A Methodology for Analog Circuit Macromodeling

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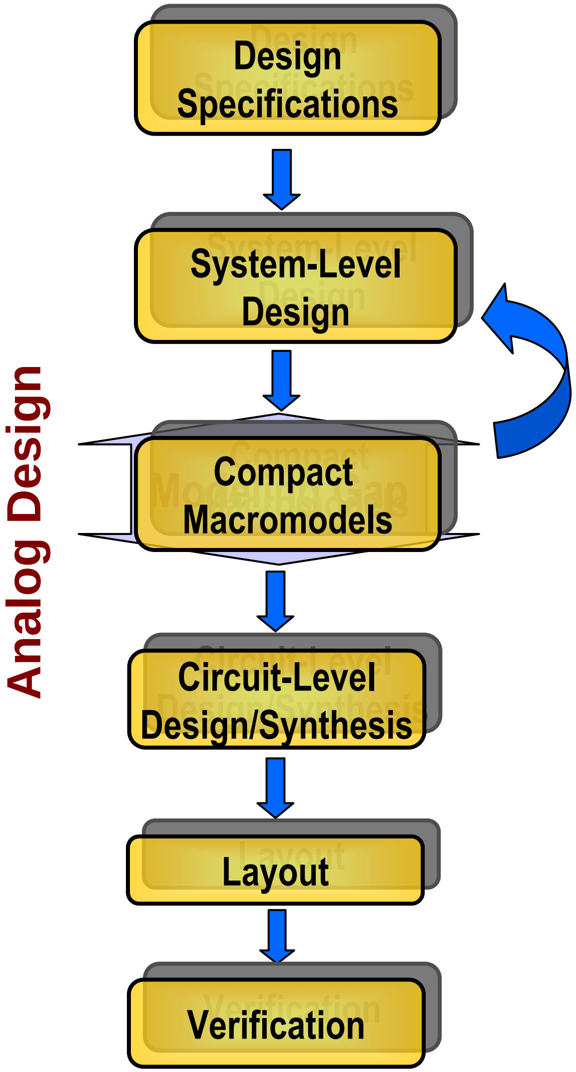
Motivation

- Compact sub-block macromodels are the key to whole-system verification
- Back-annotation of such models facilitates system-level verification
- These "reduced-order" macromodels capture the nonlinear effects
  - IIP3, THD, gain compression,…
  - Dynamic range, spectral regrowth, etc…



Agenda

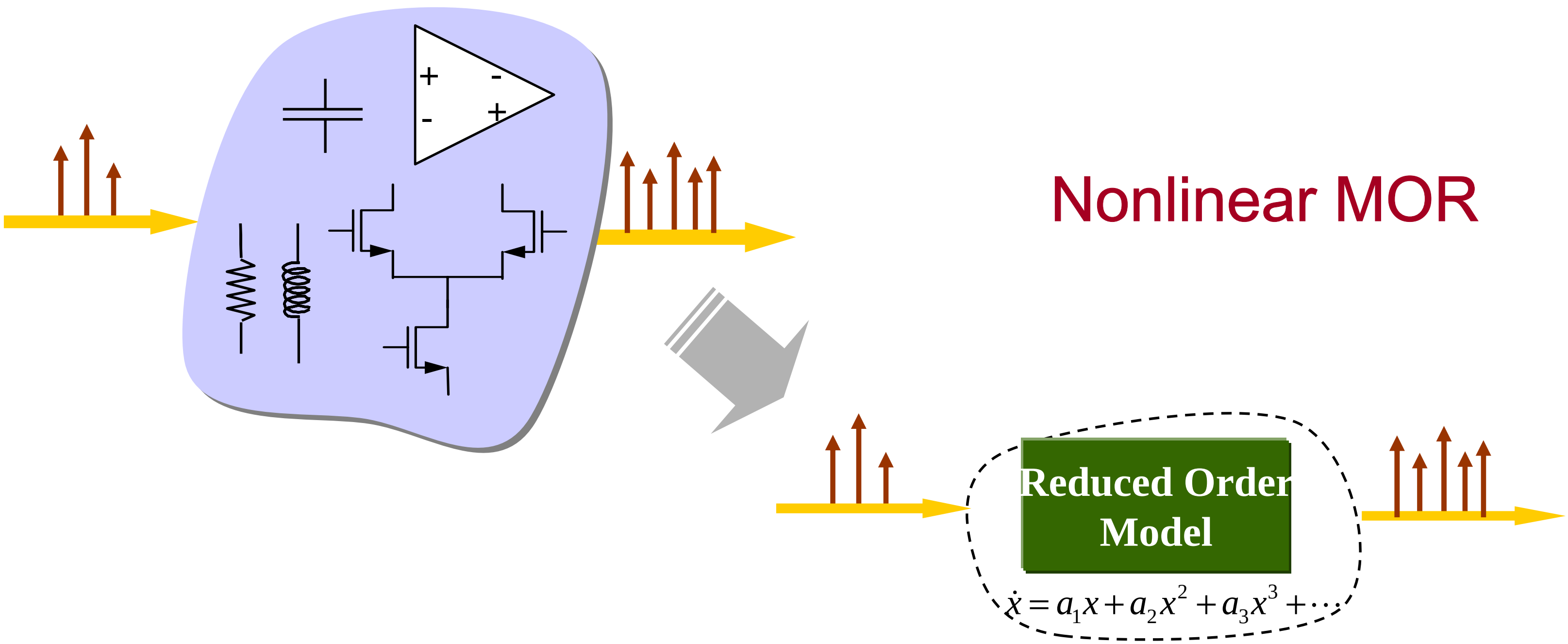
- Introduction
  - ✓ Motivation
  - Previous Work
- Nonlinear macromodeling approach
  - Background
  - Extraction of Volterra Parameters
  - Overall Macromodeling Flow
- Experimental results
- Conclusions

