
- Compact circuit models are generated via a combination of projection-based reductions for achieving improved efficiency at high-level system simulation