A General Method for Multi-Port Active Network Reduction and Realization

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Outline

- Compact modeling via model order reduction
- General hierarchical model order reduction
- Optimization considering magnitude and phase responses
- Experimental results
- Conclusions

Compact Modeling

- More parasitics for nanometer VLSIs
  - millions and hundreds of millions of RLC elements
  - Overload simulation engines
  - not all information is useful
- Compact modeling via model order reduction
  - Capture port behavior of the circuit block to speed up simulation and synthesis
  - Discard no useful information
  - Black box approach

Existing MOR Algorithms

- Projection-based
  - AWE, PVL, Matrix PVL, Arnoldi Model, PRIMA
  - Difficulty to deal with mutual inductors (M elements) and their equivalent models (with controlled sources)
- Node reduction and rational approximation
  - TICER, DTT, Circuit Crunching, Y-Delta, HMOR
  - Essentially symbolic Gaussian elimination
  - Advantage:
    - applicable to both passive and active circuits
    - No need to solve the whole circuits

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- Model order reduction
- General hierarchical model order reduction
  - Hierarchical: block Gaussian elimination
  - General: any linear circuit with any linear devices
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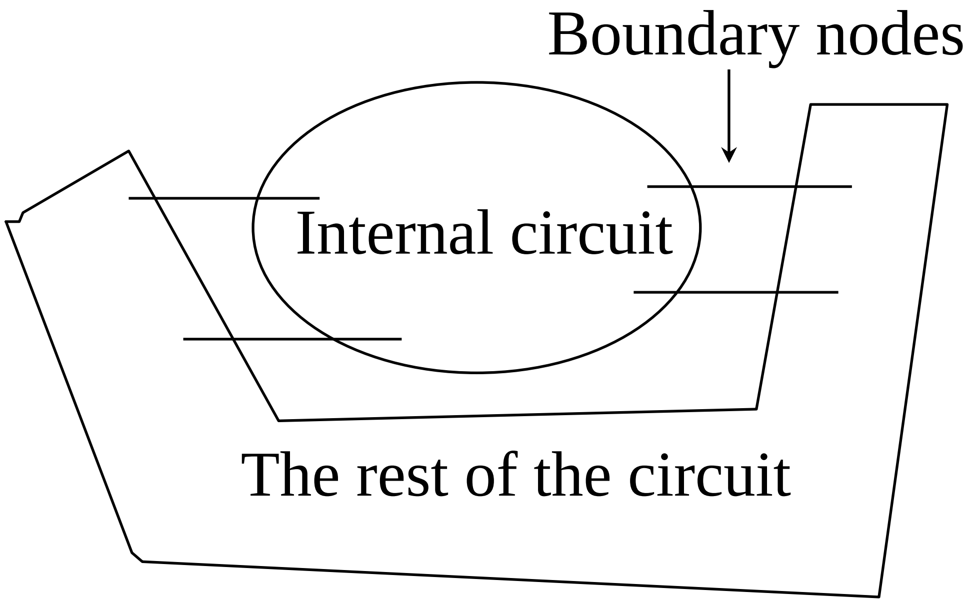
Partitioned Circuit Matrix



Two-way partition of a circuit

$$\begin{bmatrix} M^{II} & M^{IB} & \\ M^{BI} & M^{BB} & M^{BR} \\ & M^{RB} & M^{RR} \end{bmatrix} \begin{bmatrix} x^{I} \\ x^{B} \\ x^{R} \end{bmatrix} = \begin{bmatrix} b^{I} \\ b^{B} \\ b^{R} \end{bmatrix}$$

Partitioned circuit equations

Block Gaussian Elimination

- The suppressed circuit matrix becomes:

$$\begin{bmatrix} M^{BB*} & M^{BR} \\ M^{RB} & M^{RR} \end{bmatrix} \begin{bmatrix} x^B \\ x^R \end{bmatrix} = \begin{bmatrix} b^{B*} \\ b^R \end{bmatrix}$$