Previous Work

- Reduced order modeling of time-varying systems
  - [Roychowdhury TCAS 1999]
  - [Phillips CICC 2000]
- PWL/PWP and model order reduction
  - [Rewienski, White ICCAD 2001]
  - [Dong, Roychowdhury DAC 2003]
- NORM : compact model order reduction of weakly nonlinear systems
  - [Li, Pileggi DAC 2003]
- Hybrid approach to nonlinear macromodel generation
  - [Li, Xu, Li, Pileggi ICCAD 2003]
- Multivariate formulation
  - [Li, Pileggi BMAS 2003]

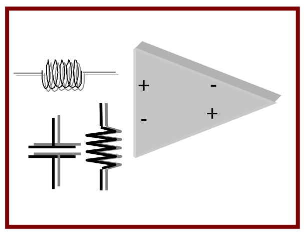
Macromodeling Problem

- **Can we build efficient analog macromodels to capture:** *linear conversion* **+** *time variance* **+** *distortion* **?**



Modeling of Nonlinear Analog Circuits

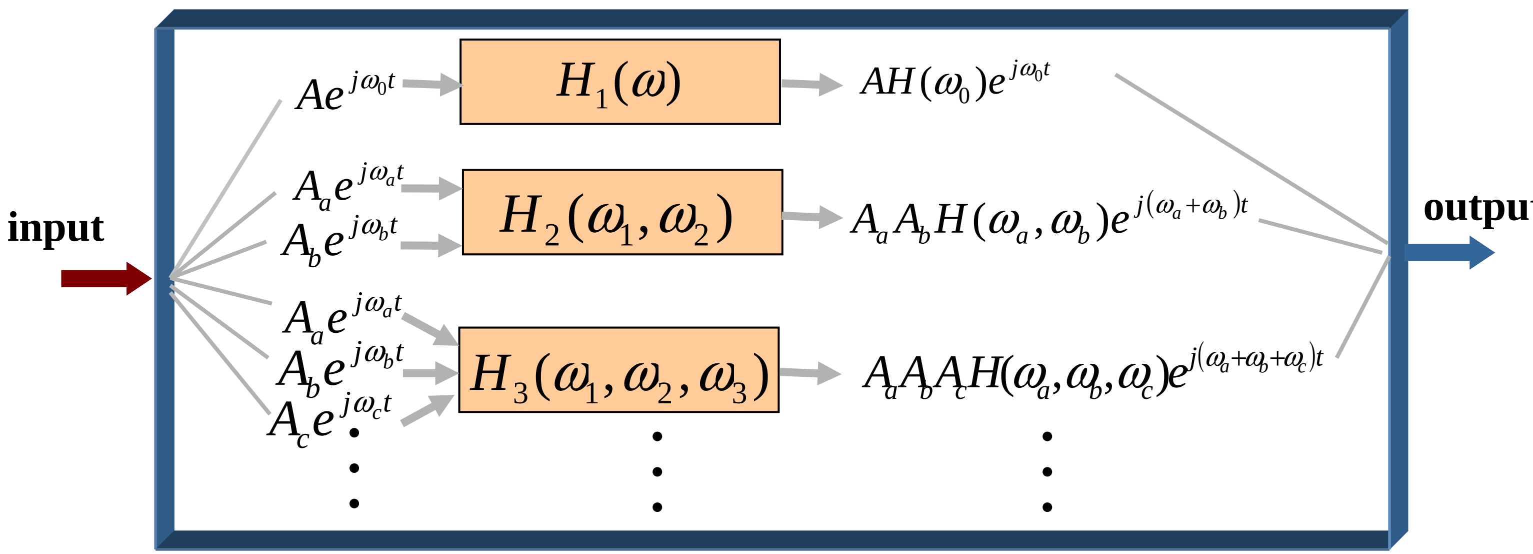
- Volterra Series to describe weakly nonlinear systems



$$x(t) = \sum_{n=1}^{\infty} x_n(t)$$

$$x_n(t) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_n(\tau_1, \ldots, \tau_n) u(t-\tau_1) \cdots u(t-\tau_n) d\tau_1 \cdots d\tau_n$$

$$H_n(f_1, \ldots, f_n) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_n(\tau_1, \ldots, \tau_n) e^{-j2\pi(f_1\tau_1 + \cdots + f_n\tau_n)} d\tau_1 \cdots d\tau_n$$

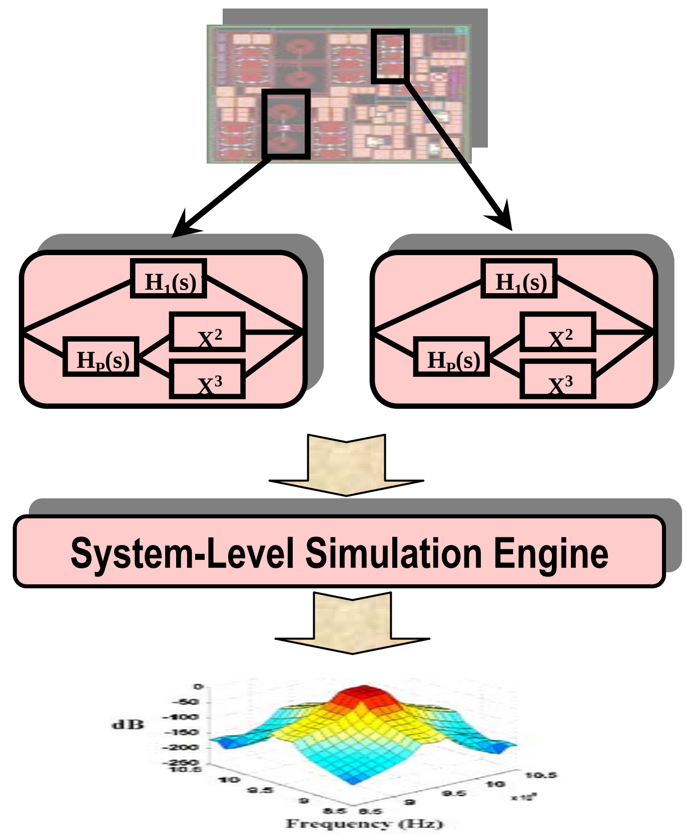


Modeling of Weakly Nonlinear Circuits

- Volterra-based descriptions are based on multi-dimensional transfer functions

- Full-models are extremely complex to be used efficiently

- Reduced-order modeling techniques are required



Nonlinear Transfer Functions

- **Nonlinear dynamic system**

$$f(x(t)) + \frac{d}{dt}(q(x(t))) = bu(t)$$

$$\frac{d}{dt}\left(C_1 x + C_2(x \otimes x) + C_3(x \otimes x \otimes x) + \cdots\right) +$$

