Modeling Memory Effects in Nonlinear Subsystems by Dymanic Volterra Series

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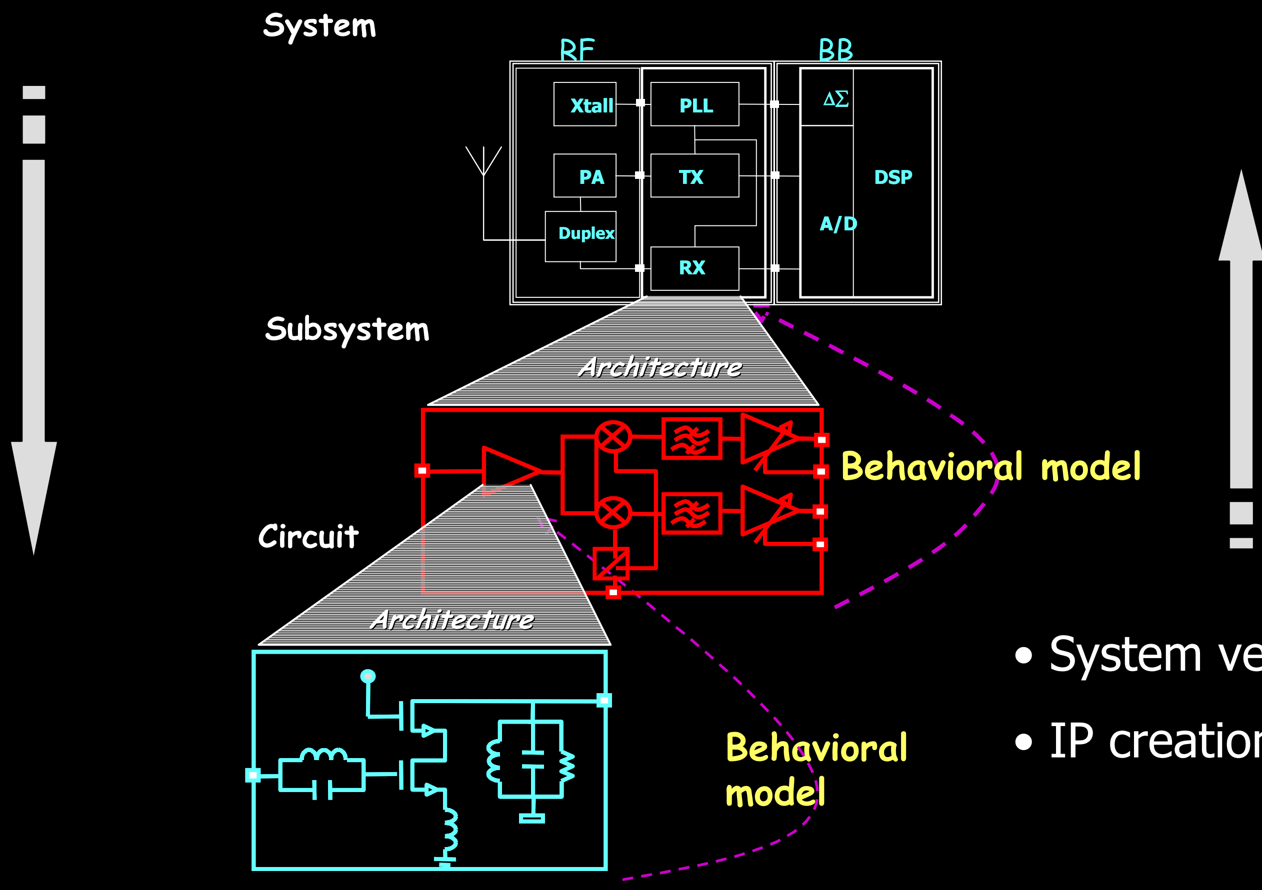
Outline

- RF subsystem modeling challenge
- Envelope domain modeling basics
- Modified Volterra series approaches for highly nonlinear systems with memory
- Examples
- Summary

Behavioral modeling for system verification and IP creation

- Top-down design



- System verification
- IP creation

Bottom-up block modeling

Kirchoff law

$$\int_0^t \boldsymbol{y}(t-\tau)\boldsymbol{v}(\tau)d\tau + \frac{d}{dt}\boldsymbol{q}\big(\boldsymbol{v}(t)\big) + \boldsymbol{i}\big(\boldsymbol{v}(t)\big) + \boldsymbol{u}(t) = 0$$