where

$$M^{BB*} = M^{BB} - M^{BI}(M^{II})^{-1} M^{IB}$$

$$b^{B*} = b^B - M^{BI}(M^{II})^{-1} b^I$$

or

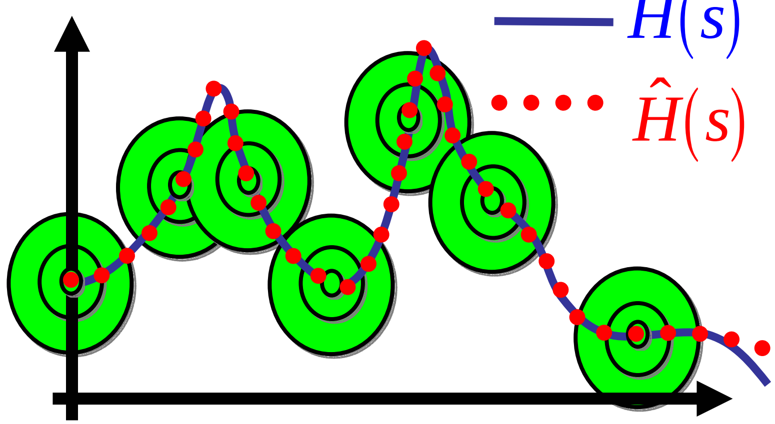
$$a_{uv}^{BB*} = a_{uv}^{B} - \frac{1}{\det(A^{II})} \sum_{k_1,k_2=1}^{m} a_{uk_1}^{BI} \Delta_{k_2k_1}^{II} a_{k_2v}^{IB}, \qquad u,v = 1,\ldots, l$$

$$b_{u}^{B*} = b_{u}^{B} - \frac{1}{\det(A^{II})} \sum_{k_1,k_2=1}^{m} a_{uk_1}^{BI} \Delta_{k_2k_1}^{II} b_{k_2}^{IB}, \qquad u,v = 1,\ldots, l$$

Hierarchical S-Domain Reduction (HMOR) (ICCAD'03)

- Basic idea
  - ✓ Compute s-polynomials for determinants and cofactors via hierarchical symbolic analysis techniques
  - ✓ Keep the exact or only limited order for each poly

- Multi-point frequency expansion for wideband accuracy



Properties of HMOR

- Theorem: HMOR method dose not change the reciprocal property of a linear system.
  - A reciprocal network is one in which the power losses are the same between any two ports regardless of direction of propagation.
  - If a system is reciprocal, the reduced system by HMOR is reciprocal.
  - If a system is not reciprocal, the reduced system by HMOR is not reciprocal.

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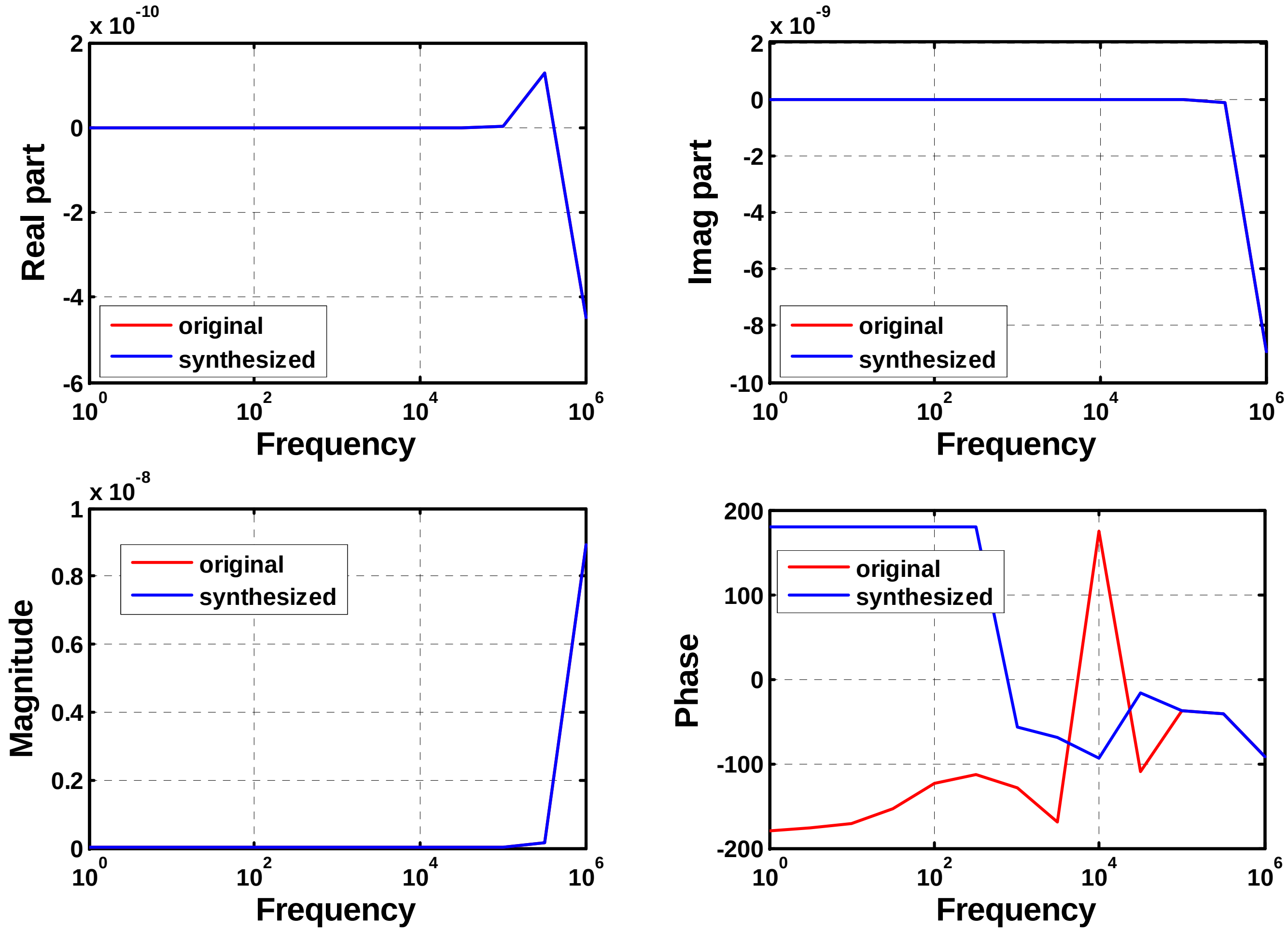
- General hierarchical model order reduction
  - Hierarchical: block Gaussian elimination
  - General: any linear passive or active circuits
- Optimization considering magnitude and phase responses
  - Motivation for considering both magnitude and phase responses
  - Constrained least square based optimization
  - Multi-port non-reciprocal circuit realization
- Experimental results
- Conclusions

