
Modeling Memory Effects in Nonlinear Subsystems by Dymanic Volterra Series

Edouard Ngoya, Arnaud Soury

IRCOM-CNRS, Université de Limoges , 123 av. A. Thomas, 87060 Limoges, France

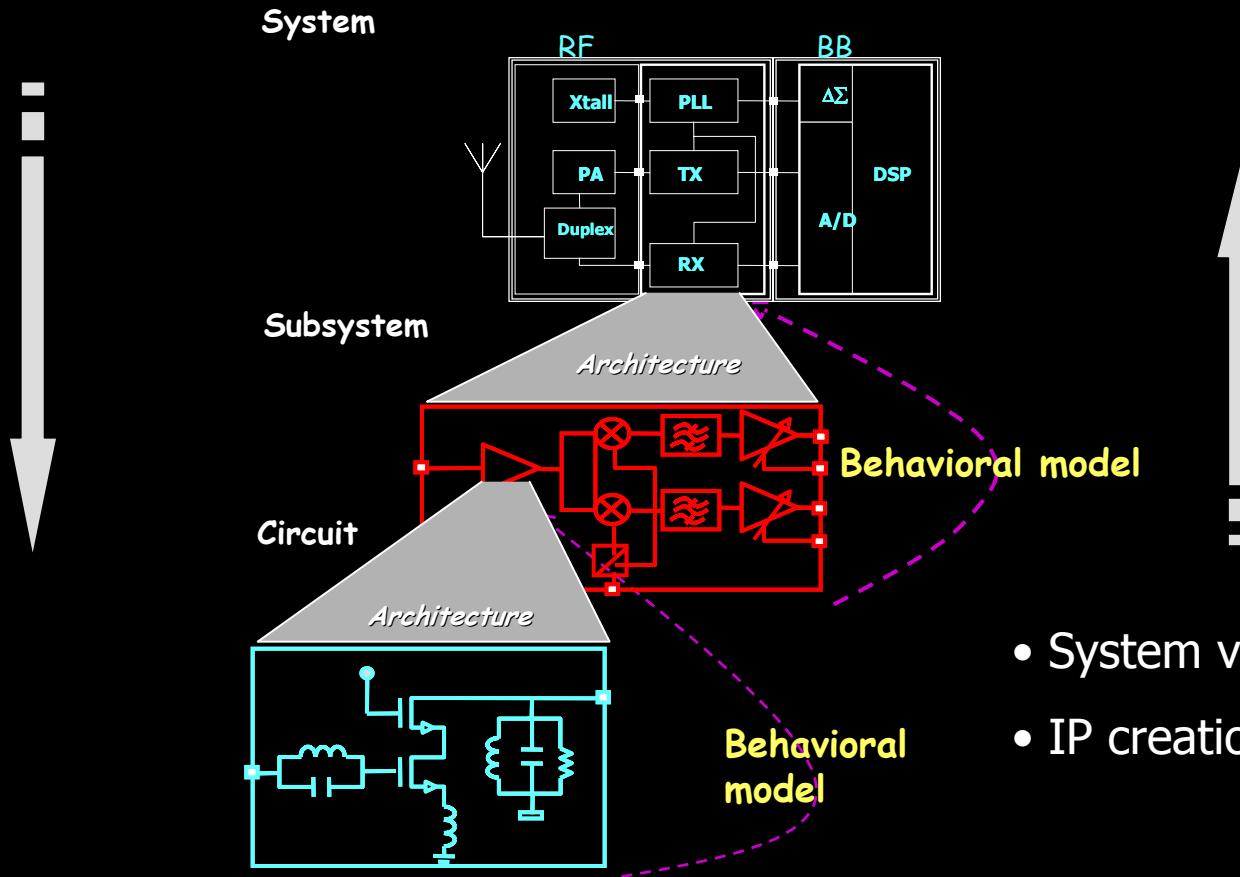
Outline

- RF subsystem modeling challenge
- Envelope domain modeling basics
- Modified Volterra series approaches for highly nonlinear systems with memory
 - Examples
 - Summary



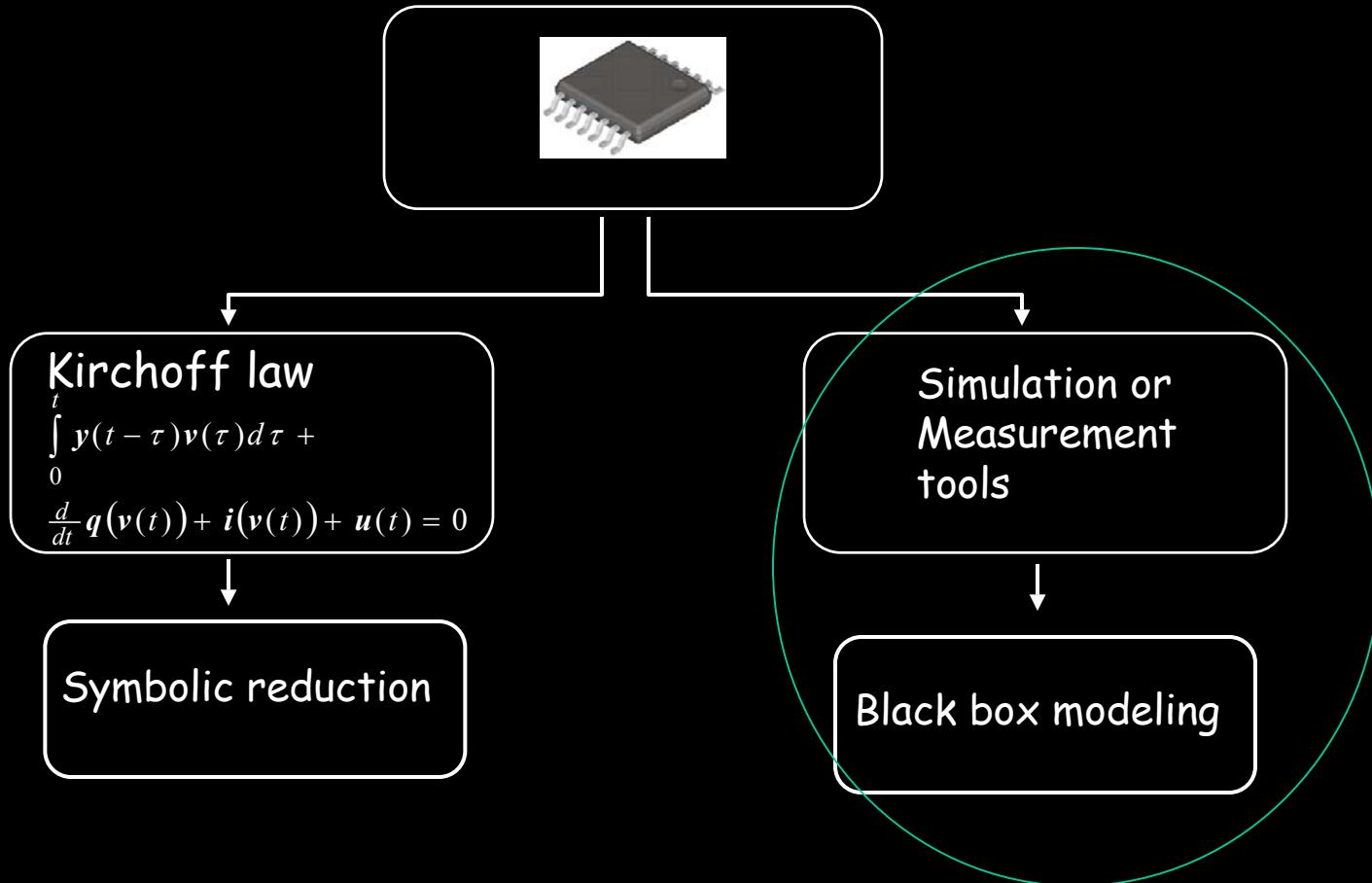
Behavioral modeling for system verification and IP creation