$$G_1 x + G_2(x \otimes x) + G_3(x \otimes x \otimes x) + \cdots = bu$$

$$G_i = \frac{1}{i!}\frac{\partial^i}{\partial x^i}(f)\bigg|_{x=x_0} \quad C_i = \frac{1}{i!}\frac{\partial^i}{\partial x^i}(q)\bigg|_{x=x_0}$$

$$x \otimes x = [x_1^2 \;\; x_1 x_2 \;\; .... \; x_1 x_n .... \; x_n x_1 .... x_n^2]^T$$

$$x \otimes x \otimes x = [x_1^3 \;\; x_1^2 x_2 \;\; .... \; x_1 x_n^2 .... x_n x_1^2 .... x_n^3]^T$$

- **1st order**

$$H_1(s) = (SC_1 + G_1)^{-1} b$$

- **2nd order**

$$(s_1 + s_2)C_1 \cdot H_2(s_1, s_2) + G_1 \cdot H_2(s_1, s_2) = -((s_1 + s_2)C_2 + G_2) \cdot \overline{H_1(s_1) \otimes H_1(s_2)}$$

$$\overline{H_1(s_1) \otimes H_1(s_2)} = \frac{1}{2}[H_1(s_1) \otimes H_1(s_2) + H_1(s_2) \otimes H_1(s_1)]$$

Moments of Nonlinear Transfer Functions

- **Moments of linear transfer functions**

$$H(s) = L^T (sC + G)^{-1} B$$

$$H(s) = M_0 + M_1 s + M_2 s^2 + \cdots$$

- **Now for nonlinear transfer functions**

$$H_2(s_1, s_2) = M_{2,0} + \underbrace{\boxed{M_{2,1,1}} s_1 + \boxed{M_{2,1,0}} s_2}_{\text{1st order}} + \underbrace{\boxed{M_{2,2,2}} s_1^2 + \boxed{M_{2,2,1}} s_1 s_2 + \boxed{M_{2,2,0}} s_2^2}_{\text{2nd order}} + \cdots$$

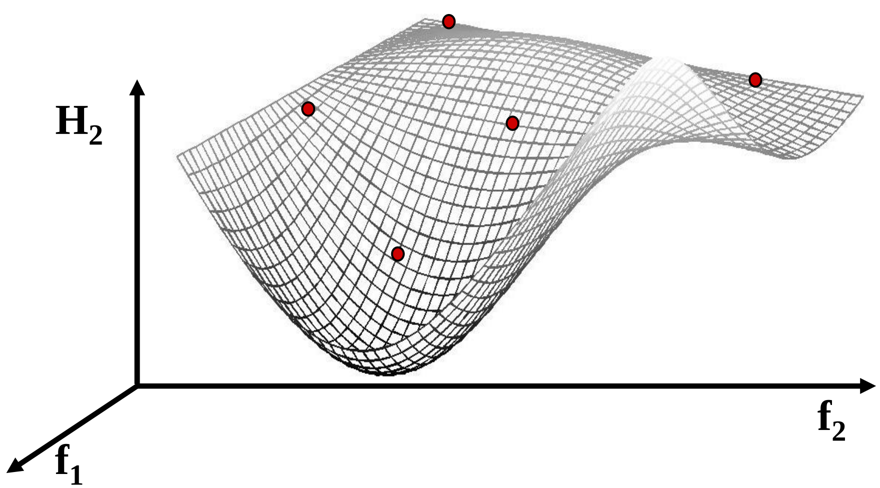
$$H_3(s_1, s_2, s_3) = M_{3,0} + \underbrace{\boxed{M_{3,1,1,0}} s_1 + \boxed{M_{3,1,0,1}} s_2 + \boxed{M_{3,1,0,0}} s_3}_{\text{1st order}} + \underbrace{\boxed{M_{3,2,2,0}} s_1^2 + \boxed{M_{3,2,1,1}} s_1 s_2 + \cdots + \boxed{M_{3,2,0,}}}_{\text{2nd order}}$$