Motivation


x 10^-10
Real part
Frequency
original
synthesized
x 10^-9
Imag part
Frequency
original
synthesized
x 10^-8
Magnitude
Frequency
original
synthesized
Phase
Frequency
original
synthesized

Optimization Problem

- Admittance matrix of the reduced order model

$$\hat{\mathbf{Y}}(s) = \begin{bmatrix} \hat{Y}_{1,1}(s) & \cdots & \hat{Y}_{1,n}(s) \\ \vdots & \ddots & \vdots \\ \hat{Y}_{n,1}(s) & \cdots & \hat{Y}_{n,n}(s) \end{bmatrix}$$

- Optimization problem (only considering magnitude)

$$min(\sum_{k=1}^{T} ||\hat{Y}_{p,q}(s_k) - \tilde{Y}_{p,q}(s_k)||_2^2)$$

Where $\tilde{Y}_{p,q}(s_k)$ is the exact values of the admittance at the entry (*p*,*q*) at the *k*-th frequency point

Optimization Problem (cont'd)

- Basic Idea:

$$\hat{Y}(s) = s\hat{Y}_\infty + \hat{Y}_0 + \sum_{m=1}^{M} \frac{rr_m}{s - pr_m} + \sum_{n=1}^{N} \left(\frac{rc_n}{s - pc_n} + \frac{rc_n^*}{s - pc_n^*} \right)$$

  - Find a set of residues such that the errors are mixed in terms of both magnitude and phases.

Optimization Problem

Some definitions

$$x = [\ x_1^r \quad \cdots \quad x_M^r \quad x_1^c \quad \cdots \quad x_{2N}^c \quad Y_0 \quad Y_\infty\ ]^T$$

$$A_k = [\ a_1^r(s_k) \quad \cdots \quad a_M^r(s_k) \quad a_1^c(s_k) \quad \cdots \quad a_{2N}^c(s_k) \quad 1 \quad s_k\ ]$$

Where $$a_m^r(s_k) = \frac{1}{s_k - pr_m}$$

$$a_n^c = \frac{1}{s_k - pc_n} + \frac{1}{s_k - pc_n^*}, a_{n+1}^c = \frac{j}{s_k - pc_n} - \frac{j}{s_k - pc_n^*}$$

Minimization for Real and Imaginary Part

Define

$$A_{lin} = \begin{bmatrix} re(A) \\ im(A) \end{bmatrix}, Y_{lin} = \begin{bmatrix} re(\tilde{Y}) \\ im(\tilde{Y}) \end{bmatrix}$$