- Top-down design

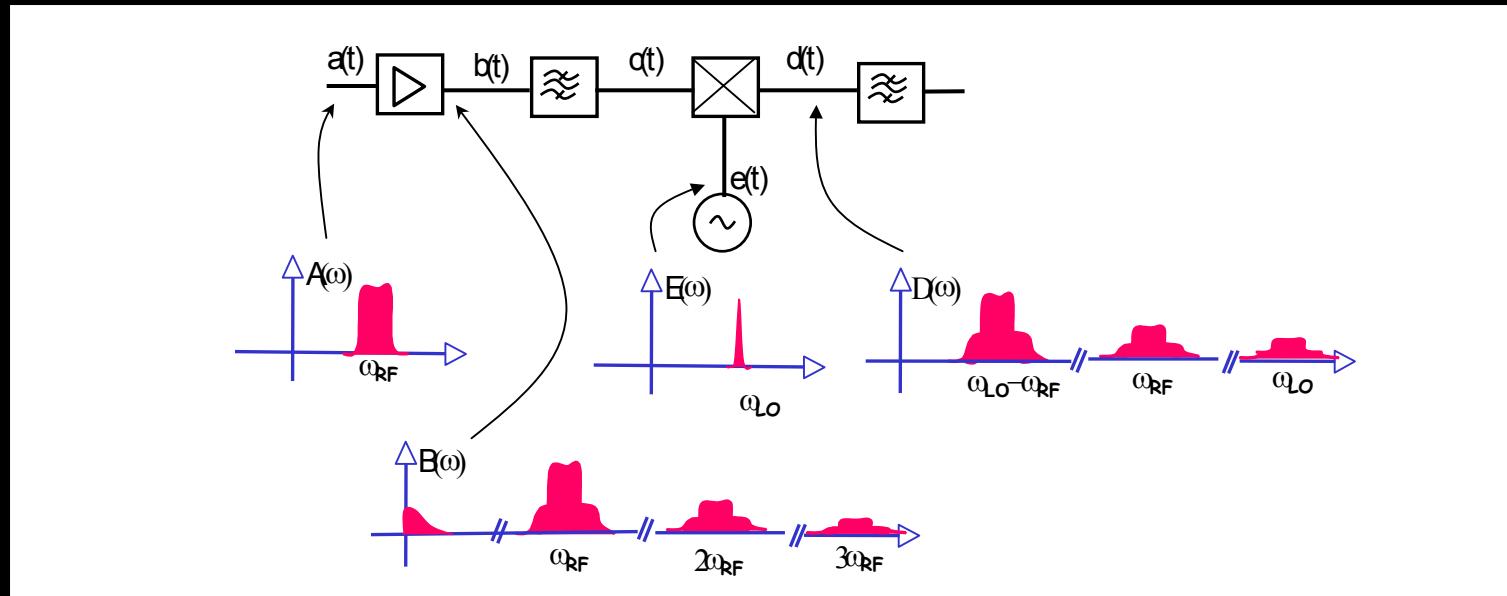


- System verification
- IP creation

Bottom-up block modeling



RF & microwave subsystem modeling challenge



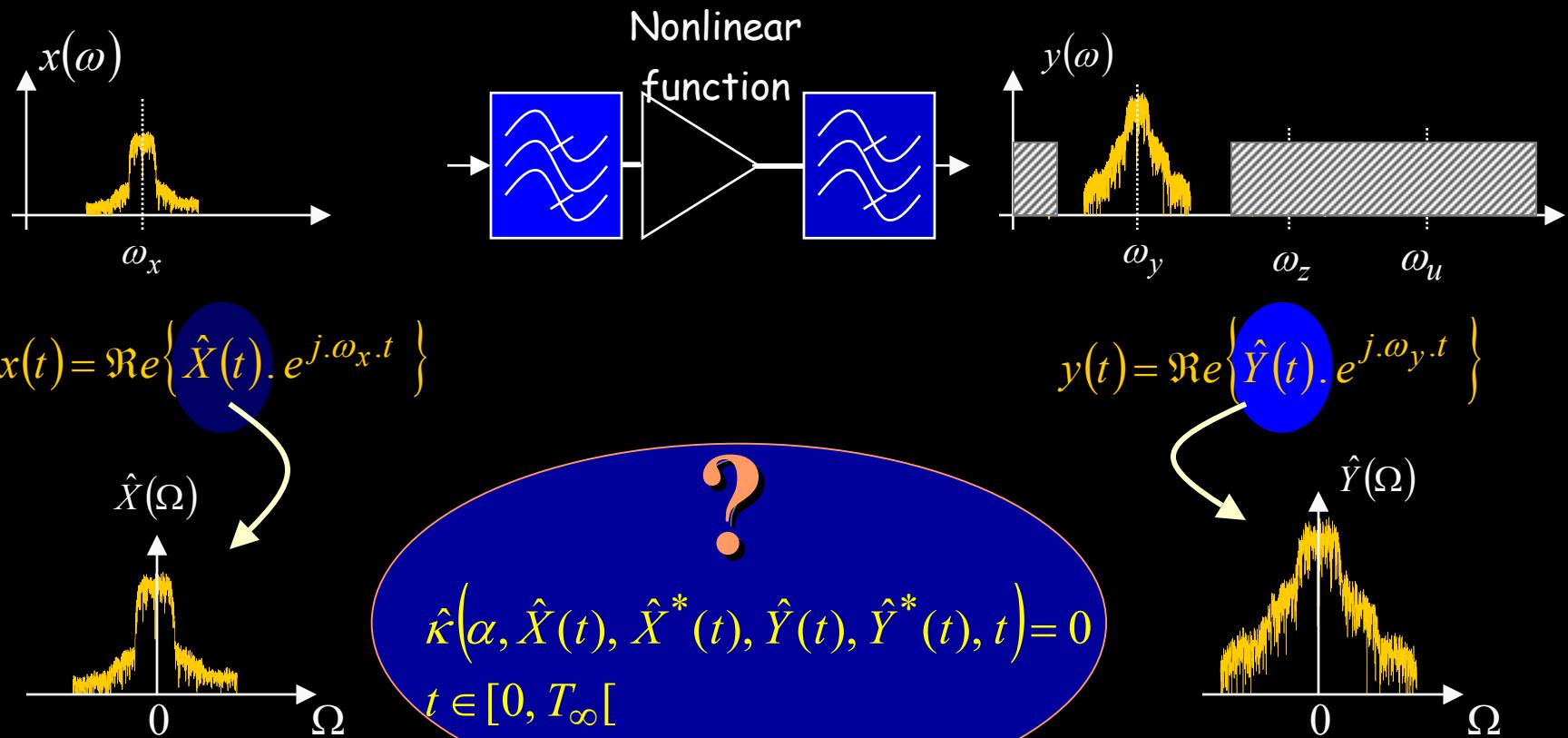
- Time scales disparity (Baseband signals, carriers, noise, spurious)
- Pass band components (High Q filters, transmission lines, ..)
- Nonlinear functions (PA, converters, mixer,..)