- **Moment matching these nonlinear transfer fcts via projection for MOR**

Reduced Order Modeling

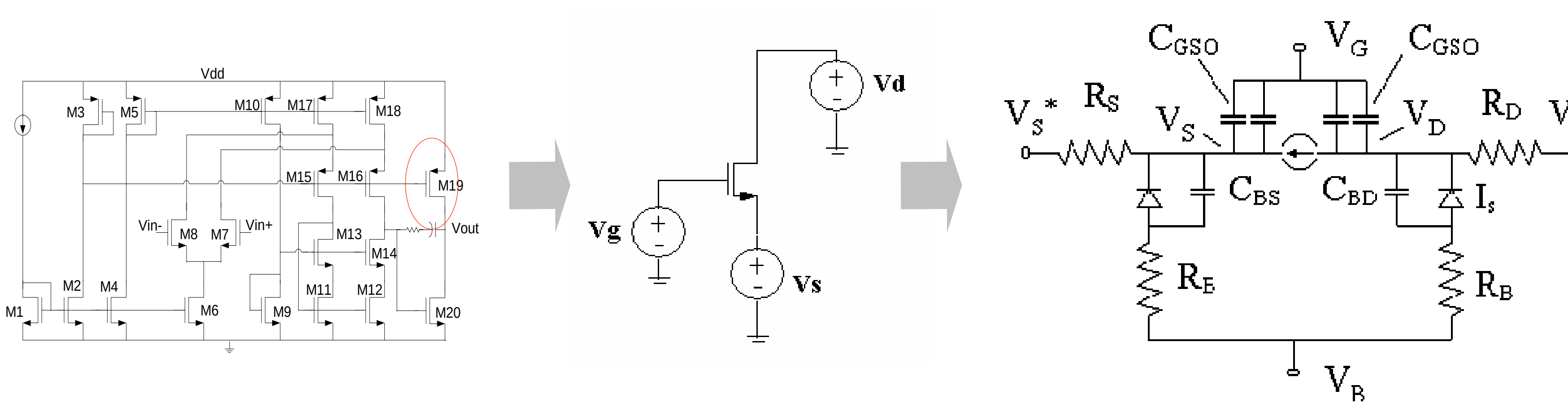
- Using a projection-based reduction – multipoint NORM

$$\tilde{G}_1 = V^T \overline{G}_1 V \quad \tilde{C}_1 = V^T \overline{C}_1 V \quad \tilde{b} = V^T \overline{b} \quad \tilde{G}_2 = V^T \overline{G}_2 (V \otimes V) \quad \tilde{C}_2 = V^T \overline{C}_2 (V \otimes V)$$



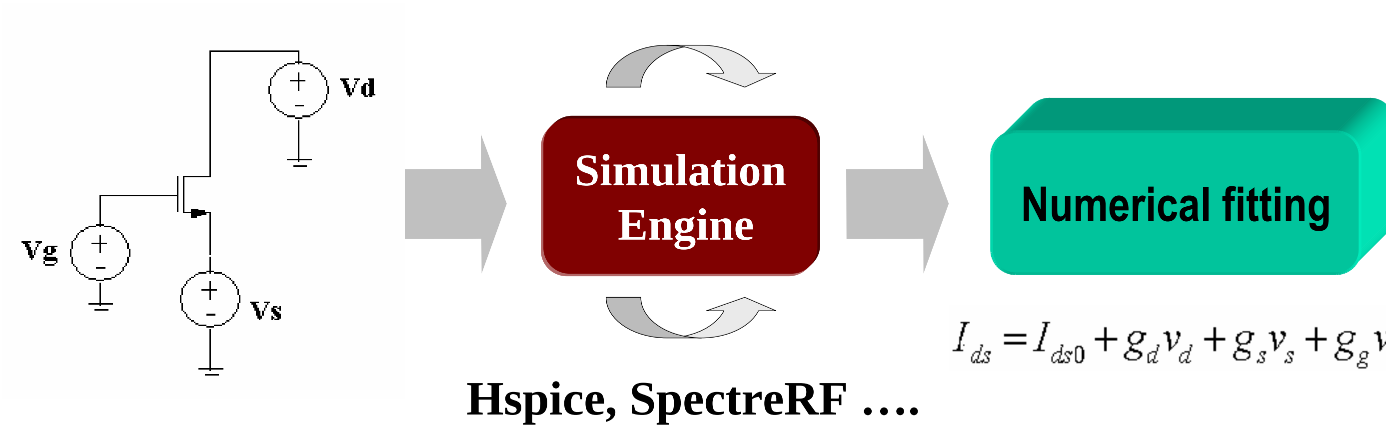
- Fully captures interactions between transfer functions of different orders

Extraction of Volterra Parameters



- **Spice models like BSIM3 include physical effects + numerical parameters which increase model complexity**
- **Infeasible to determine the coefficients by computing higher derivatives of device model equations**

Simulation Setup



$$I_{ds} = I_{ds0} + g_d v_d + g_s v_s + g_g v_g + \ldots..$$

- Commercial simulators like Hspice, SpectreRF can be used to characterize the model parameters for each transistor
- For each transistor, perturb the bias voltages to generate data-points for numerical fitting

Nonlinearities

- Second and third order fitting of drain current using least squares

$$\underbrace{I_{ds}=I_{ds0}}_{\text{dc value}}+\underbrace{g_d v_d+g_s v_s+g_g v_g}_{\text{first order}}+\ldots..+\underbrace{g_{ds}v_d v_s+g_{gd}v_g v_d}_{\text{second order}}+\ldots..\underbrace{g_{ggg}v_g^3}_{\text{third order}}+\ldots$$

- Second and third order fitting of the charge is carried out by fitting the capacitances. For instance,

$$Q_g=f_g v_g+f_d v_d+f_s v_s+f_{gd}v_g v_d+f_{dd}v_d^2+\ldots.+f_{ggd}v_g^2 v_d+f_{gdd}v_g v_d^2+\ldots.$$

- Differentiating w.r.t. drain

$$C_{gd}=\frac{dQ_g}{dV_d}=C_{gd0}+f_{gd}v_g+2f_{dd}v_d+f_{ds}v_s+f_{ggd}v_g^2+2f_{gdd}v_g v_d+\ldots...$$

- In order to fit Qg, need to fit Cgd, Cgs and Cgg

Least-squares fitting

- Y matrix contains the voltage powers and cross terms
- p contains the corresponding coefficients
- R contains the residue (Ids –Ids0)

$$Y = \begin{bmatrix} v_{d1} & v_{s1} & v_{g1} & v_{d1}^2 & v_{d1}v_{g1} & \ldots\ldots\ldots v_{s1}^3 \\ v_{d2} & v_{s2} & v_{g2} & v_{d2}^2 & v_{d2}v_{g2} & \ldots\ldots\ldots v_{s2}^3 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ v_{dn} & v_{sn} & v_{gn} & v_{dn}^2 & v_{dn}v_{gn} & \ldots\ldots\ldots v_{sn}^3 \end{bmatrix},$$

$$p = [g_d \quad g_s \quad g_g \quad g_{dg} \ \ldots\ldots\ldots\ldots g_{sss}]$$

$$R = [I_1 \ \ I_2 \ldots\ldots I_n]^T \qquad I_n = I_{dsn} - I_{ds0}$$

Least-squares fitting

- Minimize the error for each data-point, in matrix form:

$$e = [\varepsilon_1 \, \varepsilon_2 \, ...\varepsilon_n]^T$$

$$Yp - R = e$$

- The least-mean-square algorithm estimates the coefficients in p by minimizing the sum of squares errors:

$$F = e^T e = (Yp - R)^T (Yp - R)$$

$$p = (Y^T Y)^{-1}.(Y^T R)$$

Improving the fit ...