↓

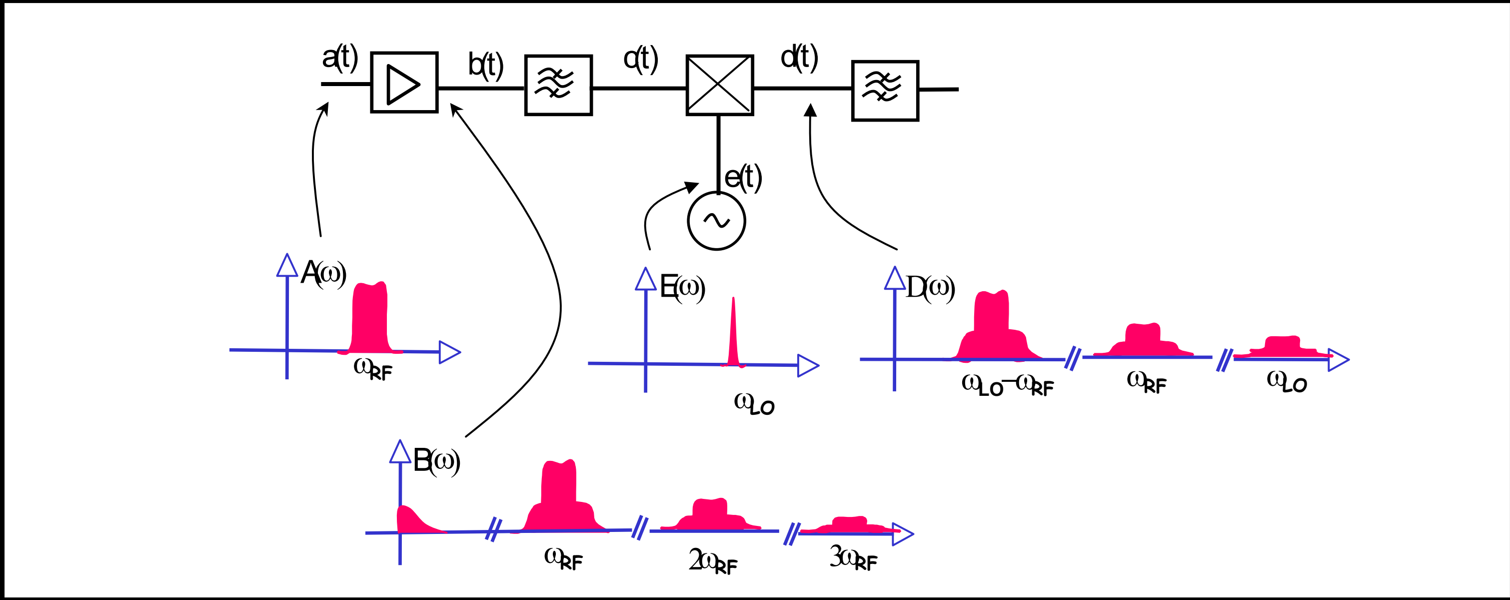
Symbolic reduction

Simulation or Measurement tools

↓

Black box modeling

RF & microwave subsystem modeling challenge

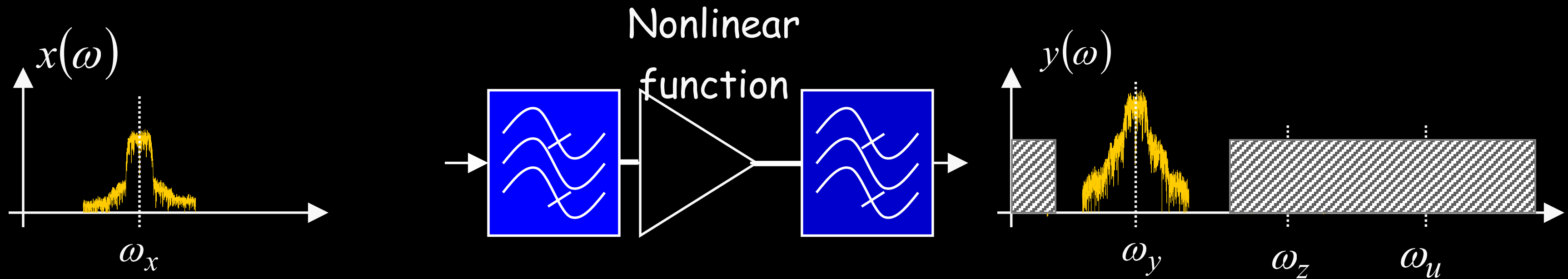


- Time scales disparity (Baseband signals, carriers, noise, spurious)
- Pass band components (High Q filters, transmission lines, ..)
- Nonlinear functions (PA, converters, mixer,..)

⇨ **Pass band – Enveloppe concept**

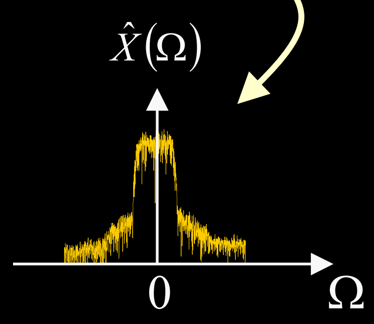
Pass band subsystem

- amplifier
- mixer
- multiplier
- VCO



$$x(t) = \Re e\left\{ \hat{X}(t) . e^{j.\omega_x.t} \right\}$$

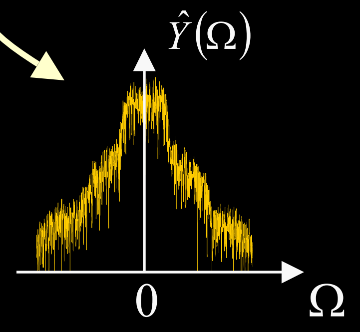
$$y(t) = \Re e\left\{ \hat{Y}(t) . e^{j.\omega_y.t} \right\}$$



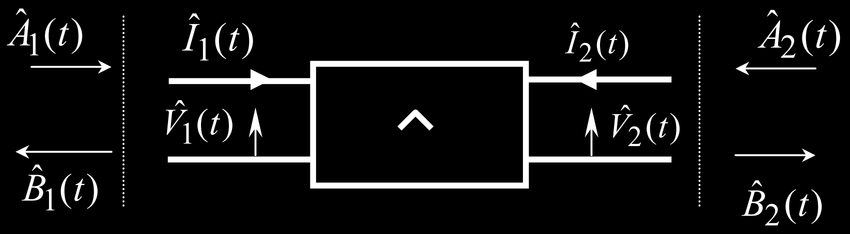
?

$$\hat{\kappa}\left(\alpha, \hat{X}(t), \hat{X}^*(t), \hat{Y}(t), \hat{Y}^*(t), t\right) = 0$$

$$t \in [0, T_\infty[$$



Functional modeling



$$\hat{\kappa}\left(\alpha, \hat{X}(t), \hat{X}^*(t), \hat{Y}(t), \hat{Y}^*(t), t\right) = 0$$
$$t \in [0, T_\infty[$$

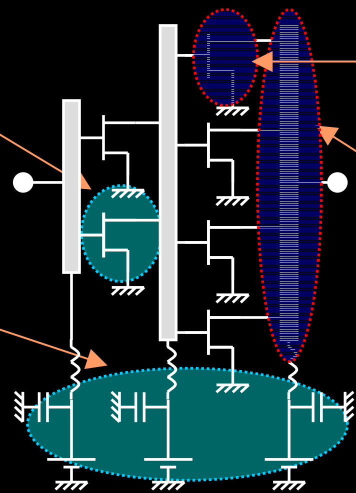
☞ **Three major problems**

- **Nonlinearity**
- **Interstage mismatch**
- **Memory effects**
  - High speed applications
  - High capacity

Severe memory effects in solid state devices

- Thermal effects
- Trapping effects

- Biasing circuits
- AGC loops



- Transistors (transit time)

- Matching networks (group delay)

