A General Method for Multi-Port Active Network Reduction and Realization

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Outline

- Compact modeling via model order reduction
- General hierarchical model order reduction
- Optimization considering magnitude and phase responses
- Experimental results
- Conclusions
Compact Modeling

- More parasitics for nanometer VLSIs
  - millions and hundreds of millions of RLC elements
  - Overload simulation engines
  - not all information is useful

- Compact modeling via model order reduction
  - Capture port behavior of the circuit block to speed up simulation and synthesis
  - Discard no useful information
  - Black box approach
Existing MOR Algorithms

- **Projection-based**
  - AWE, PVL, Matrix PVL, Arnoldi Model, PRIMA
  - Difficulty to deal with mutual inductors (M elements) and their equivalent models (with controlled sources)

- **Node reduction and rational approximation**
  - TICER, DTT, Circuit Crunching, Y-Delta, HMOR
  - Essentially symbolic Gaussian elimination
  - Advantage:
    - applicable to both passive and active circuits
    - No need to solve the whole circuits
Outline

- Model order reduction
- General hierarchical model order reduction
  - Hierarchical: block Gaussian elimination
  - General: any linear circuit with any linear devices
- Optimization considering magnitude and phase responses
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Partitioned Circuit Matrix

\[
\begin{bmatrix}
M^{II} & M^{IB} \\
M^{BI} & M^{BB} & M^{BR} \\
M^{RB} & M^{RR}
\end{bmatrix}
\begin{bmatrix}
x^I \\
x^B \\
x^R
\end{bmatrix}
= 
\begin{bmatrix}
b^I \\
b^B \\
b^R
\end{bmatrix}
\]

Boundary nodes

Internal circuit

The rest of the circuit

Two-way partition of a circuit

Partitioned circuit equations
Block Gaussian Elimination

- The suppressed circuit matrix becomes:

\[
\begin{bmatrix}
M^{BB^*} & M^{BR} \\
M^{RB} & M^{RR}
\end{bmatrix}
\begin{bmatrix}
x^B \\
x^R
\end{bmatrix}
= \begin{bmatrix}
b^{B^*} \\
b^R
\end{bmatrix}
\]

where

\[
M^{BB^*} = M^{BB} - M^{BI}(M^{II})^{-1}M^{IB}
\]

\[
b^{B^*} = b^B - M^{BI}(M^{II})^{-1}b^I
\]

or

\[
a^{BB^*}_{uv} = a^B_{uv} - \frac{1}{\det(A^{II})} \sum_{k_1,k_2=1}^m a^{BI}_{uk_1} \Delta^{II}_{k_2k_1} a^{IB}_{k_2v}, \quad u, v = 1, \ldots, l
\]

\[
b^{B^*}_{u} = b^B_{u} - \frac{1}{\det(A^{II})} \sum_{k_1,k_2=1}^m a^{BI}_{uk_1} \Delta^{II}_{k_2k_1} b^{IB}_{k_2}, \quad u, v = 1, \ldots, l
\]
Hierarchical S-Domain Reduction (HMOR) (ICCAD’03)

- Basic idea
  - Compute $s$-polynomials for determinants and cofactors via hierarchical symbolic analysis techniques
  - Keep the exact or only limited order for each poly

- Multi-point frequency expansion for wideband accuracy
Properties of HMOR

- **Theorem:** HMOR method does not change the reciprocal property of a linear system.

- A reciprocal network is one in which the power losses are the same between any two ports regardless of direction of propagation.
- If a system is reciprocal, the reduced system by HMOR is reciprocal.
- If a system is not reciprocal, the reduced system by HMOR is not reciprocal.
Outline

- Compact Modeling via model order reduction
- General hierarchical model order reduction
  - Hierarchical: block Gaussian elimination
  - General: any linear passive or active circuits
- Optimization considering magnitude and phase responses
  - Motivation for considering both magnitude and phase responses
  - Constrained least square based optimization
  - Multi-port non-reciprocal circuit realization
- Experimental results
- Conclusions
Motivation
Optimization Problem

- Admittance matrix of the reduced order model

\[
\hat{Y}(s) = \begin{bmatrix}
\hat{Y}_{1,1}(s) & \cdots & \hat{Y}_{1,n}(s) \\
\vdots & \ddots & \vdots \\
\hat{Y}_{n,1}(s) & \cdots & \hat{Y}_{n,n}(s)
\end{bmatrix}
\]

- Optimization problem (only considering magnitude)

\[
\min \left( \sum_{k=1}^{T} \left\| \hat{Y}_{p,q}(s_k) - \tilde{Y}_{p,q}(s_k) \right\|_2^2 \right)
\]

