Modeling Nonlinear Communication ICs Using a Multivariate Formulation

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Motivation

- Macromodeling is a key for whole-system verification and system-level exploration.
Motivation

- Nonlinear distortion is NOT negligible for analog & RF
  - Dynamic range, interference immunity, SNDR, THD, IM3 …

- Detailed transistor-level nonlinear analyses can lead to lengthy simulation time
  - Distortion effects must be characterized on a block/subsystem basis and reflected accurately and efficiently at system-level

- A systematic nonlinear macromodeling strategy is required – usually a very hard problem!
Outline

- Motivation
- Prior work
- Multivariate formulation
- Model order reduction
- Results
- Conclusions
Prior Work

- Black-box based approaches
  - Neural nets, data mining, describing functions, support vector machines etc.
    [Liu et al DAC02] [Root et al DAC03] [Bernardinis DAC03]
  - Can use measurement data

- White-box/equation based approaches
  - Based an explicit description of the nonlinear systems
    - Nonlinear ODEs:
      - piecewise-linear, Volterra, piecewise-polynomial representations
  - Resolve to model order reduction (MOR) or pruning for model generation
Prior Work

- Samples of recent equation-based works
  - Volterra & symbolic model generation
    - [Wambacq et al Kluwer98, DATE00]
  - Volterra & model order reduction
    - [Roychowdhury TCAS99], [Phillips CICC00, DAC00]
    - [Li/Pileggi DAC03], [Li et al, ICCAD03]
  - Piecewise-linear & model order reduction
    - [Rewienski et al ICCAD01], [Vasilyev et al DAC03]
  - Piecewise-polynomial (Volterra) & model order reduction
    - [Dong/Roychowdhury DAC03]
Volterra Series

- A commonly used system description for weakly nonlinear systems

\[ x(t) = \sum_{n=1}^{\infty} x_n(t) \]

\[ x_n(t) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_n(\tau_1, \cdots, \tau_n) u(t - \tau_1) \cdots u(t - \tau_n) d\tau_1 \cdots d\tau_n \]
Time-Varying Volterra Series

- Need to perform weakly nonlinear distortion analysis over a varying operating condition for time-varying systems

- Prior works have focused on nonlinear model order reduction based periodically time-varying Volterra series
  - E.g. mixers and SC circuits

\[ H_3(t, \omega_1, \omega_2, \omega_3) = \sum_k H_{3,k}(\omega_1, \omega_2, \omega_3)e^{j k \omega_0 t} \]

Distorted responses shifted by multiples of LO/carrier frequency

\[ i(t) = a_{1,t}v(t) + a_{2,t}v^2(t) + a_{3,t}v^3(t) \]
Proposed Work

- Multi-rate time variations are commonly present in communication ICs
  - Introduced by multiple LO’s, sampling clocks of switched-capacitor channel select filters etc.

- Weakly nonlinear distortions are coupled with multi-rate time variations
Problem Formulation

- Based upon a quasi-periodically varying operating point
- Using quasi-periodic nonlinear transfer functions for modeling weakly distortions due to the small input of interest

\[
G_1(t)H_1(t) + \frac{d}{dt}(C_1(t)H_1(t,s)) + sC_1(t)H_1(t,s) = b \quad \text{First order}
\]

\[
[G_1(t) + (s_1 + s_2)C_1(t)]H_2(t,s_1,s_2) + \frac{d}{dt}[C_1(t)H_2(t,s_1,s_2)] = -[G_2(t) + (s_1 + s_2)C_2(t)][H_1(t,s_1) \otimes H_1(t,s_2)] - \frac{d}{dt}[C_2(t)[H_1(t,s_1) \otimes H_1(t,s_2)] \quad \text{Second order}
\]

\[
i(t) = a_{1,t}v(t) + a_{2,t}v^2(t) + a_{3,t}v^3(t)
\]

Quasi-periodic nonlinear transfer functions:

\[
H_1(t, \omega) \\
H_2(t, \omega_1, \omega_2) \\
H_3(t, \omega_1, \omega_2, \omega_3) \\
\ldots \ldots
\]
Proposed Work

- Multi-rate RF/analog blocks should be modeled compactly for whole system simulation

- Divide-and-conquer
  - partition a multi-rate subsystem into chucks and model each chuck as periodically-varying (associated with a single LO/clock)

- Direct approach
  - Treat the multi-rate RF/analog block more generally as quasi-periodic
  - Cons: Larger problem size
  - Benefits
    - Allow a full consideration of interactions between different stages
      - Parasitic coupling, leakage
    - Potential increase of modeling efficiency – a single piece model for the complete multi-rate subsystem
Our Approach

- Previous works have focused on reduced-order modeling of periodically time-varying weakly nonlinear circuits (e.g. mixer)
  - Using a Volterra description