Long term memory
~µs to ms

Short term memory
~ns

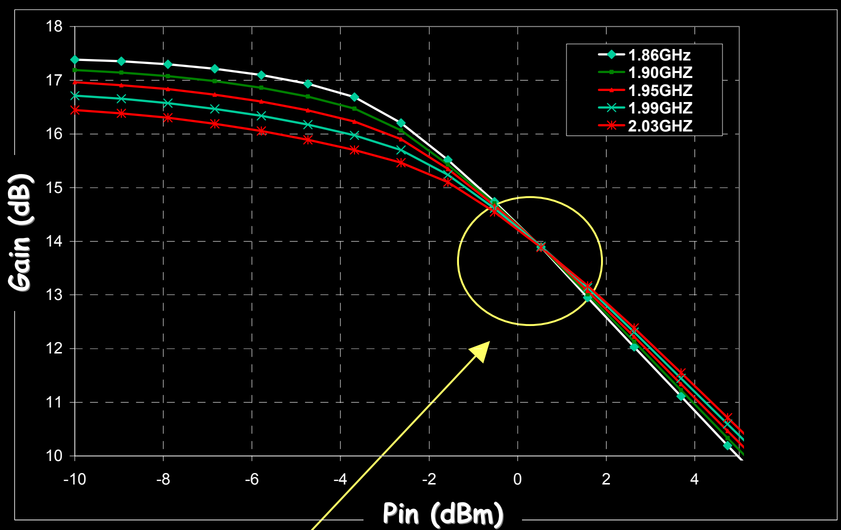
Low frequency memory effects

Nonlinear coupling

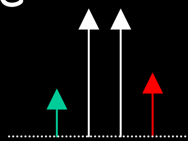
High frequency memory effects

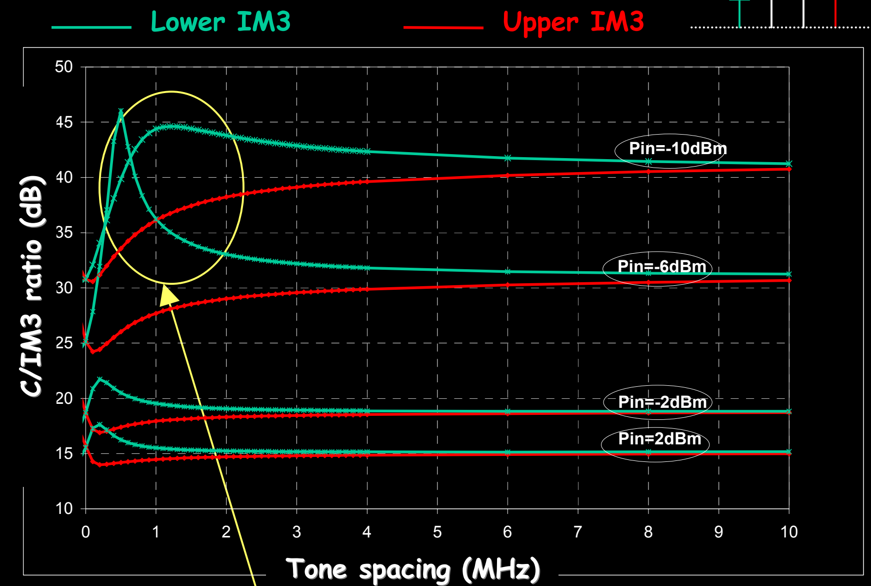
Memory effects highlights : Typical narrow band BiCMOS LNA

Short term memory : Single tone measurements



Long term memory : Two-tone measurements



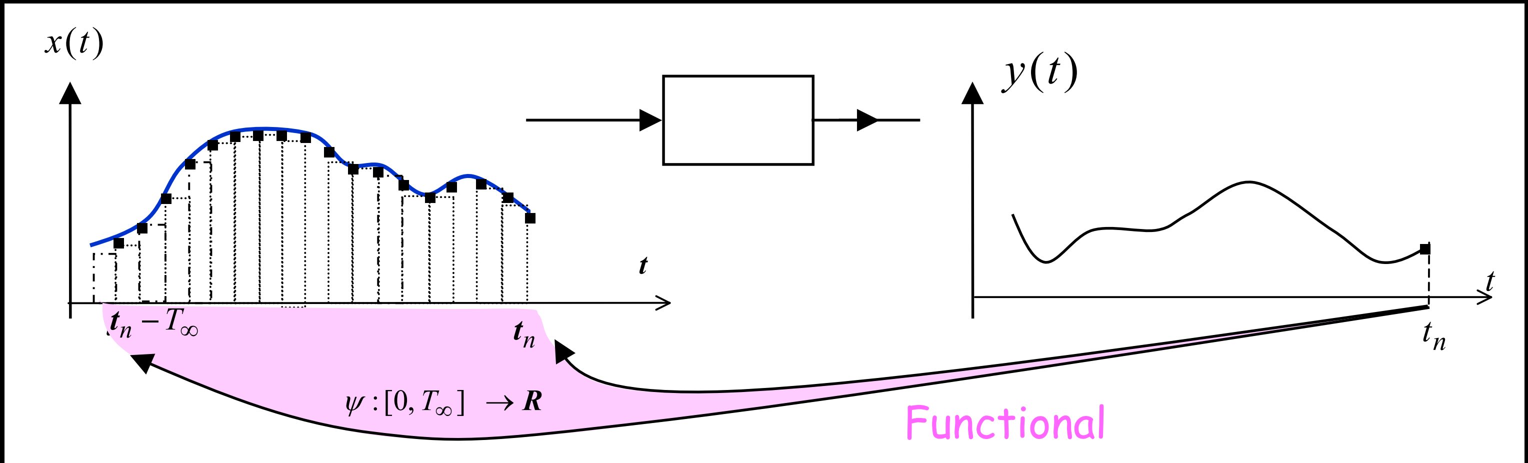


wrapping

Resonance & asymmetry

⇨ Need rigorous but yet simple modeling approach (affordable with either simulation and measurement tools)

Analytic approach: Volterra series expansion



$$\begin{cases} y(t_n) = \psi(\vec{x}) \\ \vec{x} = [x(t_n), x(t_n - \Delta t), \ldots, x(t_n - M\Delta t)]^T \end{cases}$$

T_∞ Memory duration

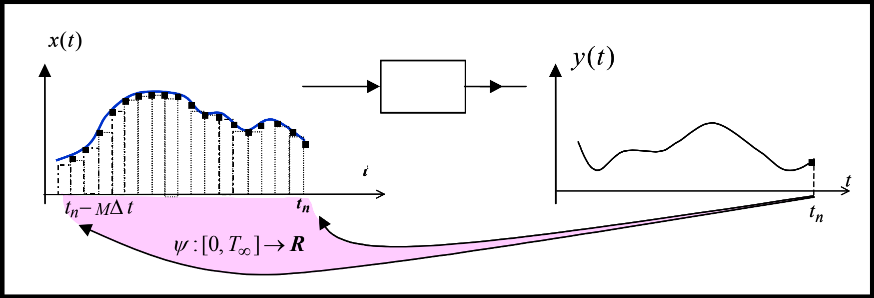
Power series expansion of the functional

$$y(t_n) = \psi(\vec{x}_0) + \Delta\psi(\vec{x}_0)^T(\vec{x} - \vec{x}_0) + \tfrac{1}{2}(\vec{x} - \vec{x}_0)^T \left[\Delta^2\psi(\vec{x}_0)\right](\vec{x} - \vec{x}_0) + \cdots$$

→ $\vec{x}_0$ is an arbitrary input record

Volterra series approach

$$\vec{x}_0 = \vec{0}, \quad \Delta t \to 0,$$



$$y(t) = \sum_{n=1}^{\infty} y_n(t)$$

$$y_1(t) = \int_0^{T_\infty} h_1(\tau_1) x(t-\tau_1) d\tau_1$$ → linear response

$$y_2(t) = \int_0^{T_\infty} \int_0^{T_\infty} h_2(\tau_1, \tau_2) x(t-\tau_1) x(t-\tau_2) d\tau_1 d\tau_2$$ → quadratic

$\ldots$

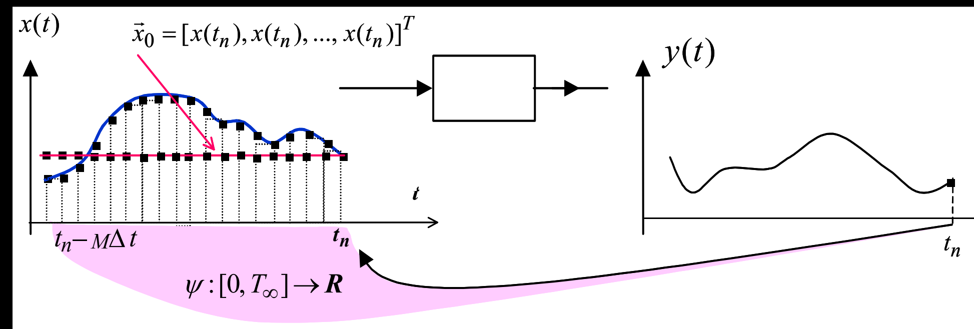
$$y_n(t) = \int_0^{T_\infty} .. \int_0^{T_\infty} h_n(\tau_1, .., \tau_n) x(t-\tau_1) \cdots x(t-\tau_n) d\tau_1 \cdots d\tau_n$$ → order n

Volterra kernel of order n

- Nonlinear circuit ⇨ consider $n > 1$ ⇨ practically inefficient

Modified Volterra series

$$\vec{x}_0 = \vec{f}(t_n) \neq \vec{0},$$



T∞

$$y(t) = \sum_{n=0}^{\infty} y_n(t)$$

$$y_0(t) = \psi(x(t))$$

$$y_1(t) = \int_0^{T_\infty} h_1(x(t), \tau_1)[x(t-\tau_1) - x(t)]\,d\tau_1$$

→ Parametric linear

...