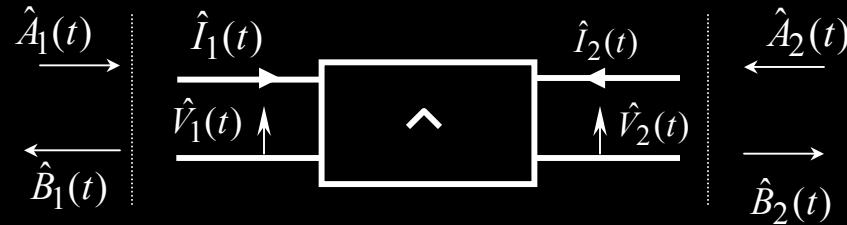
Pass band – Enveloppe concept

Pass band subsystem

amplifier
mixer
multiplier
VCO



Functional modeling



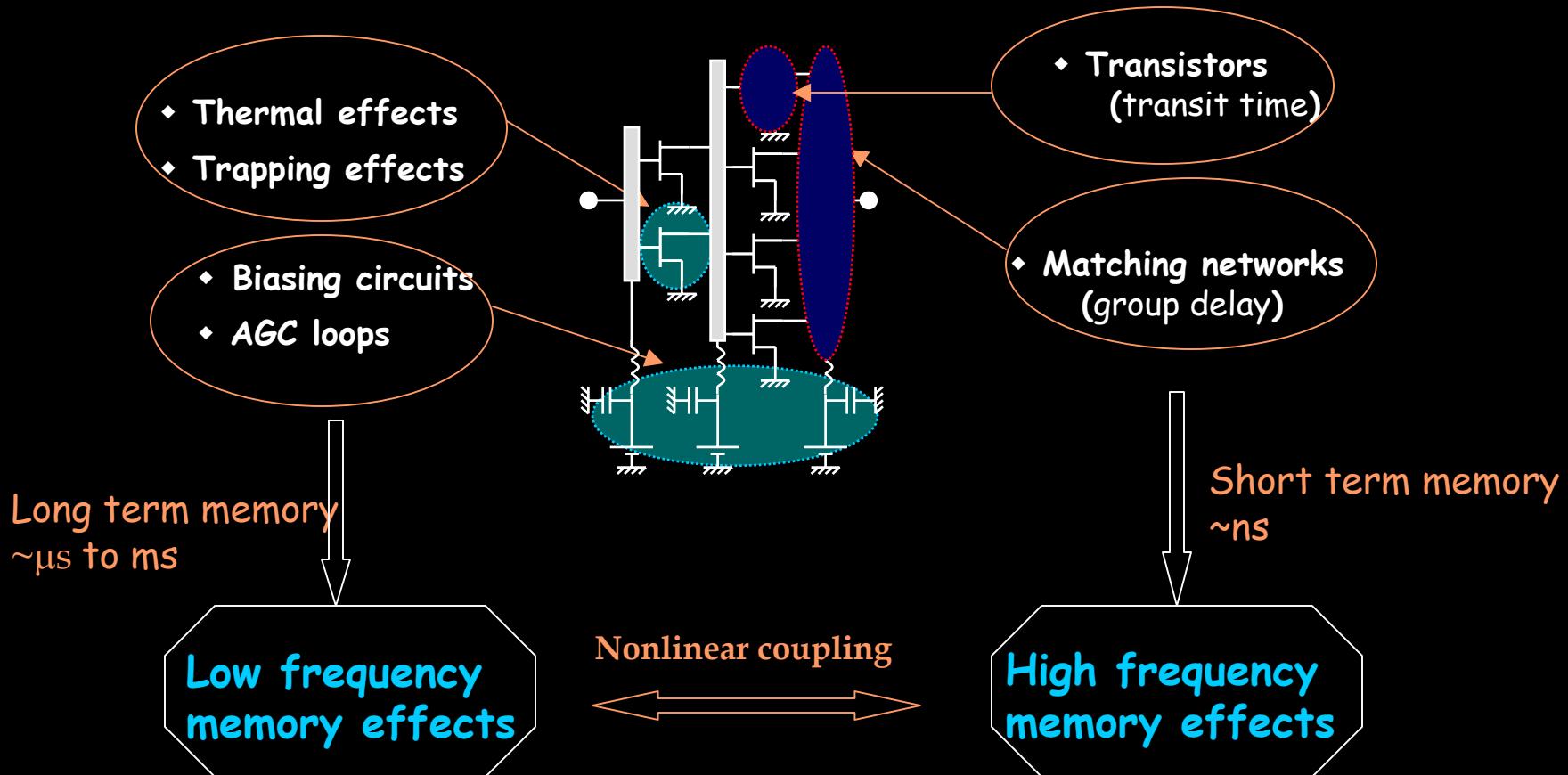
$$\hat{\kappa} \left(\alpha, \hat{X}(t), \hat{X}^*(t), \hat{Y}(t), \hat{Y}^*(t), t \right) = 0$$

$$t \in [0, T_\infty[$$

👉 Three major problems

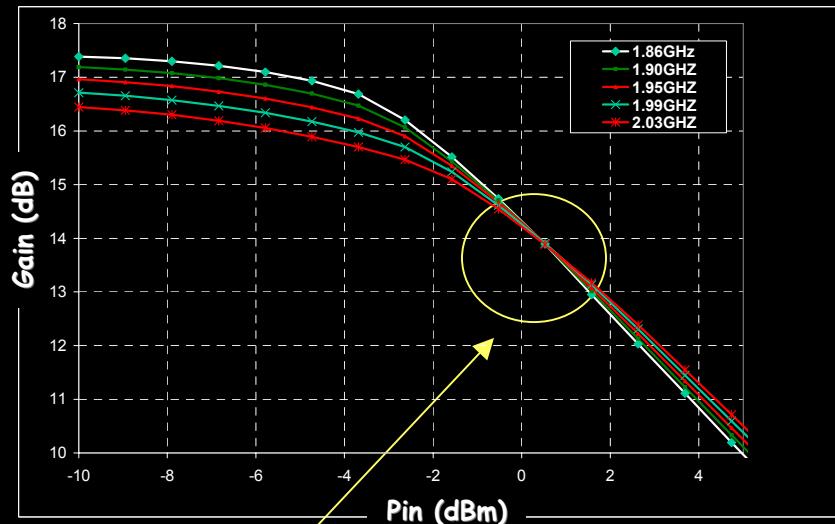
- Nonlinearity
- Interstage mismatch
- Memory effects
 - High speed applications
 - High capacity