- We need to consider more general quasi-periodically time-varying systems to accommodate multi-rates

\[ \tilde{x} = A_4\tilde{x} + A_2(\tilde{x} \otimes \tilde{x}) + \cdots \]

Macromodel

Co-Simulate With Digital
Problem Formulation

- Model the weakly nonlinear distortions under a multi-rate varying operating condition

- Need to formulate proper quasi-periodic boundary conditions for nonlinear transfer functions
  - Formulating matrix-form system description for reduced-order modeling

- Can use a boundary condition formulation very similar to that used in steady-state analyses
  - Delay operator [Kundert et al JSCC89] [Feng et al DAC99]
  - MPDE approach [Roychowdhury TCAS01]
Multivariate Formulation

- Introduce one time-variable for each large time-variation
- Can simply enforce periodic boundary condition for each time-variable
- Biperiodic case

\[
\begin{align*}
G_1(t) & \quad G_2(t) \\
C_1(t) & \quad C_2(t) \\
H_1(t, \omega) & \\
H_2(t, \omega_1, \omega_2) & \\
G_1(t)H_1(t) + \frac{d}{dt}(C_1(t)H_1(t, s)) + sC_1(t)H_1(t, s) &= b
\end{align*}
\]

**Quasi-periodic**

\[
\begin{align*}
\hat{G}_1(t_1, t_2) & \quad \hat{G}_2(t_1, t_2) \\
\hat{C}_1(t_1, t_2) & \quad \hat{C}_2(t_1, t_2) \\
\hat{H}_1(t_1, t_2, \omega) & \\
\hat{H}_2(t_1, t_2, \omega_1, \omega_2) & \\
\hat{G}_1(t_1)\hat{H}_1(t_1, t_2) + \frac{\partial}{\partial t_1}(\hat{C}_1(t_1, t_2)\hat{H}_1(t_1, t_2, s)) + \\
\frac{\partial}{\partial t_2}(\hat{C}_1(t_1, t_2)\hat{H}_1(t_1, t_2, s)) + s\hat{C}_1(t_1, t_2)\hat{H}_1(t_1, t_2, s) &= b
\end{align*}
\]

**Biperiodic:**

\[
\hat{X}(t_1 + T_1, t_2 + T_2) = \hat{X}(t_1, t_2)
\]
Multivariate Formulation

- Use finite-difference discretizations lead to a set of linear algebraic equations for transfer functions
- Bivariate case
  - Discretize the PDEs (in terms of Volterra) on a 2D grid
  - Substitute in biperiodic boundary conditions

\[
\begin{align*}
[\tilde{J}_{1} + s\tilde{C}_{1}]\tilde{H}_{1}(s) &= \tilde{b} \\
[\tilde{J}_{1} + (s_{1} + s_{2})\tilde{C}_{1}]\tilde{H}_{2}(s_{1}, s_{2}) &= \\
- [\tilde{J}_{2} + (s_{1} + s_{2})\tilde{C}_{2}]\tilde{H}_{11} &= \\
\vdots &= \vdots
\end{align*}
\]
Model Order Reduction

- Discretized multivariate transfer functions can be reduced by recently developed nonlinear model order reduction algorithm NORM [DAC 03]
  - Nonlinear transfer function moments are matched by projection-based nonlinear Padé approximations

- But for high-Q circuits, a full projection approach becomes ineffective for modeling high-order distortions [Li et al, ICCAD03]
  - Many projection vectors are needed for modeling the full frequency range
  - Large model size
Reduced Hybrid Model

- Low-order responses are projected onto the reduced coordinates and then mapped back.

- High-order response is evaluated in the original coordinates based on sparsified matrices and the propagation through reduced adjoint network.
Results

- A heterodyne receiver
  - $f_{lo1} = 880$MHz, $f_{lo2} = 70$MHz
  - Quasi-periodic operating condition is computed by 2-tone steady-state analysis
  - The system is modeled using multivariate Volterra series using 40,000 unknowns
  - The extracted full model is then reduced by a combination of projection-based reductions
Results

- Model structure
  - First and second order nonlinear responses
    - Approximated using a nonlinear reduced model of size 36 produced by multi-point NORM
  - Third order response
    - Approximated using the above model and a reduced adjoint model of size 20

- Achieved 13x runtime speedup using the reduced order model
  - Further model simplification is possible via a pruning processing in the original system coordinates
Results

- One harmonic of the 1\textsuperscript{st} order transfer function at the output
  - Specifying the linear conversion gain
Results

- One harmonic of the time-varying third order nonlinear transfer function at the output
  - Specifying the amount of IM3 translated to the base band
Conclusions

- A nonlinear model generation approach is presented for modeling multirate nonlinear communication ICs.

- Nonlinear distortion effects are characterized using a multivariate formulation of Volterra nonlinear transfer functions.

- Compact circuit models are generated via a combination of projection-based reductions for achieving improved efficiency at high-level system simulation.