→ **Modified Volterra kernel of order n**

$$y_n(t) = \int_0^{T_\infty} .. \int_0^{T_\infty} h_n(x(t), \tau_1,.., \tau_n)[x(t-\tau_1) - x(t)] \cdots [x(t-\tau_n) - x(t)]\,d\tau_1 \cdots d\tau_n$$

← Signal speed

truncate to $n = 1$

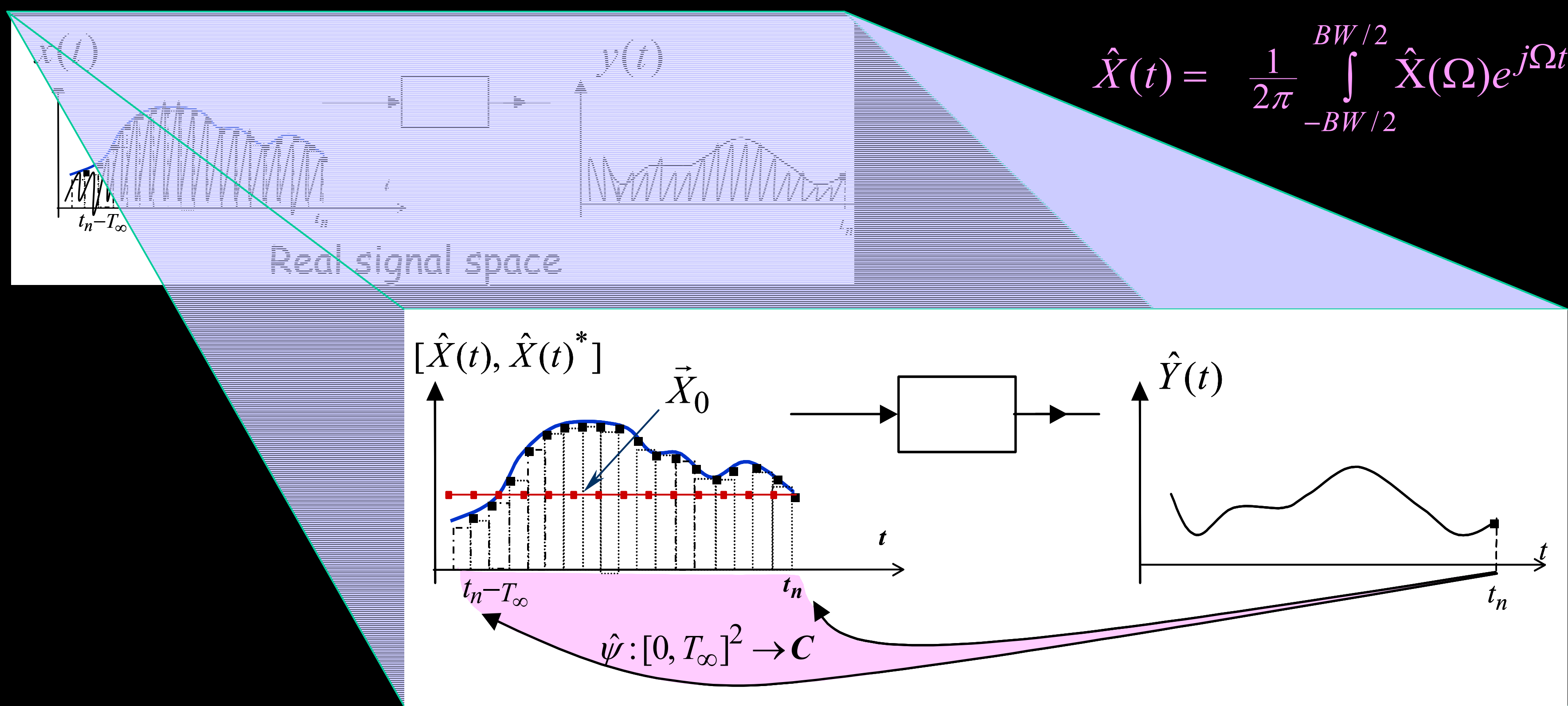
- Short memory duration:
  T∞ << signal period

achieve some practical efficiency

Envelope domain modeling enforces short memory duration conditions

$x(t) = \Re e[\hat{X}(t) e^{j\omega_x t}]$ $\qquad$ $y(t) = \Re e[\hat{Y}(t) e^{j\omega_y t}]$

$$\hat{X}(t) = \frac{1}{2\pi} \int_{-BW/2}^{BW/2} \hat{X}(\Omega) e^{j\Omega t} d\Omega$$



Short memory duration = $T_\infty \ll \frac{1}{BW}$ = more realistic assumption

Envelope domain modified Volterra model

$$\hat{Y}(t)=\sum_{n=0}^{\infty}\hat{Y}_n(t)$$

$$\hat{Y}_0(t)=\hat{\psi}\left(\hat{X}(t),\hat{X}(t)^*\right)$$ → Order 0: AM/AM, AM/PM model

$$\hat{Y}_1(t)=\int_0^{T_\infty} h_I\left(\hat{X}(t),\hat{X}(t)^*,\tau\right)\cdot\left(X(t-\tau)-X(t)\right)d\tau+\int_0^{T_\infty} h_Q\left(X(t),X(t)^*,\tau\right)\cdot\left(X(t-\tau)^*-X(t)^*\right)d\tau$$ → Parametric linear (I-Q)

…

$$\hat{Y}_n(t)=\int_0^{T_\infty}..\int_0^{T_\infty}\sum_{i_1=1}^{2}..\sum_{i_n=1}^{2}\hat{h}_n^{i_1..i_n}\left(\hat{X}(t),\hat{X}(t)^*,\tau_1,..,\tau_n\right)\cdot\left(X^{i_1}(t-\tau_1)-X^{i_1}(t)\right)\cdot\cdot\left(X^{i_n}(t-\tau_1)-X^{i_n}(t)\right)d\tau_1\cdot\cdot d\tau_n$$ → Parametric order n

$$X^1(t)\overset{\Delta}{=}\hat{X}(t),\quad X^2(t)\overset{\Delta}{=}\hat{X}(t)^*$$