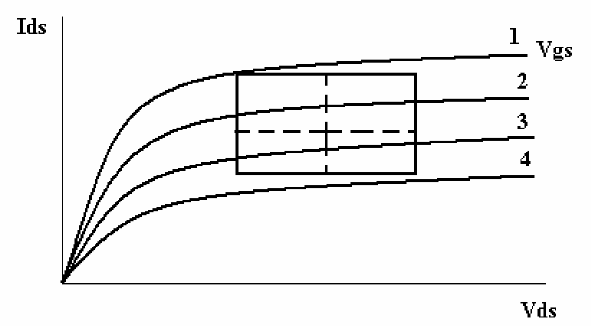
- Weighted-least squares approach

$$w_i(I_{dsi} - I_{ds0}) = g_d v_{di} w_i + g_s v_{si} w_i + \ldots + g_{ds} v_{di} v_{si} w_i + \ldots.$$

$$F = (Yp - R)^T W (Yp - R)$$

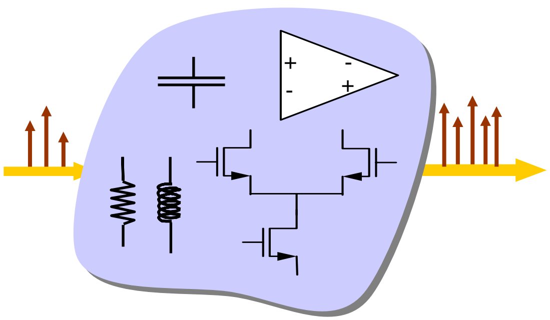
$$p = (Y^T W Y)^{-1} . (Y^T W R)$$

- Fitting range
  - Large enough to encompass nonlinearities
  - Should not cover effects outside signal swing range*



* [“Distortion in RF power amplifiers”, Vuolevi and Rahkonen, Artech House, 2003]

Overall Macromodeling Flow



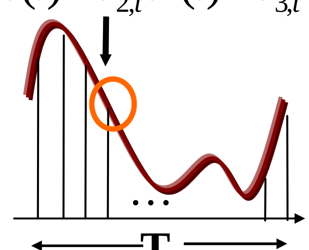
Nonlinear analog, RF,
MEMS circuit netlist

LO/clock

Simulation Engine

Hspice, SpectreRF ….

$$i(t)=a_{1,t}v(t)+a_{2,t}v^2(t)+a_{3,t}v^3(t)$$



Periodic Time-varying Op

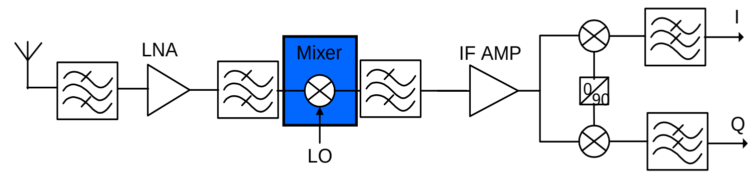
Nonlinear MOR

$$\frac{d}{dt}\left(q(\hat{C}_1,\hat{C}_2,\hat{C}_3,\hat{x})\right) = f(\hat{C}_1,\hat{C}_2,\hat{C}_3,\hat{x},\hat{b},u)$$