Severe memory effects in solid state devices



Memory effects highlights : Typical narrow band BiCMOS LNA

Short term memory : Single tone measurements

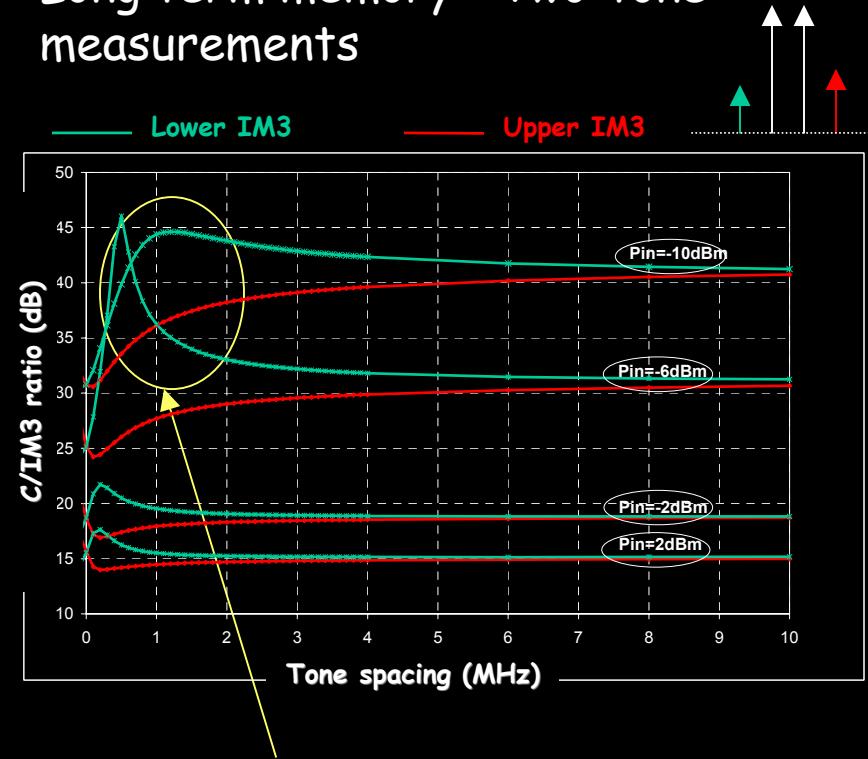


wrapping



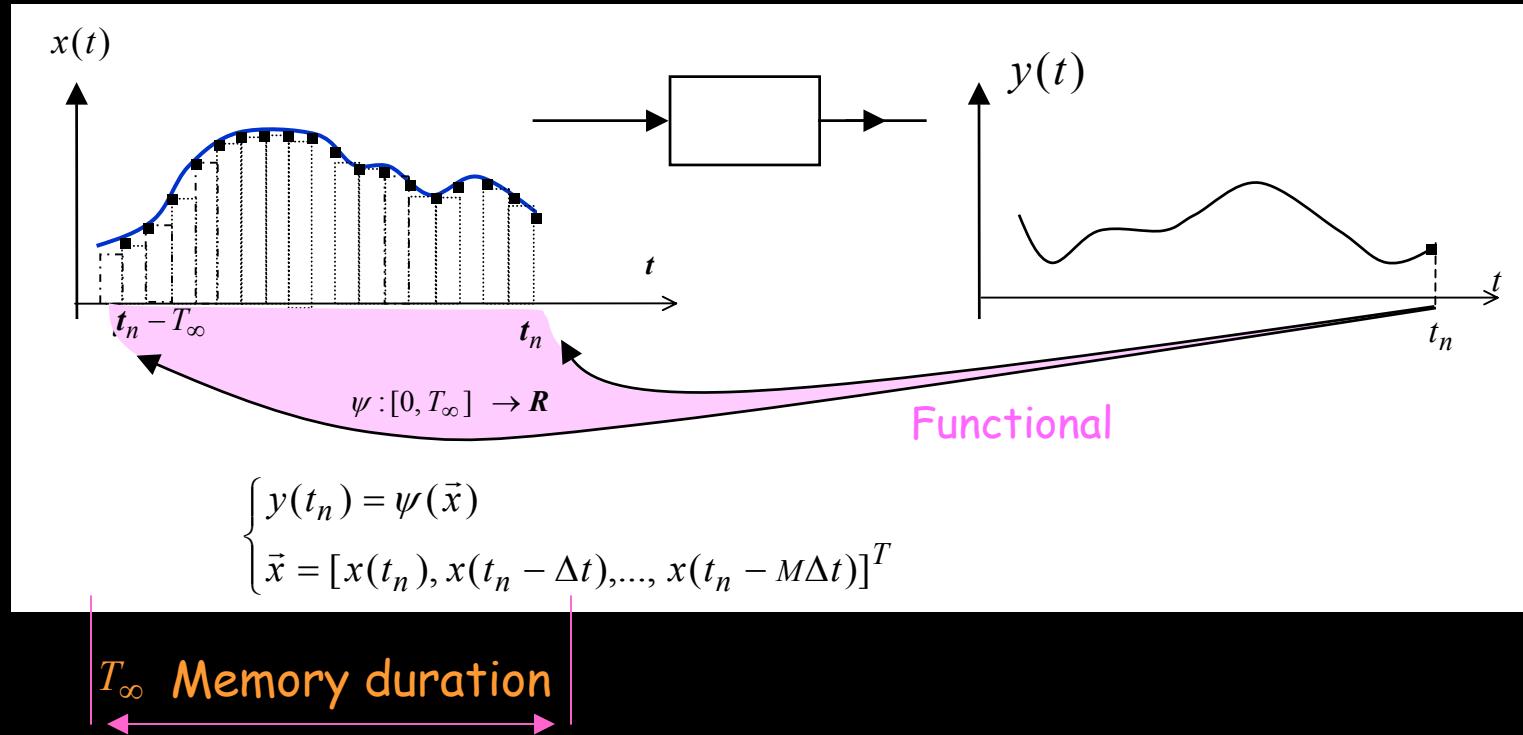
Need rigorous but yet simple modeling approach (affordable with either simulation and measurement tools)

Long term memory : Two-tone measurements



Resonance & asymmetry

Analytic approach: Volterra series expansion



Power series expansion of the functional

$$y(t_n) = \psi(\vec{x}_0) + \Delta\psi(\vec{x}_0)^T (\vec{x} - \vec{x}_0) + \frac{1}{2} (\vec{x} - \vec{x}_0)^T [\Delta^2\psi(\vec{x}_0)] (\vec{x} - \vec{x}_0) + \dots$$

→ \vec{x}_0 is an arbitrary input record

Volterra series approach

$$\vec{x}_0 = \vec{0}, \quad \Delta t \rightarrow 0,$$

$$y(t) = \sum_{n=1}^{\infty} y_n(t)$$

$$y_1(t) = \int_0^{T_\infty} h_1(\tau_1) x(t - \tau_1) d\tau_1$$

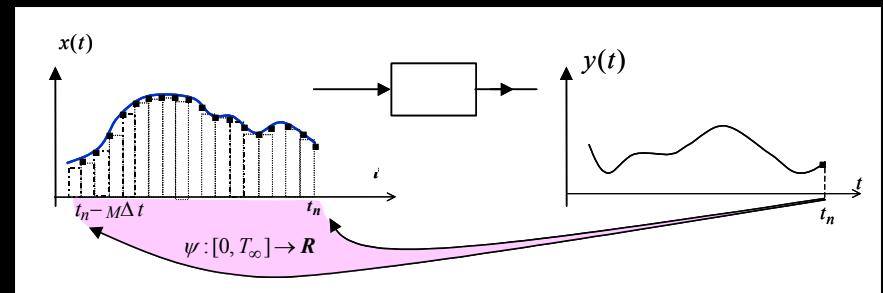
$$y_2(t) = \int_0^{T_\infty} \int_0^{T_\infty} h_2(\tau_1, \tau_2) x(t - \tau_1) x(t - \tau_2) d\tau_1 d\tau_2 \quad \rightarrow \text{quadratic}$$

...

$$y_n(t) = \int_0^{T_\infty} \dots \int_0^{T_\infty} h_n(\tau_1, \dots, \tau_n) x(t - \tau_1) \dots x(t - \tau_n) d\tau_1 \dots d\tau_n \quad \rightarrow \text{order } n$$

Volterra kernel of order n

- Nonlinear circuit \Rightarrow consider $n > 1 \Rightarrow$ practically inefficient



linear
response



Modified Volterra series

$$\vec{x}_0 = \vec{f}(t_n) \neq \vec{0},$$

$$y(t) = \sum_{n=0}^{\infty} y_n(t)$$

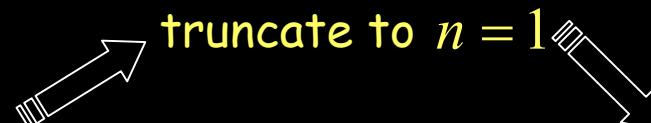
$$y_0(t) = \psi(x(t))$$

$$y_1(t) = \int_0^{T_\infty} h_1(x(t), \tau_1) [x(t - \tau_1) - x(t)] d\tau_1 \quad \rightarrow \text{Parametric linear}$$

...

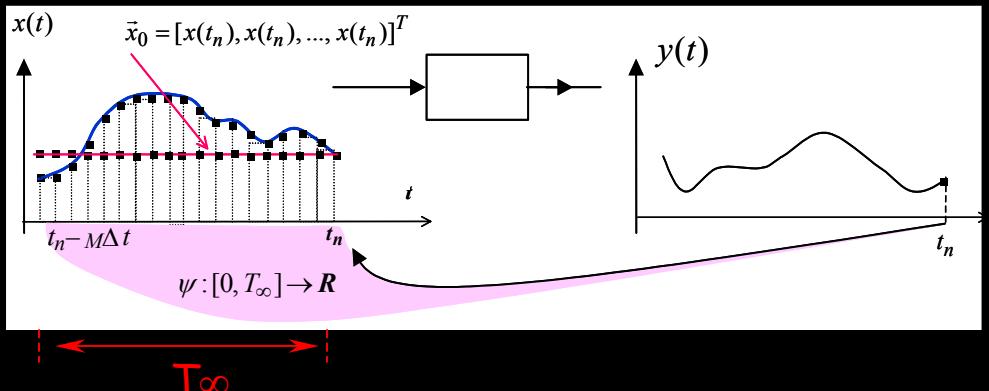
$$y_n(t) = \int_0^{T_\infty} \dots \int_0^{T_\infty} h_n(x(t), \tau_1, \dots, \tau_n) [x(t - \tau_1) - x(t)] \dots [x(t - \tau_n) - x(t)] d\tau_1 \dots d\tau_n \quad \rightarrow \text{Modified Volterra kernel of order } n$$

Signal speed



- Short memory duration:
 $T_\infty \ll \text{signal period}$

achieve some practical efficiency

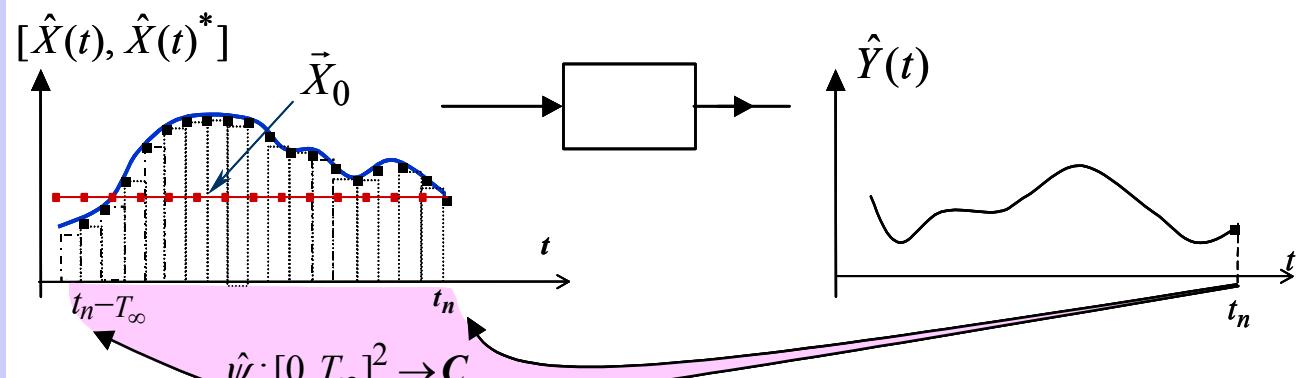


Envelope domain modeling enforces short memory duration conditions

$$x(t) = \Re e[\hat{X}(t)e^{j\omega_x t}]$$

$$y(t) = \Re e[\hat{Y}(t)e^{j\omega_y t}]$$

$$\hat{X}(t) = \frac{1}{2\pi} \int_{-BW/2}^{BW/2} \hat{X}(\Omega) e^{j\Omega t} d\Omega$$