Short term memory conditions ⇒ $n=1$

$$T_\infty \ll \frac{1}{BW}$$

$$\hat{Y}(t)=\hat{\psi}(\hat{X}(t),\hat{X}^*(t)) + \int h_I\left(\hat{X}(t),\hat{X}^*(t),\tau\right)\times\left(\hat{X}(t-\tau)-\hat{X}(t)\right)d\tau + \int h_Q\left(\hat{X}(t),\hat{X}^*(t),\tau\right)\times\left(\hat{X}^*(t-\tau)-\hat{X}^*(t)\right)d\tau$$

Amplifier
Mixer
VCO

1rst order modified Volterra model : summary

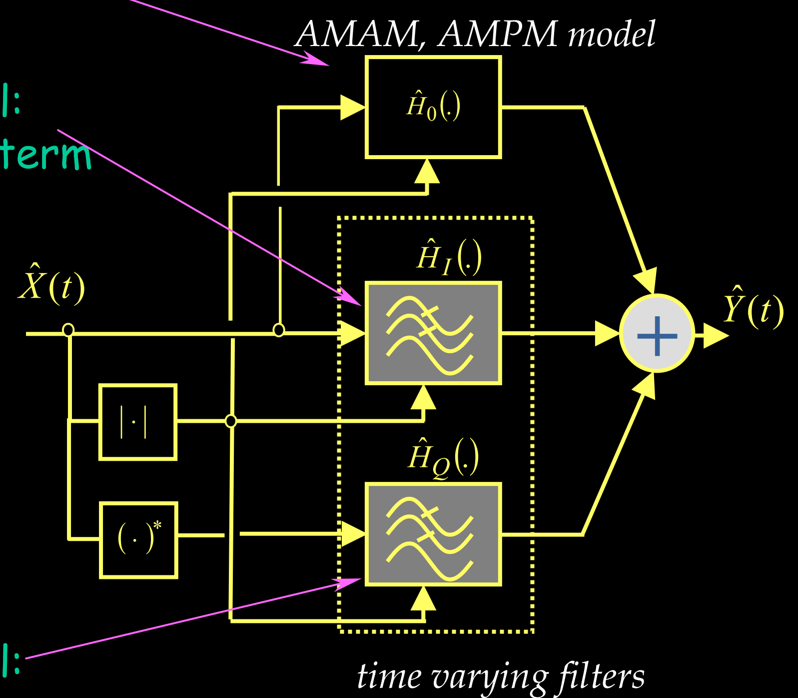
Static kernel:
capture saturation

Dynamic kernel:
capture short term
memory

Dynamic kernel:
capture long term
memory

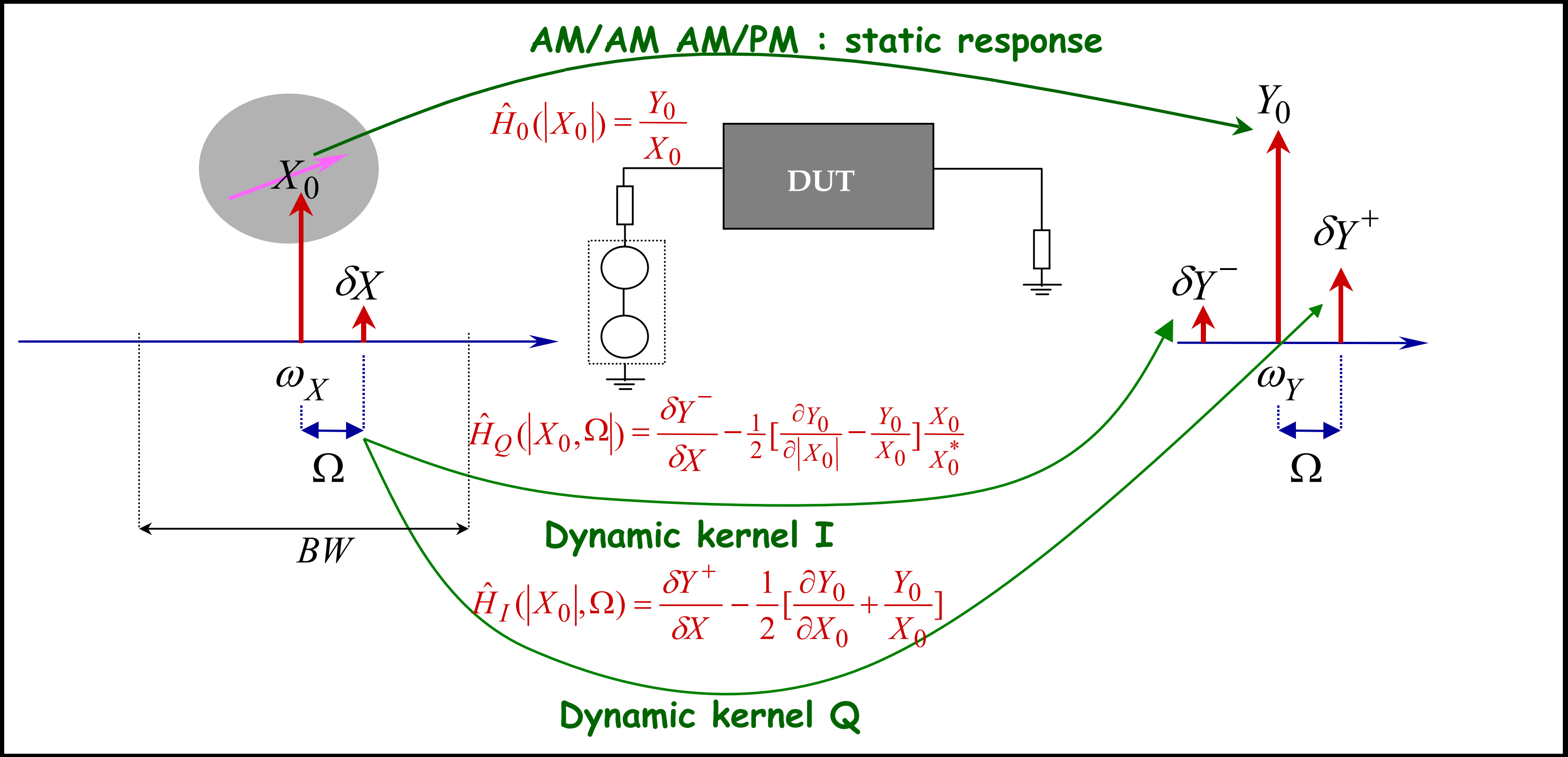
$$\hat{Y}(t) = \hat{H}_0\left(\left|\hat{X}(t)\right|\right)\times \hat{X}(t) + \int_{-BW/2}^{BW/2} \hat{H}_I\left(\left|\hat{X}(t)\right|,\Omega\right)\times \hat{\mathbf{X}}(\Omega)e^{j\Omega t}\,d\Omega/2\pi + \int_{-BW/2}^{BW/2} \hat{H}_Q\left(\left|\hat{X}(t)\right|-\Omega\right)\frac{\hat{X}(t)}{\hat{X}(t)^*}\times \hat{\mathbf{X}}^*(\Omega)e^{-j\Omega t}\,d\Omega/2\pi$$

$\Longleftrightarrow$



1rst order modified Volterra model : Kernel extraction

- Two-tone measurements



⇒ Harmonic Balance, shooting simulation or Experimental characterization with VNA or digital scope

Constant envelope systems (gmsk, radar, ..)