$$y=[\ldots,e^0,e^{j2\pi f_0 t},\ldots]\cdot\Gamma\cdot\hat{d}^T\cdot\hat{x}$$

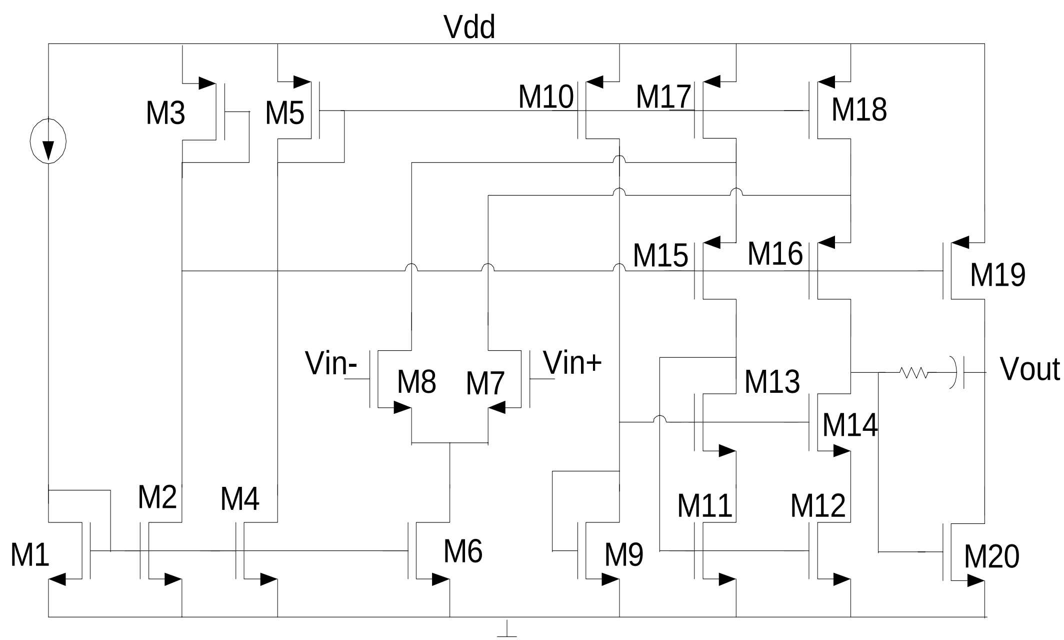
Reduced-order Model

Back-annotation
of models



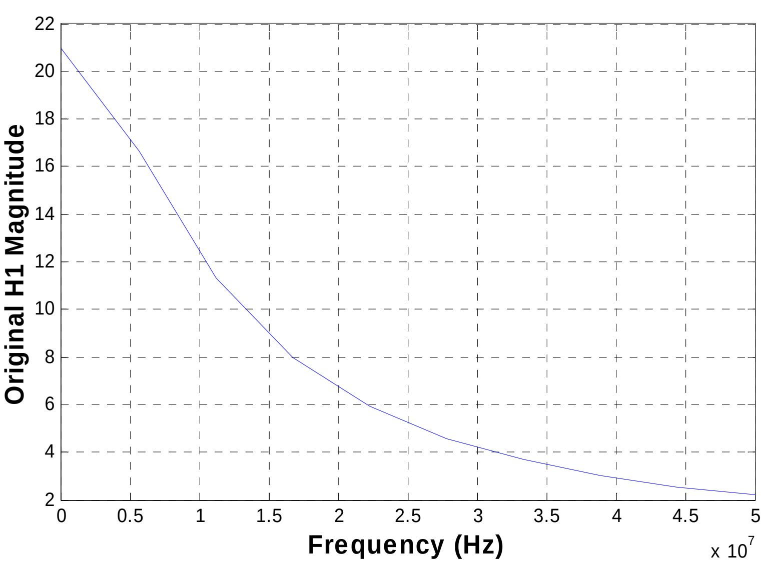
An Opamp

- Modeled as a time-invariant system, linearized at the DC bias point to fit second and third-order coefficients for each transistor
- Second-order nonlinearities are much higher than third-order nonlinearities for single-ended output



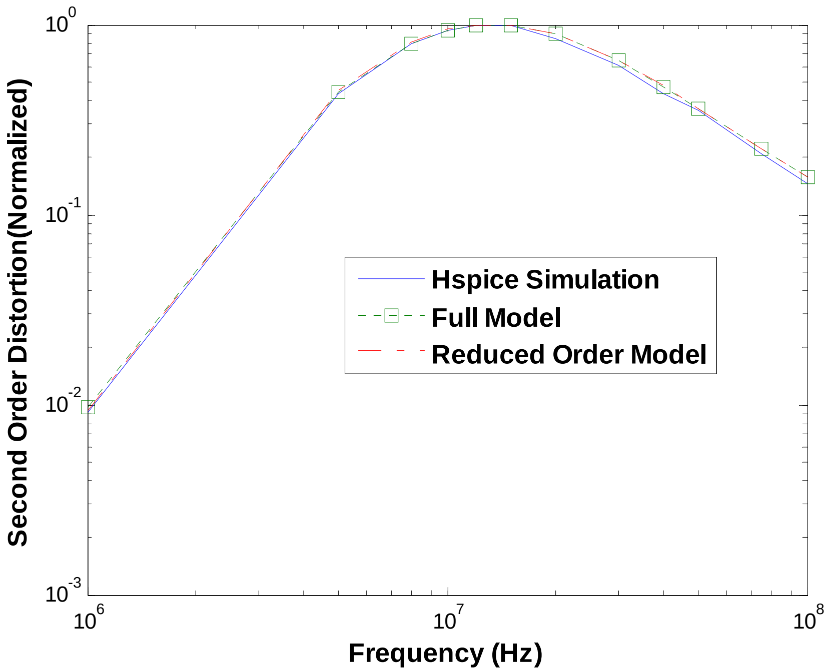
An Opamp

- Perform transient analysis in Hspice followed by fourier transform to compare with model results
- First-order (small-signal) results
  - Max error between full model generated using small-signal parameters and reduced-order model is 0.07%



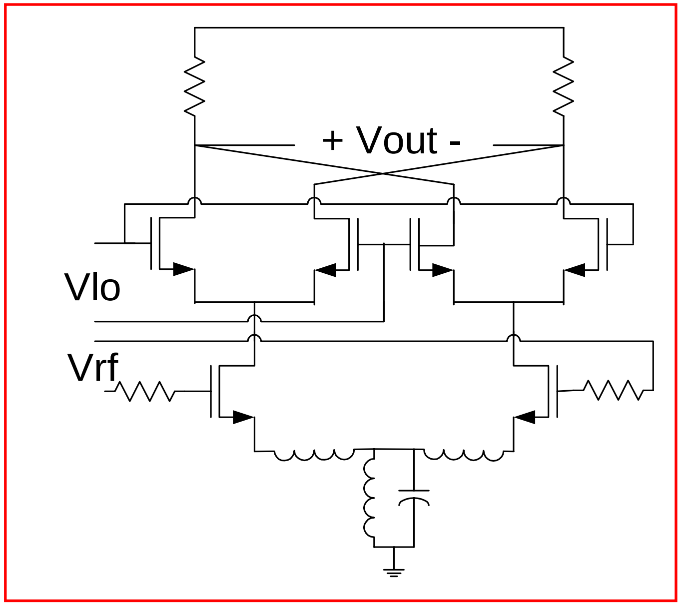
An Opamp

- Second-order distortion as a function of frequency
  - Max error between transient simulation and full model : 5.1%
  - Max error between full model and reduced model : 2.9%
  - Max error between transient simulation and reduced model : 5.3%