Then $$min(\|A_{lin}x - Y_{lin}\|_2^2)$$

Constraints for Sign of Phase

define

$$Y_D = diag(Y_{lin})$$

$$D_{lin} = Y_D A_{lin}$$

Then $$D_{lin} x \geq 0$$

Constraints for Value of Phase

- Define

$$Y_I = diag\left(\begin{bmatrix} im(\tilde{Y}) \\ re(\tilde{Y}) \end{bmatrix}\right)$$

$$I_{lin} = \begin{bmatrix} 1 & \cdots & 0 & -1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 & \cdots & -1 \end{bmatrix}$$

$$C_{lin} = I_{lin} Y_I A_{lin}$$

Then $C_{lin} x = 0$

Optimization Problem

- The constrained linear least square optimization problem considering both magnitude and phase

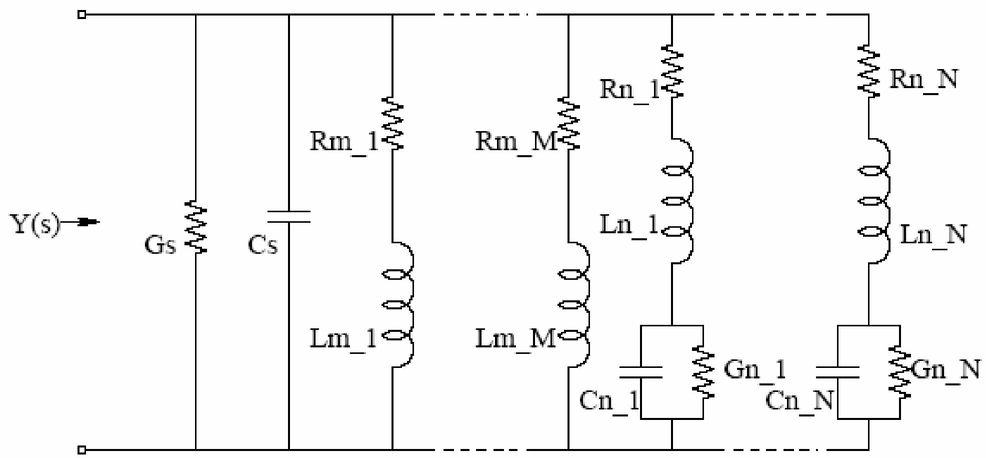
$$min(\|A_{lin}x - Y_{lin}\|_2^2) \quad \text{subject to} \quad \begin{array}{c} D_{lin}x \geq 0 \\ C_{lin}x = 0 \end{array}$$

Realization (one-port)

- Foster's canonical form for 1x1 admittance Y(s)

$$Y(s) = sY_\infty + Y_0 + \sum_{m=1}^{M} \frac{x_m^r}{s - pr_m} + \sum_{n=1}^{N} \left(\frac{x_n^c + x_{n+1}^c j}{s - pc_n} + \frac{x_n^c - x_{n+1}^c j}{s - pc_n^*} \right)$$