- Long term memory is not excited

$$\hat{Y}(t) = \hat{H}_0\left(\left|\hat{X}(t)\right|\right)\times \hat{X}(t) + \int_{-BW/2}^{BW/2} \hat{H}_I\left(\left|\hat{X}(t)\right|,\Omega\right)\times \hat{\mathbf{X}}(\Omega)e^{j\Omega t}d\Omega/2\pi + \int_{-BW/2}^{BW/2} \hat{H}_Q\left(\left|\hat{X}(t)\right|-\Omega\right)\frac{\hat{X}(t)}{\hat{X}(t)^*}\times \hat{\mathbf{X}}^*(\Omega)e^{-j\Omega t}d\Omega/2\pi$$

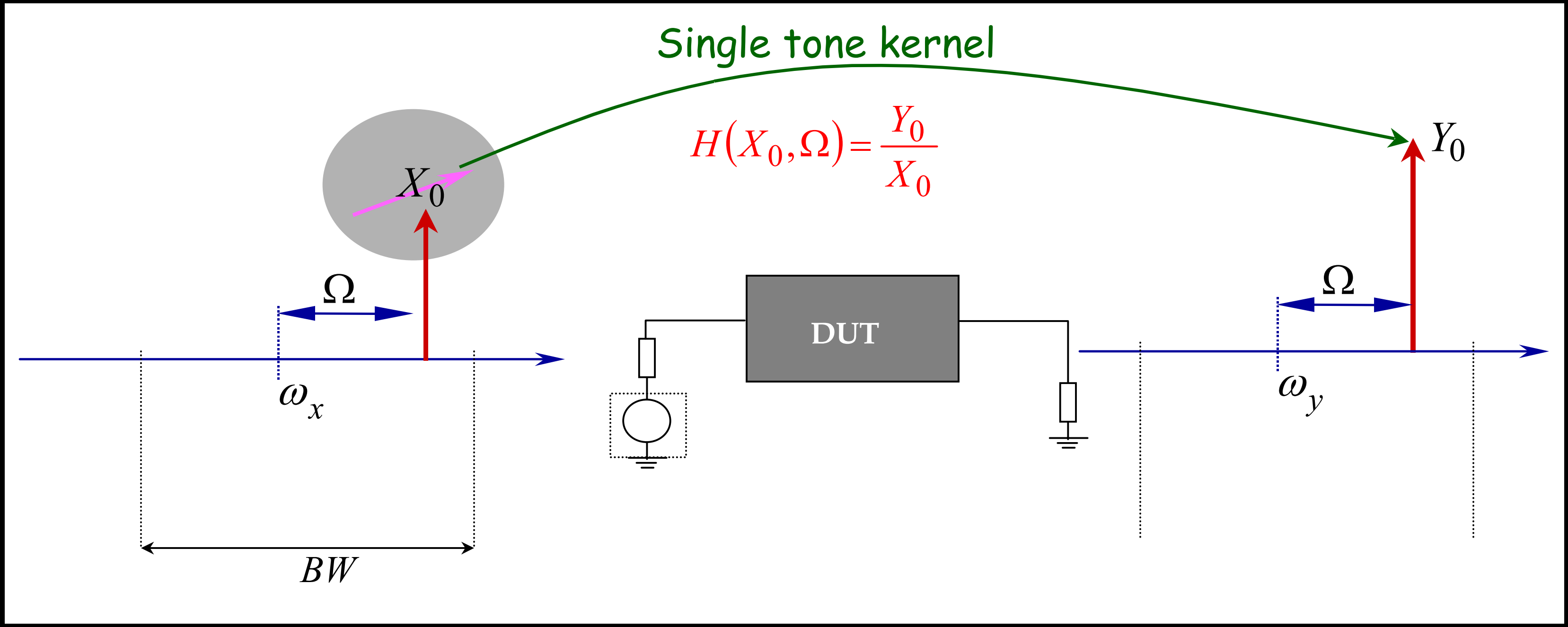
- Very effective model for constant envelope systems

$$\hat{Y}(t) = \int_{-BW/2}^{BW/2} \hat{H}\left(\left|\hat{X}(t)\right|,\Omega\right)\times \hat{\mathbf{X}}(\Omega)e^{j\Omega t}\frac{d\Omega}{2\pi} = \int_{0}^{T_\infty} \hat{h}\left(\left|\hat{X}(t)\right|,\tau\right)\times \hat{X}(t-\tau)d\tau$$

Kernel = Single tone complex gain vs power and frequency

Quasi-constant envelope systems (gmsk, radar, ..)

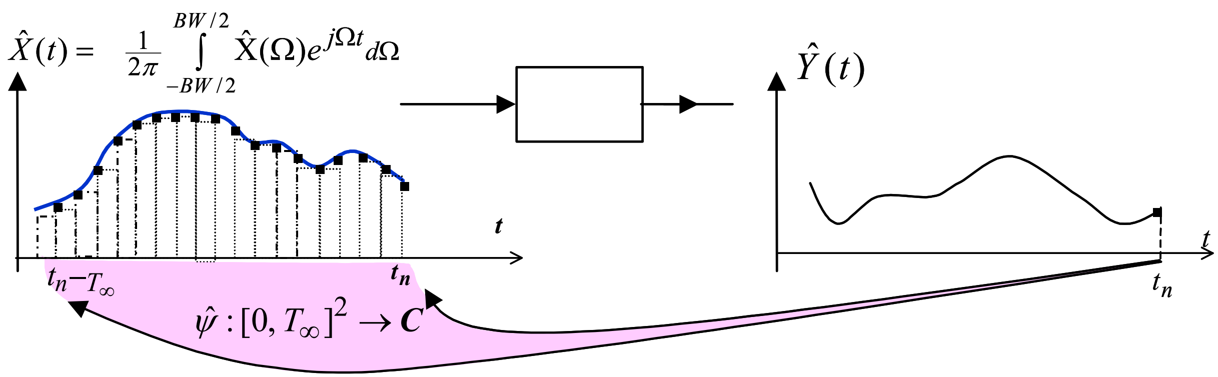
Very simple kernel extraction



Certain circuits do not satisfy short memory duration conditions

Long term memory effects: $T_\infty > \dfrac{1}{BW}$

- Poorly designed biasing circuits - AGC loops - Thermal effects



$$\begin{cases} \hat{Y}(t_n) = \hat{\psi}(\vec{X}) \\ \vec{X} = [\hat{X}(t_n), \hat{X}(t_n)^*, \hat{X}(t_n - \Delta t), \hat{X}(t_n - \Delta t)^*, \ldots, \hat{X}(t_n - M\Delta t), \hat{X}(t_n - M\Delta t)^*]^T \end{cases}$$

