SOM-LS: Selective Orthogonal Matrix Least-Squares Method for Macromodeling Multiport Networks Characterized by Sampled Data

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Which function is dominant?
The least-squares method for system identification **selecting the dominant basis functions** is presented.

This method is based on a learning algorithm of neural network.

Macromodeling of networks characterized by sampled data via electromagnetic analysis.

The macromodels are described in the format of **Verilog-A**.
Selective Orthogonal Matrix Least-Squares Method

Device Model:

\[ G(x) = \sum_{i=1}^{N} K_i f_i(x) \]

- \( K_i \) : constant matrix
- \( f_i(x) \) : basis function
- \( x \) : design variable

The device model is made by least-squares fitting of sampled data
The over-determined matrix equation is solved by orthogonal least-squares method.

Over-determined Equation:

\[
P K = F
\]

\[
P = \begin{bmatrix}
f_1(x_1) & f_2(x_1) & \cdots & f_M(x_1) \\
f_1(x_2) & f_2(x_2) & \cdots & f_M(x_2) \\
\vdots & \vdots & \ddots & \vdots \\
f_1(x_N) & f_2(x_N) & \cdots & f_M(x_N)
\end{bmatrix}
\]

\[
F = \left[ \mathbf{G}(x_1), \mathbf{G}(x_2), \ldots, \mathbf{G}(x_N) \right]^T
\]
The coefficient matrix is rewritten by

\[ P = [p_1, p_2, p_3, p_4, \ldots, p_k, \ldots, p_N] \]

The number of basis functions is equal to the number of orthogonal vectors.

The key issue is how to select the column vectors and orthogonalize them.

\[ P = [w_1, w_2, w_3, w_4, \ldots, w_{k-1}, p_k, \ldots, p_N] \]

\[ P = [p_1, \ldots, p_{N-k+1}, w_1, w_2, w_3, w_4, \ldots, w_{k-1}] \]
After \( k \)th step in the orthogonalization, the residual matrix is defined as

\[
Z = F - W_k G_k
\]

\( W_k \): orthogonal matrix
\( G_k \): intermediate solution

The 2-norm of the residual matrix is proven to be monotonously decreasing function.
Selectively Orthogonalization

Evaluating the 2-norm of the residual matrix, the columns of the matrix $P$ are orthogonalized so that the 2-norm largely decreases at each step.

$$P = [w_1, w_2, p_3, \ldots, p_k, \ldots, p_N]$$
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The SOM-LS method is used for approximating the sampled data with the rational matrix.
Sampled Data \rightarrow Rational Function \rightarrow Stable Poles

$Y_{ij}(j\omega_i) = \frac{b_0 + b_1(j\omega_i) + \cdots + b_m(j\omega_i)^m}{1 + a_1(j\omega_i) + \cdots + a_n(j\omega_i)^n}$

(\omega_i = 1, \ldots, N)

Least-Squares Fitting (scalar approximation)

Root Finding
Using the SOM-LS method, the dominant poles are extracted and the compact model is obtained.
Orthogonal Least-Squares Method

\[ PK = F \]

\[ P = \begin{bmatrix}
1 & 1 & \cdots & 1 \\
1 & j\omega_1 - p_1 & \cdots & j\omega_1 - p_Q \\
1 & j\omega_2 - p_1 & \cdots & j\omega_2 - p_Q \\
\vdots & \vdots & \ddots & \vdots \\
1 & j\omega_N - p_1 & \cdots & j\omega_N - p_Q \\
\end{bmatrix} \]
2-norm of Residual Matrix

![Graph showing the 2-norm of Residual Matrix with 'Normal' and 'Selective' curves.](#)
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Examples

- The marcromodels are described by Verilog-A.
Laplace Transform Description by Verilog-A

\[
V_{out}(s) = H(s) \cdot V_{in}(s)
\]

\[
H(s) = \frac{b_0 + b_1 \cdot s}{a_0 + a_1 \cdot s + a_2 \cdot s^2}
\]

module transfer_func(in, out);

inout in, out;
electrical in, out;

analog begin

V(out) <+ laplace_nd(V(in), [b_0,b_1], [a_0, a_1, a_2]);

end

endmodule
Verilog-A Model Generation

module model_name(in, out);
  inout in, out;
  electrical in, out;
  analog begin
    V(out) <+ laplace_nd(V(in), [b_0, b_1], [a_0, a_1, a_2]);
    V(out) <+ laplace_nd(V(in), [b_0, b_1], [a_0, a_1, a_2]);
    : 
    V(out) <+ laplace_nd(V(in), [b_0, b_1], [a_0, a_1, a_2]);
  end
endmodule
We computed the responses of simple PCB models using *Cadence Spectre*. The results using the proposed macromodels were compared with the FDTD method on *Spectre*. The computational speed with the proposed macromodels is two magnitudes faster than the FDTD method on *Spectre*. 
Example PCB Model

(a) Ground Plane

Bottom surface

Port 1

Port 2

Port 3

24mm

10kΩ

1pF

(b) +5V

V₁

V₂

1pF
Frequency-Domain Response

![Graph showing frequency-domain response with dB and GHz axes. The graph compares sampled data and proposed models.](image-url)
Time-Domain Response

![Graph showing Time-Domain Response with three curves: Input, FDTD, and proposed. The x-axis represents time in nsec, and the y-axis represents voltage [V]. The graph compares the response of different methods over time.]
## CPU Time Comparison

<table>
<thead>
<tr>
<th>Example</th>
<th>FDTD</th>
<th>Proposed Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example1</td>
<td>550.94 (sec)</td>
<td>4.27 (sec)</td>
</tr>
<tr>
<td>Example2</td>
<td>596.19 (sec)</td>
<td>5.96 (sec)</td>
</tr>
<tr>
<td>Example3</td>
<td>416.96 (sec)</td>
<td>4.09 (sec)</td>
</tr>
</tbody>
</table>
Summary

- The selective orthogonal matrix least-squares method is presented.
- This method is applied to macromodeling of networks characterized by sampled data.
- The proposed models are described in the format of Verilog-A.
- Future work: Passivity consideration of the macromodel.