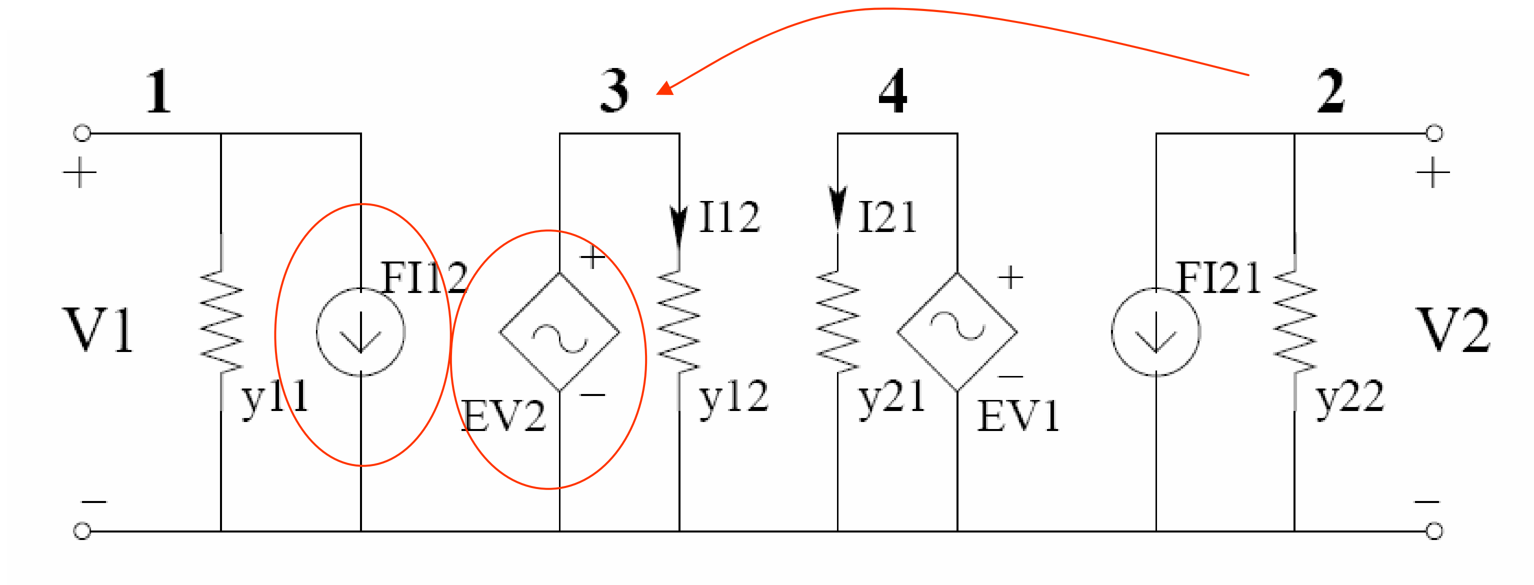
- Realization of Foster's canonical form



Multi-port non-reciprocal Realization

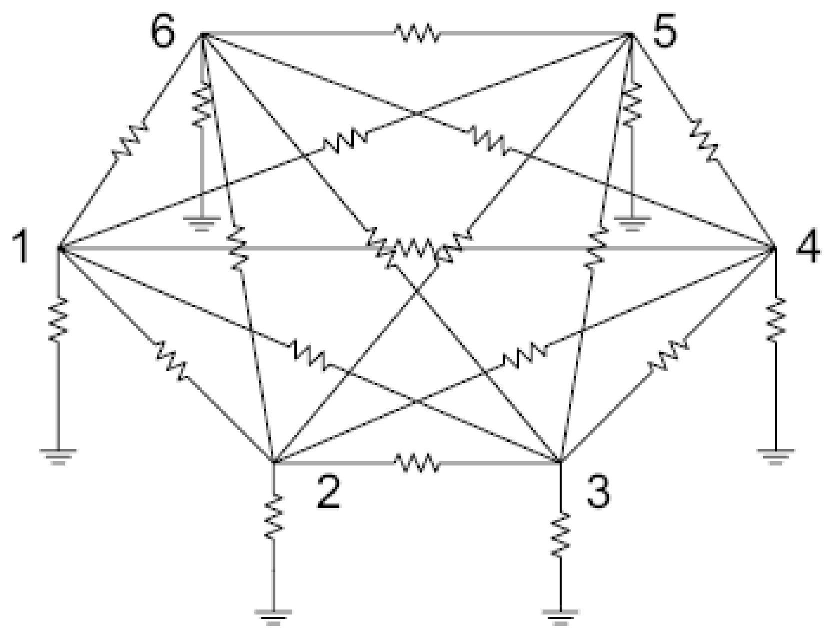
- 2 port

$$Y_{2\times 2}(s) = \begin{bmatrix} y_{11}(s) & y_{12}(s) \\ y_{21}(s) & y_{22}(s) \end{bmatrix},$$