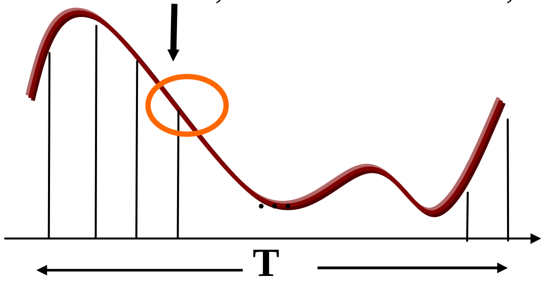


A Double-Balanced Mixer

- Characterized using time-varying Volterra series w.r.t. RF input based on 1350 time-sampled circuit variables
- Each nonlinearity is modeled as a third-order polynomial about the time varying operating point due to large-signal LO



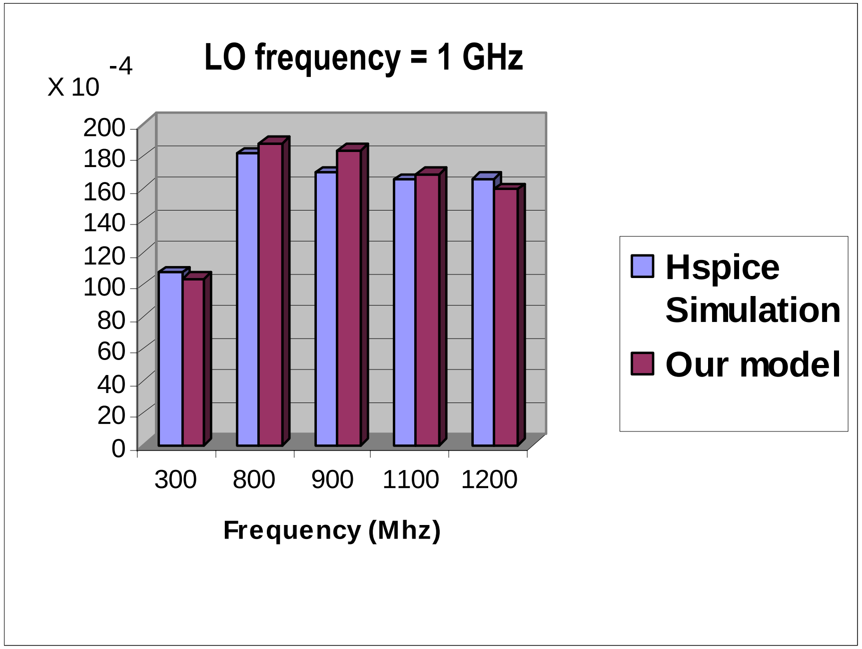
$$i(t) = a_{1,t} v(t) + a_{2,t} v^2(t) + a_{3,t} v^3(t)$$



Periodic Time-varying Op

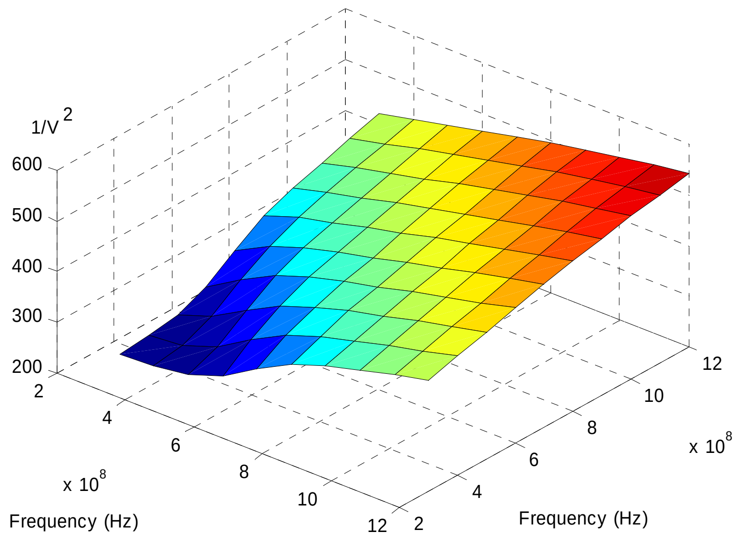
A Double-Balanced Mixer

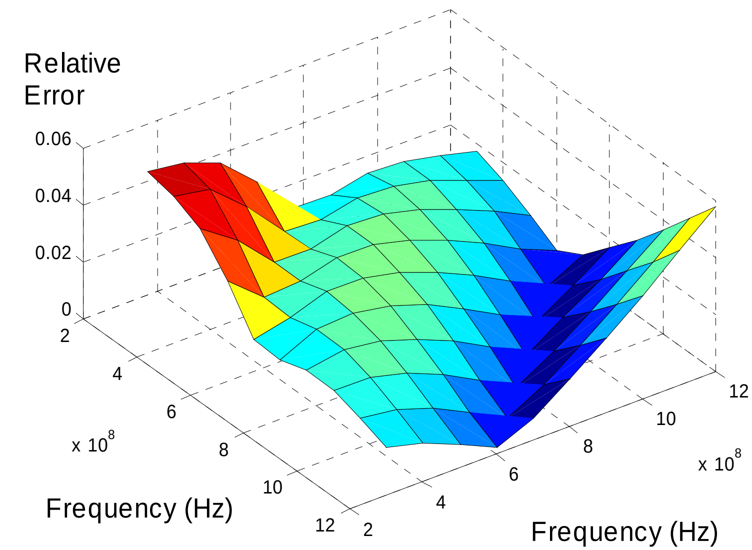
- Third-order results
  - Single tone RF input frequency varied from 300MHz to 1200MHz
  - Third-order harmonic of the RF input down-converted w.r.t LO
  - Max error between transient simulation and full model : 8%



A Double-Balanced Mixer

- Reduced order model
  - 14 circuit variables as compared to 1350 in the full model
  - Third-order transfer function: 300Mhz ≤ f_1, f_2 ≤ 1.2Ghz and f_{LO} = 1Ghz