Where \( \hat{Y}_{p,q}(s_k) \) is the exact values of the admittance at the entry \((p,q)\) at the \(k\)-th frequency point.
Optimization Problem (cont’d)

- Basic Idea:

\[ \hat{Y}(s) = s \hat{Y}_\infty + \hat{Y}_0 + \sum_{m=1}^{M} \frac{r r_m}{s - p r_m} + \sum_{n=1}^{N} \left( \frac{r c_n}{s - p c_n} + \frac{r c_n^*}{s - p c_n^*} \right) \]

- Find a set of residues such that the errors are mixed in terms of both magnitude and phases.
Optimization Problem

Some definitions

\[ x = \begin{bmatrix} x_1^r & \cdots & x_M^r & x_1^c & \cdots & x_{2N}^c & Y_0 & Y_\infty \end{bmatrix}^T \]

\[ A_k = \begin{bmatrix} a_1^r(s_k) & \cdots & a_M^r(s_k) & a_1^c(s_k) & \cdots & a_{2N}^c(s_k) & 1 & s_k \end{bmatrix} \]

Where

\[ a_m^r(s_k) = \frac{1}{s_k - pr_m} \]

\[ a_n^c = \frac{1}{s_k - pc_n} + \frac{1}{s_k - pc_n^*}, a_{n+1}^c = \frac{j}{s_k - pc_n} - \frac{j}{s_k - pc_n^*} \]
Minimization for Real and Imaginary Part

Define

\[ A_{\text{lin}} = \begin{bmatrix} \text{re}(A) \\ \text{im}(A) \end{bmatrix}, \quad Y_{\text{lin}} = \begin{bmatrix} \text{re}(\tilde{Y}) \\ \text{im}(\tilde{Y}) \end{bmatrix} \]

Then

\[ \min(\|A_{\text{lin}}x - Y_{\text{lin}}\|_2^2) \]
Constraints for Sign of Phase

- define

\[ Y_D = \text{diag}(Y_{lin}) \]

\[ D_{lin} = Y_D A_{lin} \]

- Then

\[ D_{lin}x \geq 0 \]
Constraints for Value of Phase

- Define

\[ Y_I = \text{diag} \left( \begin{bmatrix} \text{im}(\tilde{Y}) \\ \text{re}(\tilde{Y}) \end{bmatrix} \right) \]

\[ I_{lin} = \begin{bmatrix} 1 & \cdots & 0 & -1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 & \cdots & -1 \end{bmatrix} \]

\[ C_{lin} = I_{lin} Y_I A_{lin} \]

Then \[ C_{lin} x = 0 \]
Optimization Problem

- The constrained linear least square optimization problem considering both magnitude and phase

\[
\min(\|A_{\text{lin}}x - Y_{\text{lin}}\|_2^2) \quad \text{subject to} \quad D_{\text{lin}}x \geq 0 \\
C_{\text{lin}}x = 0
\]
Realization (one-port)

- Foster’s canonical form for 1x1 admittance $Y(s)$

$$Y(s) = sY_\infty + Y_0 + \sum_{m=1}^{M} \frac{x_m^r}{s - pr_m} + \sum_{n=1}^{N} \left( \frac{x_n^c + x_{n+1}^c j}{s - pc_n} + \frac{x_n^c - x_{n+1}^c j}{s - pc_n^*} \right)$$

- Realization of Foster’s canonical form
Multi-port non-reciprocal Realization

- 2 port

\[ Y_{2\times2}(s) = \begin{bmatrix} y_{11}(s) & y_{12}(s) \\ y_{21}(s) & y_{22}(s) \end{bmatrix}, \]
General Multi-Port Network Realization

- For general $n \times n$ network
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A Folded Cascode COMS OPAMP

- $Y_{12}$ frequency responses of the original circuit and the reduced model
An active Sallen-Key low-pass filter
An active Sallen-Key low-pass filter

- $Y_{12}$ Frequency-domain pulse responses
  - Considering phase
  - Without Considering phase

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An active Sallen-Key low-pass filter

- Time-domain responses excited by different sources
An active Sallen-Key low-pass filter

- Reduction efficiency:
  - Original model: 636 elements
  - Realized reduced order model: 88 RLC, 2 VCVS, 2 CCCS elements
  - Reduction ratio: 85.5%.

Realized circuits have very regular structure, which can be utilized for further reduction or acceleration in time-domain simulation
Conclusions

- A general multi-point hierarchical model order reduction and realization flow for active circuits

- Constrained linear least square optimization technique considering both magnitude and phase responses for any linear active network

- Multi-port non-reciprocal circuit realization

- SPICE-in SPICE-out reduction/realization for maximum model portability and flexibility