General Multi-Port Network Realization

- For general *n x n* network



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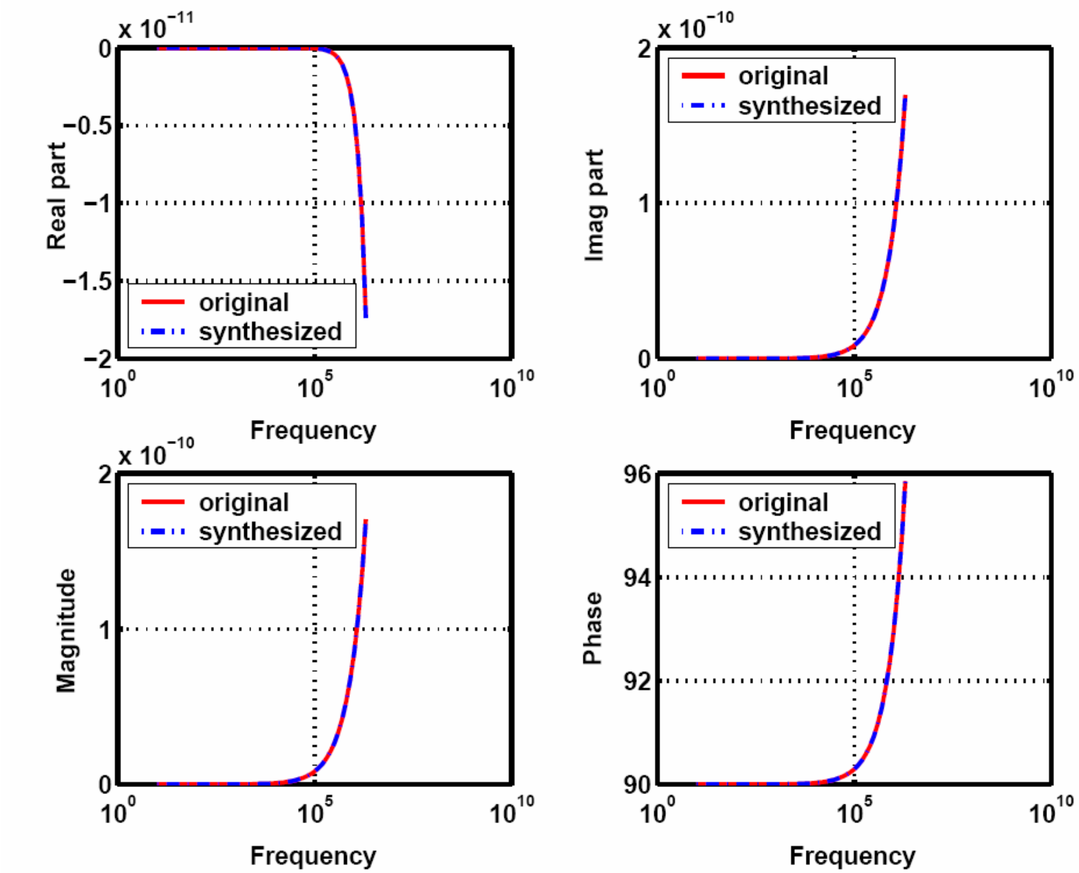
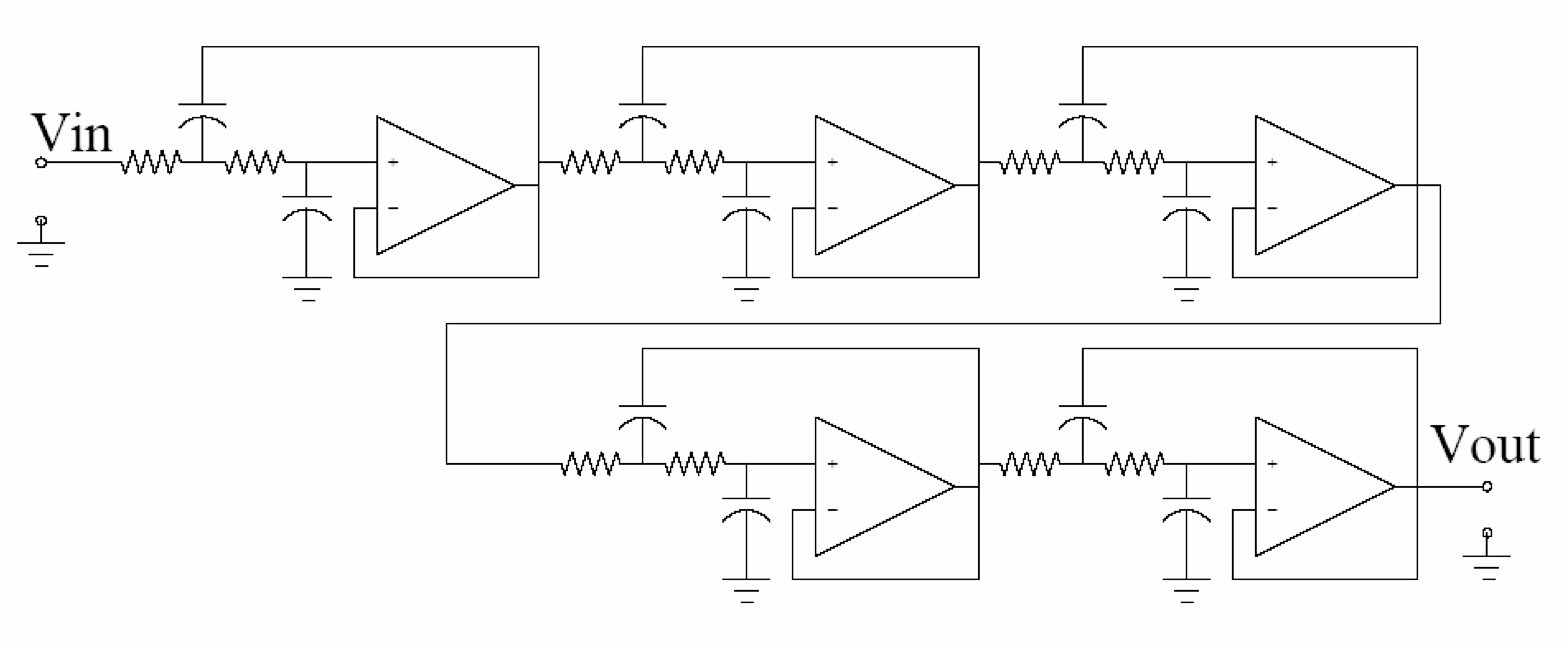
- Y_{12} frequency responses of the original circuit and the reduced model