Short memory duration = $T_\infty \ll \frac{1}{BW}$ = more realistic assumption

Envelope domain modified Volterra model

$$\hat{Y}(t) = \sum_{n=0}^{\infty} \hat{Y}_n(t)$$

$$\hat{Y}_0(t) = \hat{\psi}(\hat{X}(t), \hat{X}(t)^*)$$

$$\hat{Y}_1(t) = \int_0^{T_\infty} h_I(\hat{X}(t), \hat{X}(t)^*, \tau) \cdot (X(t-\tau) - X(t)) d\tau + \int_0^{T_\infty} h_Q(X(t), X(t)^*, \tau) \cdot (X(t-\tau)^* - X(t)^*) d\tau$$

...

Order 0: AM/AM, AM/PM
model

Parametric linear (I-Q)

$$\hat{Y}_n(t) = \int_0^{T_\infty} \dots \int_0^{T_\infty} \sum_{i_1=1}^2 \dots \sum_{i_n=1}^2 \hat{h}_{i_1 \dots i_n}(\hat{X}(t), \hat{X}(t)^*, \tau_1, \dots, \tau_n) \cdot (X^{i_1}(t-\tau_1) - X^{i_1}(t)) \cdot \dots \cdot (X^{i_n}(t-\tau_1) - X^{i_n}(t)) d\tau_1 \dots d\tau_n$$

$$X^1(t) \stackrel{\Delta}{=} \hat{X}(t), \quad X^2(t) \stackrel{\Delta}{=} \hat{X}(t)^*$$

Parametric order n

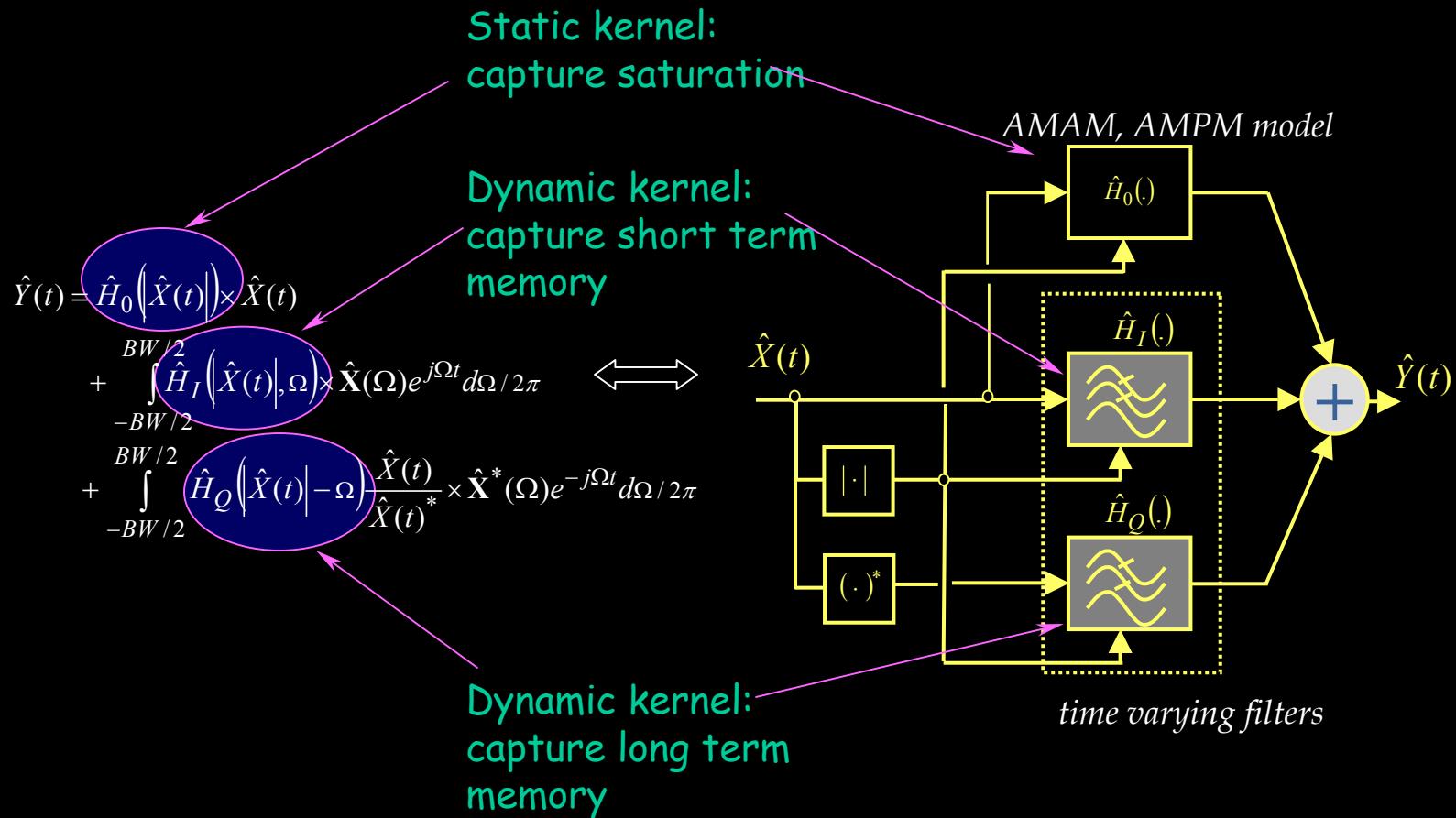
Short term memory conditions $\Rightarrow n = 1$

$$T_\infty \ll \frac{1}{BW}$$

$$\begin{aligned} \hat{Y}(t) = \hat{\psi}(\hat{X}(t), \hat{X}^*(t)) &+ \int h_I(\hat{X}(t), \hat{X}^*(t), \tau) \times (\hat{X}(t-\tau) - \hat{X}(t)) d\tau \\ &+ \int h_Q(\hat{X}(t), \hat{X}^*(t), \tau) \times (\hat{X}^*(t-\tau) - \hat{X}^*(t)) d\tau \end{aligned}$$