⇨ **Reconsider the functional decomposition :**

Use more effective basis functions instead of power expansions

- **Previously modified Volterra series**

Basis function order n : $B_n(X) = X^n$

Change basis functions

$$
\begin{aligned}
\hat{Y}(t) = \; & \psi(\hat{X}(t), \hat{X}(t)^*) \\
& + \sum_{k=0}^{\infty} [h_I^1(\hat{X}(t), \hat{X}(t)^*, k\Delta t) \quad h_Q^1(\hat{X}(t), \hat{X}(t)^*, t_k)] \cdot \begin{bmatrix} \hat{X}(t-k\Delta t) - \hat{X}(t) \\ \hat{X}(t-k\Delta t)^* - \hat{X}(t)^* \end{bmatrix} \\
& + \sum_{k_1}^{\infty} \sum_{k_2}^{\infty} [\hat{X}(t-k_1\Delta t) - \hat{X}(t) \quad \hat{X}(t-k_1\Delta t)^* - \hat{X}(t)^*] \cdot \begin{bmatrix} \hat{h}_{II}^2(\tau_1,\tau_2) & \hat{h}_{IQ}^2(\tau_1,\tau_2) \\ \hat{h}_{QI}^2(\tau_1,\tau_2) & \hat{h}_{QQ}^2(\tau_1,\tau_2) \end{bmatrix} \cdot \begin{bmatrix} \hat{X}(t-k_2\Delta t) - \hat{X}(t) \\ \hat{X}(t-k_2\Delta t)^* - \hat{X}(t)^* \end{bmatrix} \\
& + \\
& \circ \\
& \circ
\end{aligned}
$$

First order term is forced to be linear as to signal speed

- **Extension**

Basis function order n : $B_n(X) = \sum_{k=0}^{Kn} \alpha_{n,k} X^n$

$$
\begin{aligned}
\hat{Y}(t) = \; & \psi(\hat{X}(t), \hat{X}(t)^*) \\
& + \sum_{k=0}^{\infty} f^1\left(\hat{X}(t), \hat{X}(t)^*, \hat{X}(t-k\Delta t), \hat{X}(t-k\Delta t)^*, k\Delta t\right) \\
& + \sum_{k_1}^{\infty} \sum_{k_2}^{\infty} f^2\left(\hat{X}(t), \hat{X}(t)^*, \hat{X}(t-k_1\Delta t), \hat{X}(t-k_1\Delta t)^*, \hat{X}(t-k_2\Delta t), \hat{X}(t-k_2\Delta t)^*, k_1\Delta t, k_2\Delta t\right) \\
& + \\
& \circ \\
& \circ
\end{aligned}
$$

First order term is allowed to be nonlinear as to signal speed

1rst order truncation => Nonlinear impulse response

$$\hat{X}(t) = \left|\hat{X}(t)\right| e^{j\int_0^t \Omega_x(\tau)\,d\tau}$$

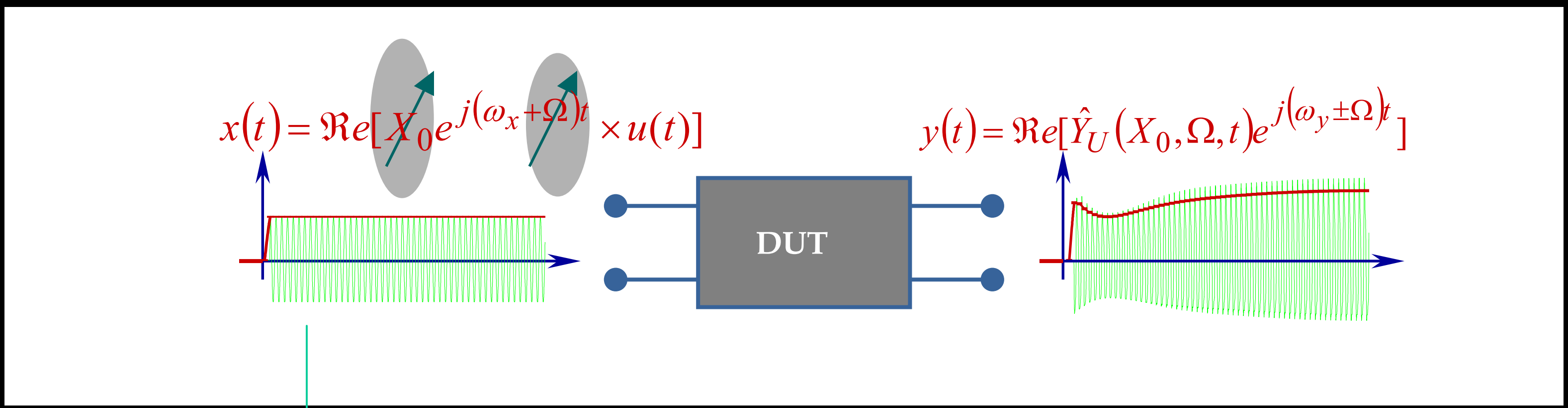
Time varying amplitude

Time varying frequency

$$\hat{Y}(t) = \int_0^{T_\infty} \hat{h}\left(\left|\hat{X}(t-\tau)\right|, \Omega_x(t-\tau), \tau\right) \times \hat{X}(t-\tau)\,d\tau$$

Nonlinear impulse response extraction

- Unit step envelope drive:

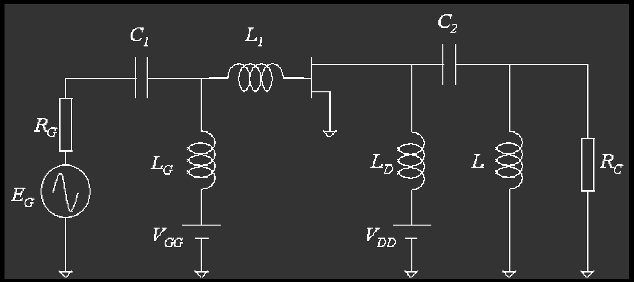


$$\hat{Y}(t) = \int_0^{T_\infty} \hat{h}\left(\left|\hat{X}(t-\tau)\right|, \Omega(t-\tau), \tau\right) \times \hat{X}(t-\tau)\, d\tau$$

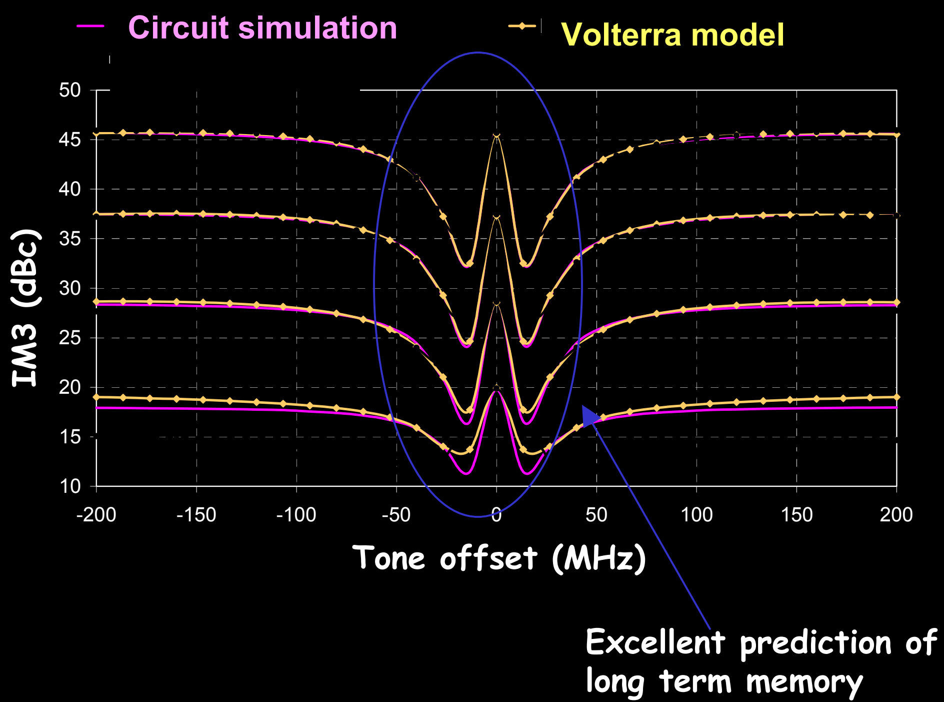
$$\hat{h}(X_0, \Omega, t) = \frac{1}{X_0} \frac{\partial \hat{Y}_U(X_0, \Omega, t)}{\partial t} e^{\pm j\Omega t}$$