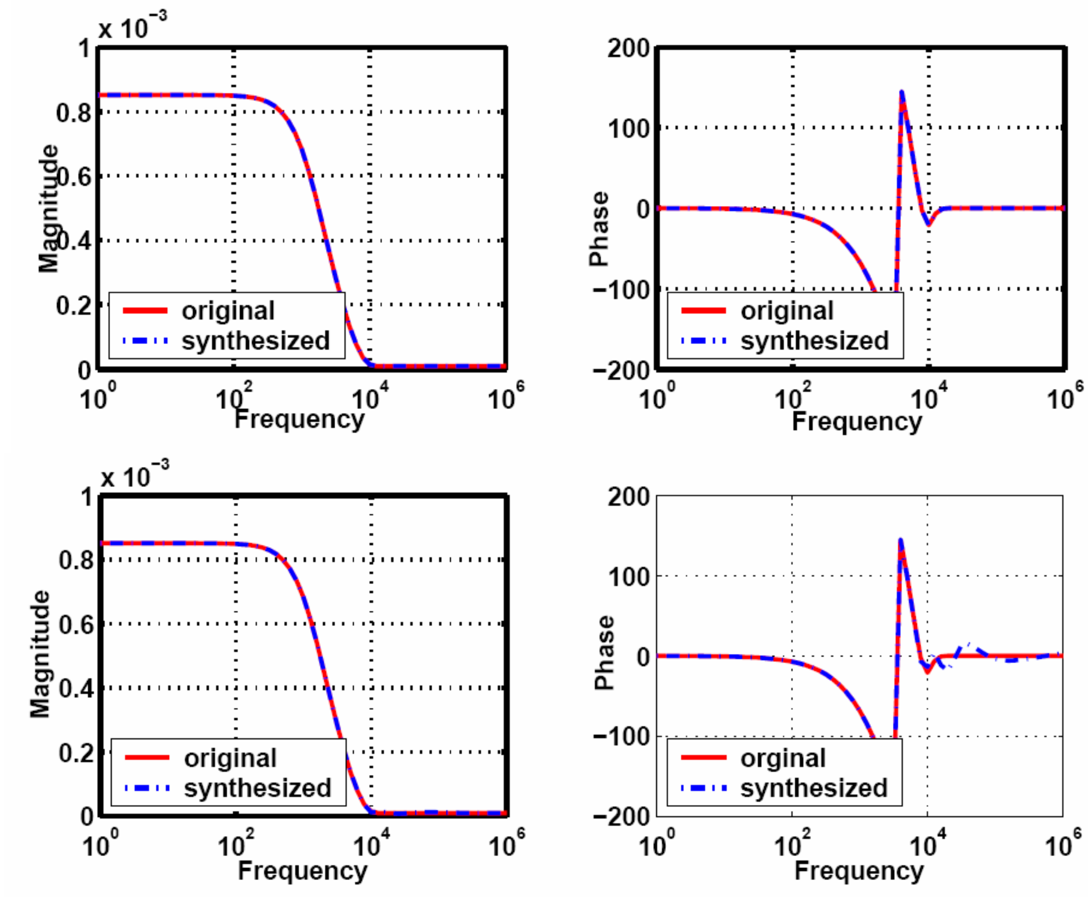


An active Sallen-Key low-pass filter



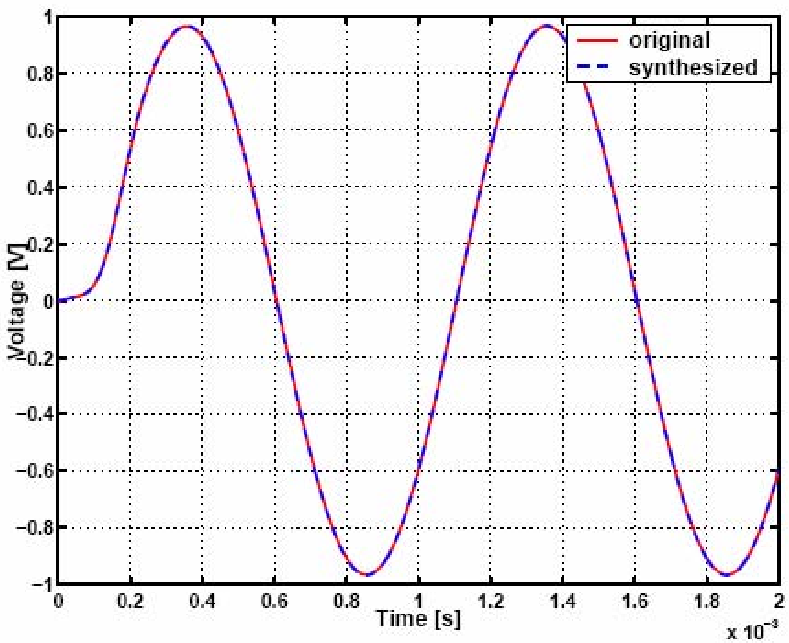
An active Sallen-Key low-pass filter

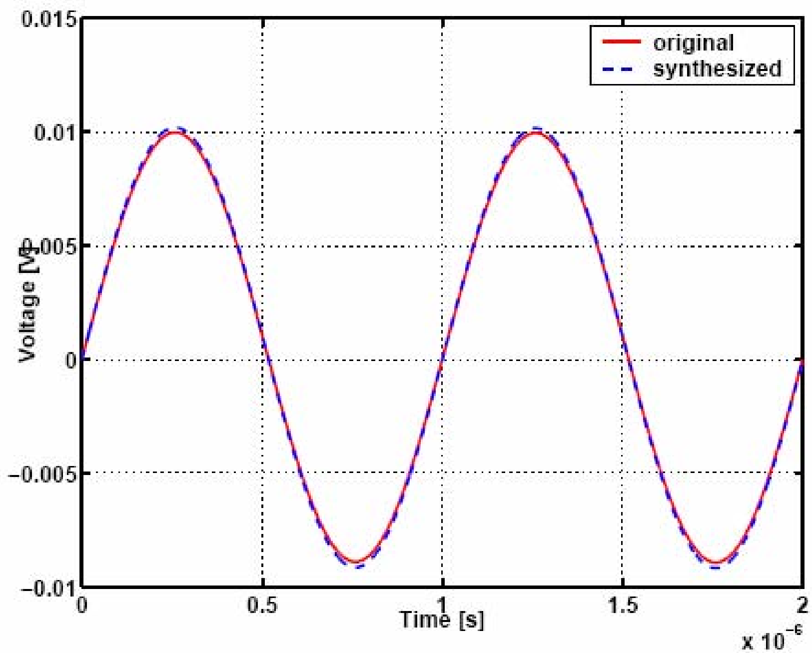
- Y_{12} Frequency-domain pulse responses
  - Considering phase
  - Without Considering phase



An active Sallen-Key low-pass filter

- Time-domain responses excited by different sources





An active Sallen-Key low-pass filter

- Reduction efficiency:
  - Original model: 636 elements
  - Realized reduced order model: 88 RLC, 2 VCVS, 2CCCS elements
  - Reduction ratio: 85.5%.

Realized circuits have very regular structure, which can be utilized for further reduction or acceleration in time-domain simulation

Conclusions

- A general multi-point hierarchical model order reduction and realization flow for active circuits
- Constrained linear least square optimization technique considering both magnitude and phase responses for any linear active network
- Multi-port non-reciprocal circuit realization
- SPICE-in SPICE-out reduction/realization for maximum model portability and flexibility