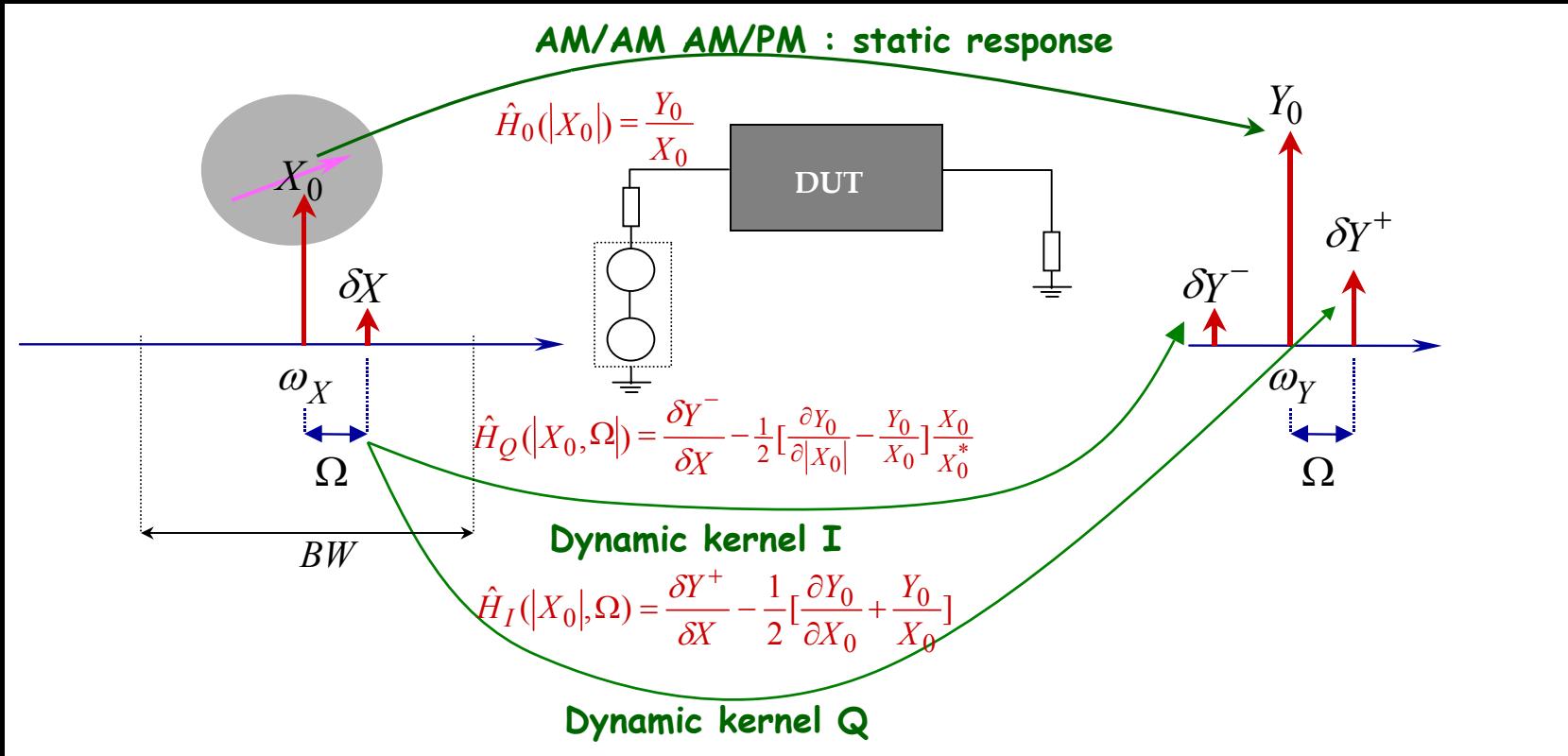


1rst order modified Volterra model : summary



1rst order modified Volterra model : Kernel extraction

- Two-tone measurements



→ Harmonic Balance, shooting simulation or Experimental characterization with VNA or digital scope

Constant envelope systems (gmsk, radar, ..)

→ Long term memory is not excited

$$\begin{aligned}\hat{Y}(t) &= \hat{H}_0 \left(\hat{X}(t) \right) \times \hat{X}(t) \\ &+ \int_{-BW/2}^{BW/2} \hat{H}_I \left(\hat{X}(t), \Omega \right) \times \hat{\mathbf{X}}(\Omega) e^{j\Omega t} d\Omega / 2\pi \\ &+ \int_{-BW/2}^{BW/2} \hat{H}_Q \left(\hat{X}(t), -\Omega \right) \hat{X}(t)^* \times \hat{\mathbf{X}}^*(\Omega) e^{-j\Omega t} d\Omega / 2\pi\end{aligned}$$

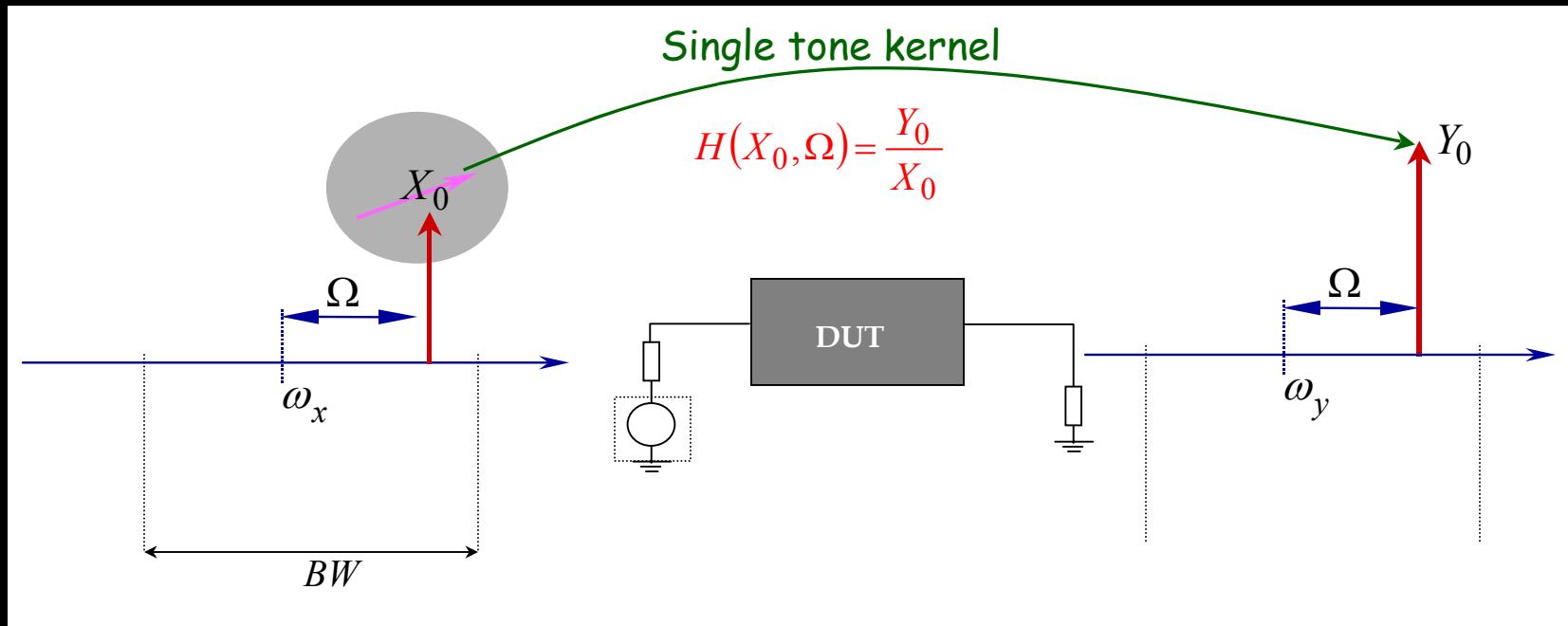
→ Very effective model for constant envelope systems

$$\hat{Y}(t) = \int_{-BW/2}^{BW/2} \hat{H} \left(\hat{X}(t), \Omega \right) \times \hat{\mathbf{X}}(\Omega) e^{j\Omega t} \frac{d\Omega}{2\pi} = \int_0^{T_\infty} \hat{h} \left(\hat{X}(t), \tau \right) \times \hat{X}(t - \tau) d\tau$$

Kernel = Single tone complex gain vs power and frequency

Quasi-constant envelope systems (gmsk, radar, ..)

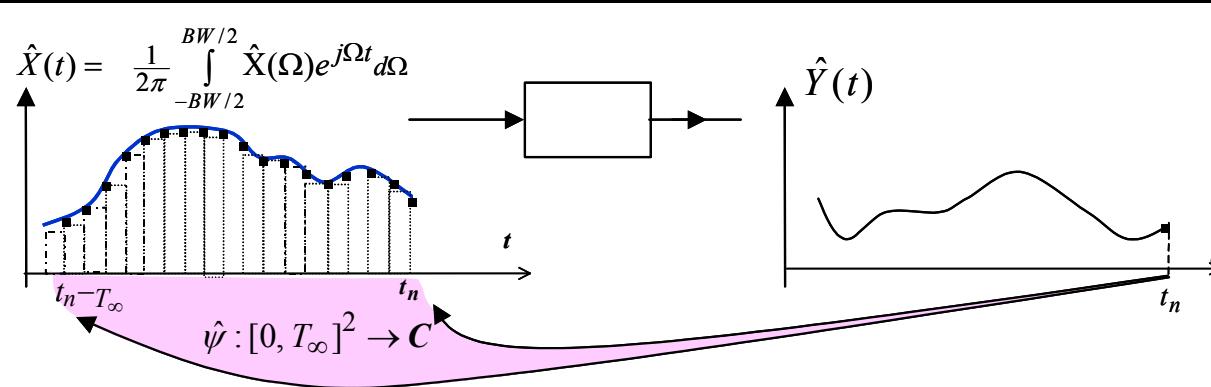
→ **Very simple kernel extraction**



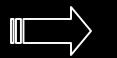
Certain circuits do not satisfy short memory duration conditions