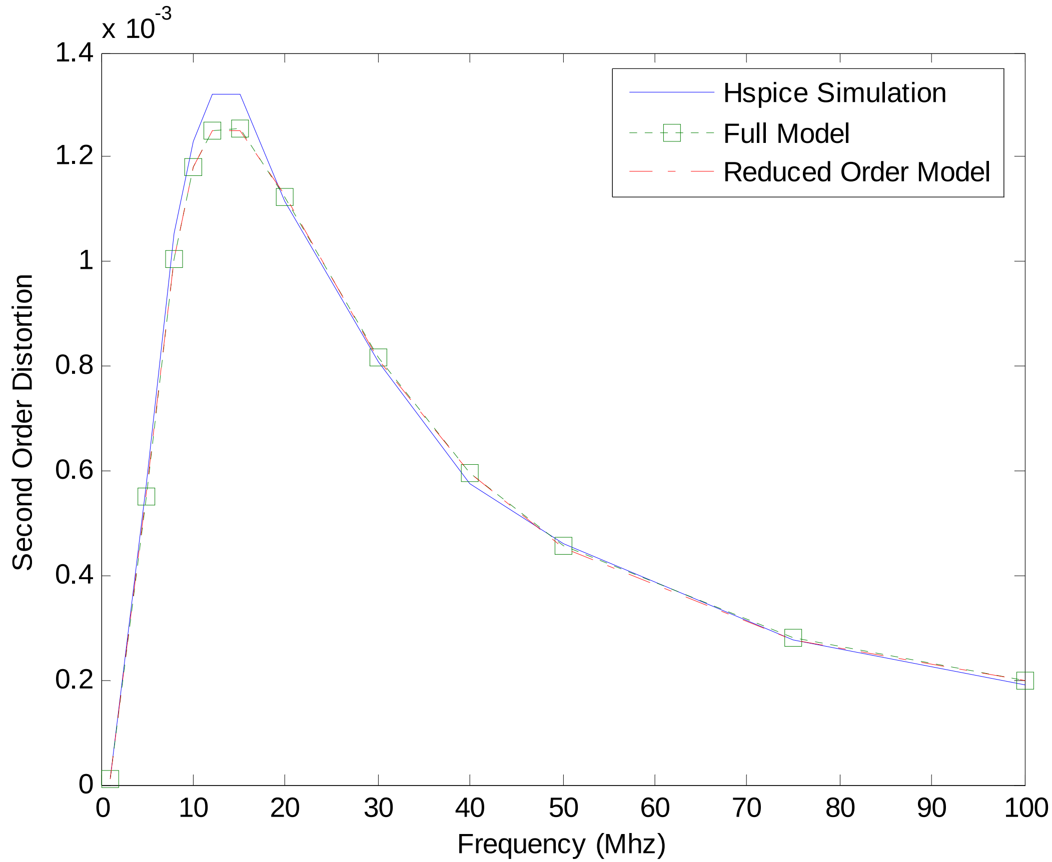


Conclusions

- Reduced-order models for weakly nonlinear analog circuits can be generated from transistor-level netlists
- The accuracy is comparable to transistor-level simulation using commercial simulators
- Explore the adoption of these compact reduced-order models in behavioral languages like Verilog-A

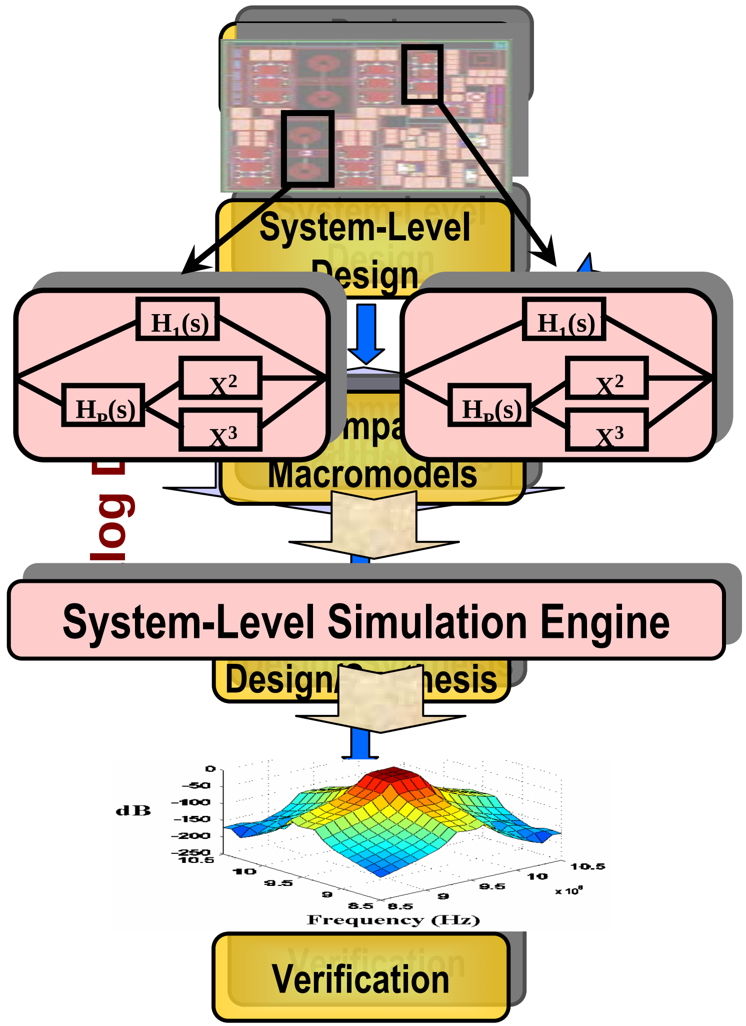
Backup Slides

Opamp Results



Compact Nonlinear Macromodels

- The purpose of developing compact macromodels is two-fold
  - A library of reduced-order models can facilitate system-level design exploration by "re-use"
  - Verification of the complete system based on these compact macromodels



Overall Macromodeling Flow