⇒ Envelope transient simulation or Experimental envelope characterization with digital scope

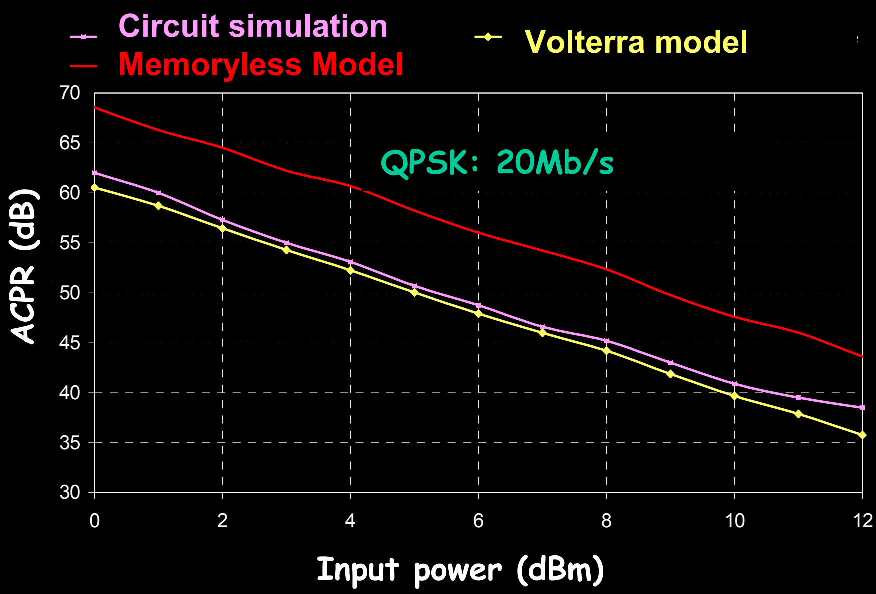
Elementary amplifier modeling



- Two-tone test



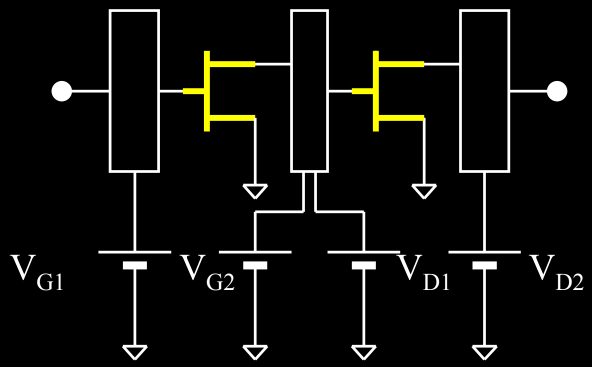
- Multitone test



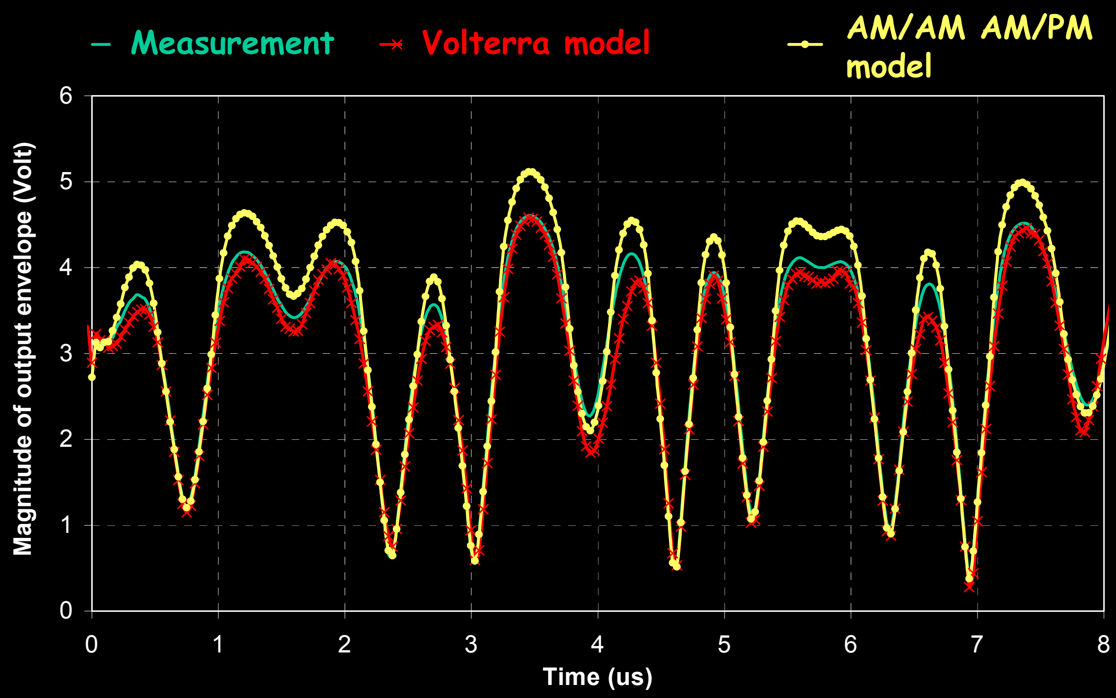
Experimental modeling

Experimentally extracted model from time domain envelope measurement setup (Tektronix AWG2021 and TDS 754D)

space qualified
2stage 1.2mm HFET amplifier
f_0 = 1.6 GHz
Pout = 350 mW
BW = 80 MHz



QPSK signal @ 1 MB/s rate,
-16 dBm input



Summary

- Envelope domain modeling + Modified Volterra series concept give an effective solution for RF & microwave circuit modeling
- Models can be extracted from either circuit simulation tools and physical measurement equipments
- Model equations can be implemented in analog HDL

Model justification

$$\hat{Y}(t) = \int_0^{T_\infty} \hat{h}\left(\left|\hat{X}(t-\tau)\right|, \Omega(t-\tau), \tau\right) \times \hat{X}(t-\tau)\, d\tau$$