Long term memory effects: $T_\infty > \frac{1}{BW}$

- Poorly designed biasing circuits - AGC loops - Thermal effects



$$\begin{cases} \hat{Y}(t_n) = \hat{\psi}(\vec{X}) \\ \vec{X} = [\hat{X}(t_n), \hat{X}(t_n)^*, \hat{X}(t_n - \Delta t), \hat{X}(t_n - \Delta t)^*, \dots, \hat{X}(t_n - M\Delta t), \hat{X}(t_n - M\Delta t)^*]^T \end{cases}$$



Reconsider the functional decomposition :
Use more effective basis functions instead of power expansions

- Previously modified Volterra series

Basis function order n : $B_n(X) = X^n$

$$\hat{Y}(t) = \psi(\hat{X}(t), \hat{X}(t)^*)$$

$$\begin{aligned}
 & + \sum_{k=0}^{\infty} [h_I^1(\hat{X}(t), \hat{X}(t)^*, k\Delta t) - h_Q^1(\hat{X}(t), \hat{X}(t)^*, t_k)] \cdot \begin{bmatrix} \hat{X}(t - k\Delta t) - \hat{X}(t) \\ \hat{X}(t - k\Delta t)^* - \hat{X}(t)^* \end{bmatrix} \\
 & + \sum_{k_1}^{\infty} \sum_{k_2}^{\infty} [\hat{X}(t - k_1\Delta t) - \hat{X}(t) - \hat{X}(t - k_1\Delta t)^* + \hat{X}(t)^*] \cdot \begin{bmatrix} \hat{h}_{II}^2(\tau_1, \tau_2) & \hat{h}_{IQ}^2(\tau_1, \tau_2) \\ \hat{h}_{QI}^2(\tau_1, \tau_2) & \hat{h}_{QQ}^2(\tau_1, \tau_2) \end{bmatrix} \cdot \begin{bmatrix} \hat{X}(t - k_2\Delta t) - \hat{X}(t) \\ \hat{X}(t - k_2\Delta t)^* - \hat{X}(t)^* \end{bmatrix} \\
 & + \\
 & \circ \\
 & \circ
 \end{aligned}$$

Change basis functions

First order term is forced to be linear as to signal speed

- Extension

Basis function order n : $B_n(X) = \sum_{k=0}^{Kn} \alpha_{n,k} X^n$

$$\hat{Y}(t) = \psi(\hat{X}(t), \hat{X}(t)^*)$$

$$+ \sum_{k=0}^{\infty} f^1(\hat{X}(t), \hat{X}(t)^*, \hat{X}(t - k\Delta t), \hat{X}(t - k\Delta t)^*, k\Delta t)$$

$$+ \sum_{k_1}^{\infty} \sum_{k_2}^{\infty} f^2(\hat{X}(t), \hat{X}(t)^*, \hat{X}(t - k_1\Delta t), \hat{X}(t - k_1\Delta t)^*, \hat{X}(t - k_2\Delta t), \hat{X}(t - k_2\Delta t)^*, k_1\Delta t, k_2\Delta t)$$

+

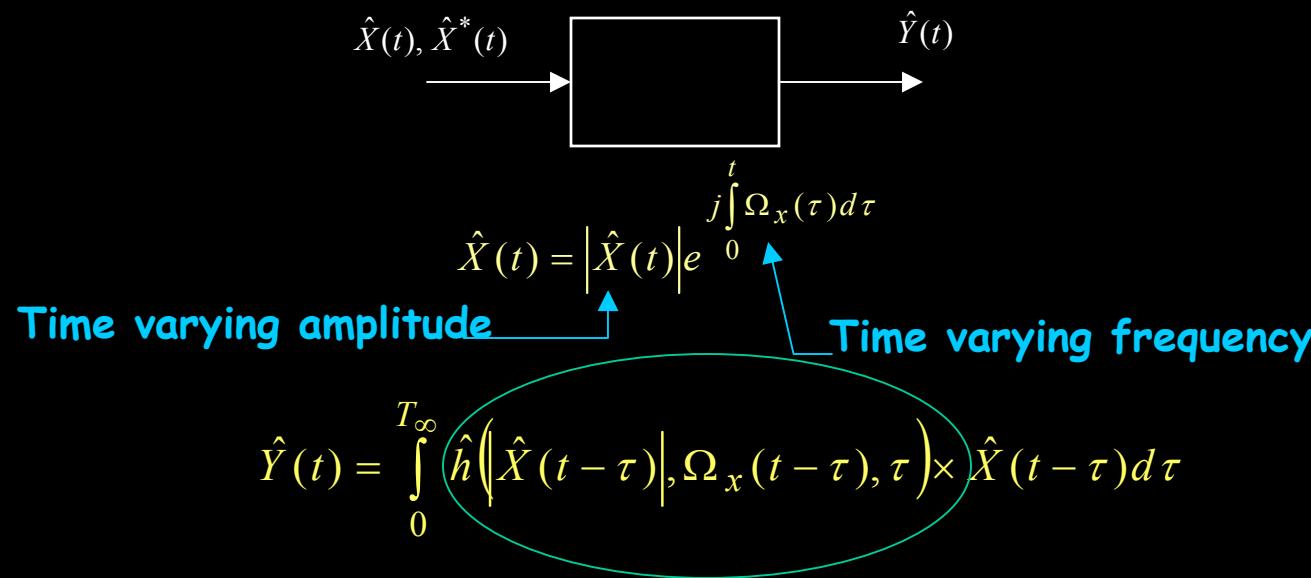
◦

◦



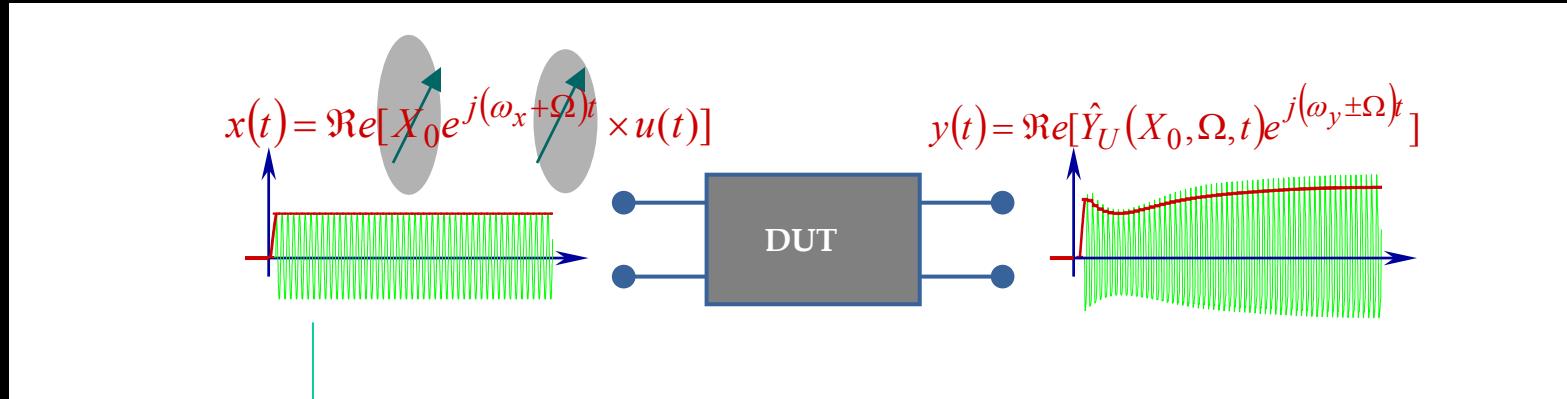
First order term is allowed to be nonlinear as to signal speed

1rst order truncation => Nonlinear impulse response



Nonlinear impulse response extraction

- Unit step envelope drive:

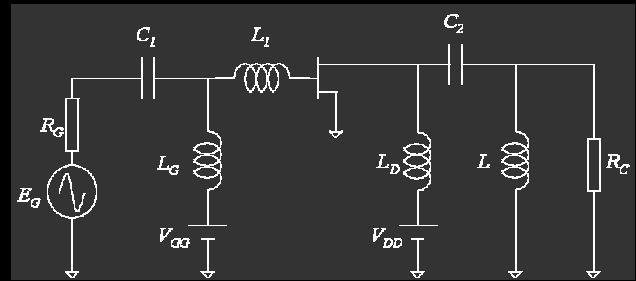


$$\hat{Y}(t) = \int_0^{T_\infty} \hat{h}\left(\hat{X}(t-\tau), \Omega(t-\tau), \tau\right) \times \hat{X}(t-\tau) d\tau$$

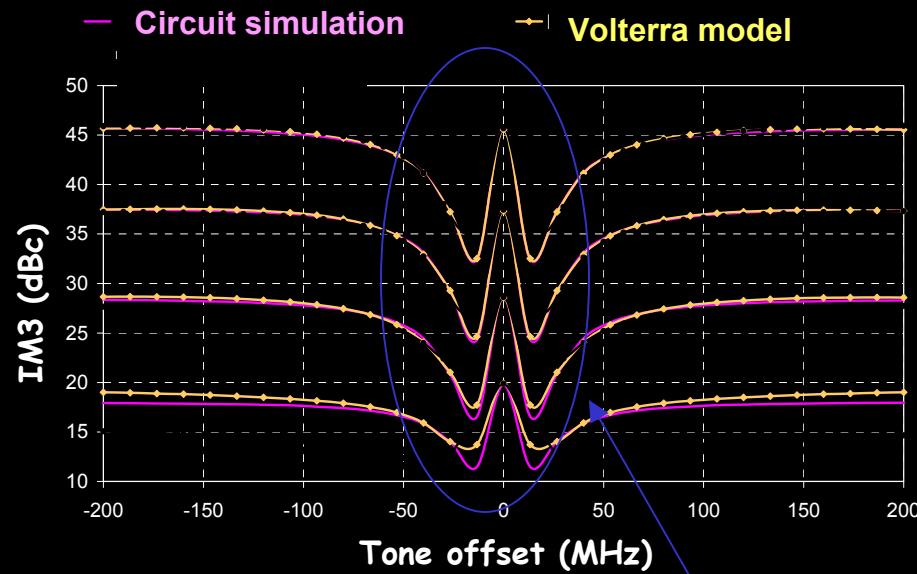
$$\hat{h}(X_0, \Omega, t) = \frac{1}{X_0} \frac{\partial \hat{Y}_U(X_0, \Omega, t)}{\partial t} e^{\pm j\Omega t}$$

→ Envelope transient simulation or Experimental envelope characterization with digital scope

Elementary amplifier modeling

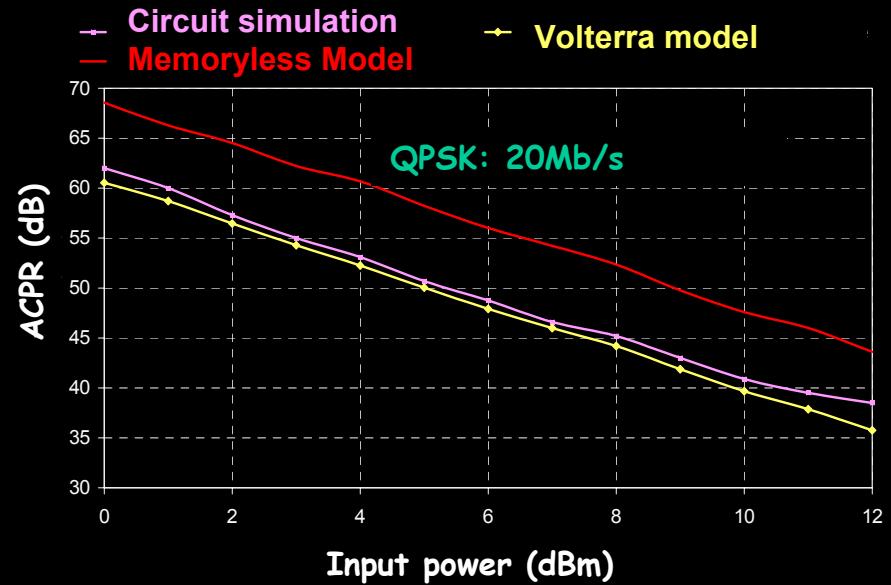


- Two-tone test



Excellent prediction of
long term memory

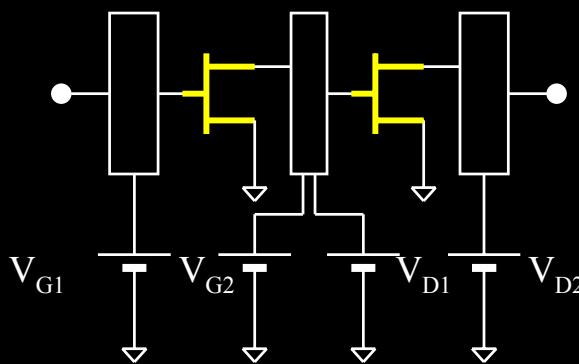
- Multitone test



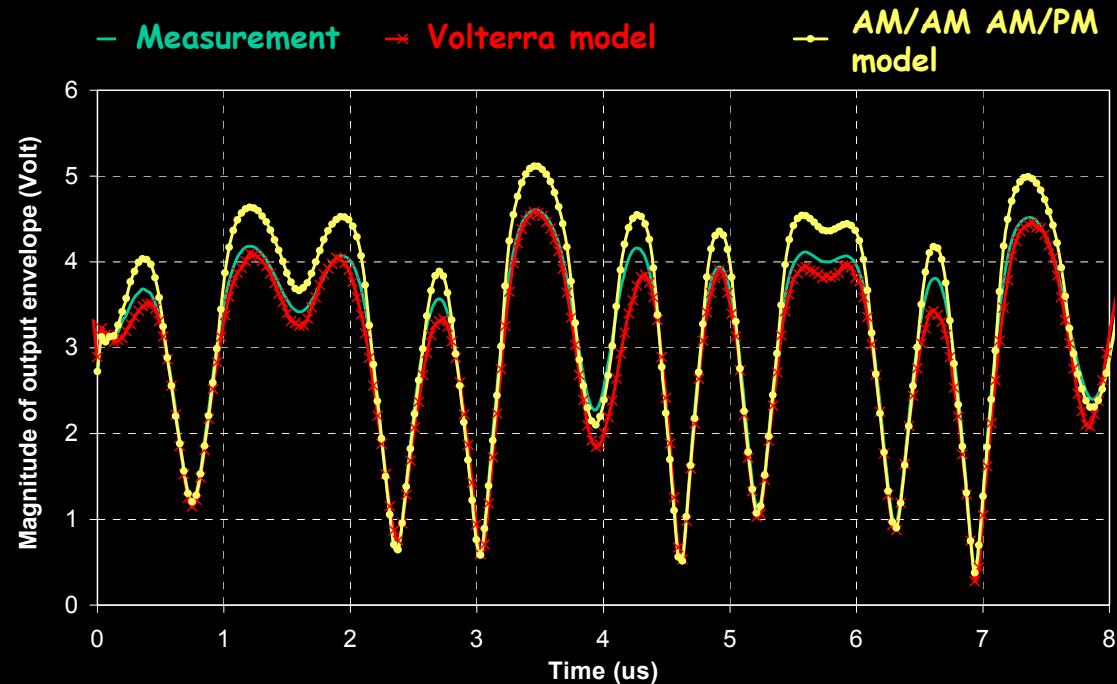
Experimental modeling

Experimentally extracted model from time domain envelope measurement setup
(Tektronix AWG2021 and TDS 754D)

space qualified
2stage 1.2mm HFET amplifier
 $f_0 = 1.6$ GHz
 $P_{out} = 350$ mW
 $BW = 80$ MHz



QPSK signal @ 1 MB/s rate,
-16 dBm input



Summary

- Envelope domain modeling + Modified Volterra series concept give an effective solution for RF & microwave circuit modeling
- Models can be extracted from either circuit simulation tools and physical measurement equipments
- Model equations can be implemented in analog HDL



Model justification

$$\hat{Y}(t) = \int_0^{T_\infty} \hat{h}\left(\hat{X}(t-\tau), \Omega(t-\tau), \tau\right) \times \hat{X}(t-\tau) d\tau$$

