# Variational Compact Modeling and Simulation for Linear Dynamic Systems

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# Outline

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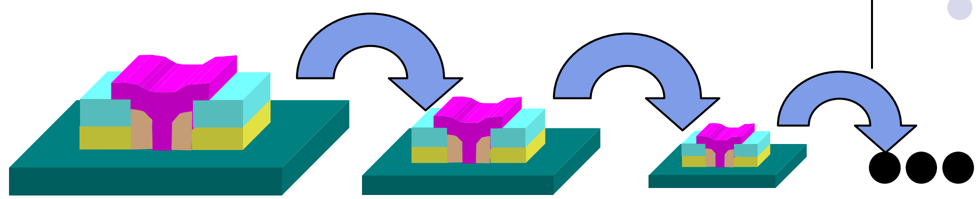
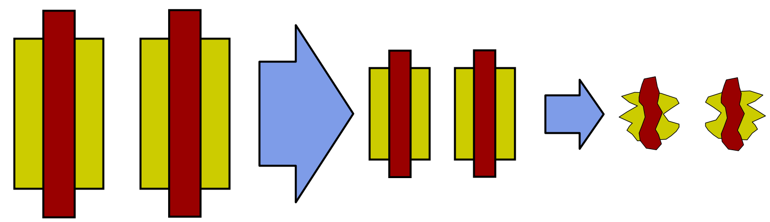
# Manufacturing Variations is REAL

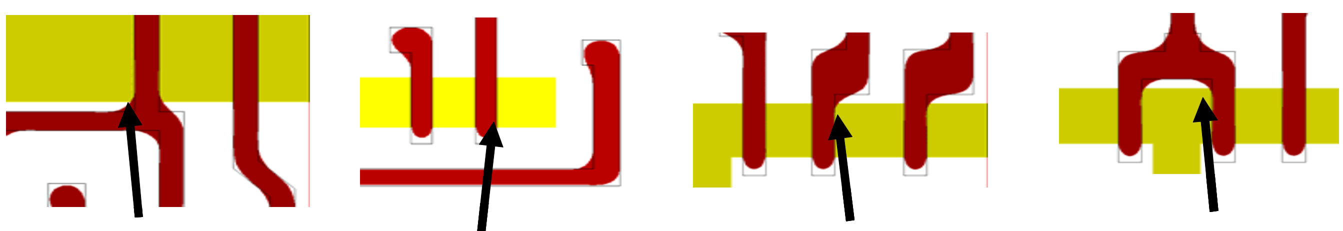
- CMOS scaling…
  - **Good for speed**
  - **Good for density**



  - **Bad for variability**
  - **Bad for manufacturability**
  - **Bad for predictability**



- What you design and verify pre-silicon is *not* the same as what you get post-silicon



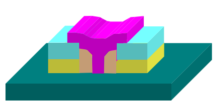
**Sensitive to grow due to defocus** | **Sensitive to shrink due to defocus** | **Sensitive to exposure variation** | **Sensitive to resist effects**

[Source: Rob Rutenbar and Andrzej Strojwas, CMU]

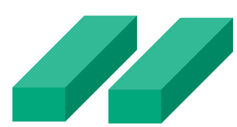
# Putting Variations in Context: Devices + Wires

- 3σ of variations normalized to nominal values **[Nassif, CICC'2001]**
- Manipulating things that are getting smaller
- Inevitable fluctuations are getting *relatively* larger
  - **Variations on (spatially nearby) devices/wires are *correlated***

| Year | $L_{eff}$ ±3σ (% nom) | $T_{ox}$ ±3σ (% nom) | $V_T$ ±3σ (% nom) | W ±3σ (% nom) | H ±3σ (% nom) | ρ ±3σ (% nom) |
|---|---|---|---|---|---|---|
| 1997 | 32% | 8% | 10% | 25% | 25% | 22% |
| 1999 | 33% | 8% | 10% | 26% | 30% | 24% |
| 2002 | 35% | 10% | 10% | 28% | 30% | 27% |
| 2005 | 40% | 12% | 11% | 30% | 34% | 32% |
| 2006 | 47% | 16% | 13% | 33% | 36% | 33% |



Devices



Interconnects

# Review of existing approaches for statistical MOR methods

- Perturbation based MOR (L. Daniel, TCAD04, X. Li, ICCAD05)
  - – inter-die variation only
- Interval-valued modeling (J. Ma:ICCAD'04, ISPD'05)
  - – accumulated errors can become unmanageable
- Statistical spectrum analysis (J. Wang, ICCAD04, P. Ghanta, DATE'05)
  - solve for coefficients of orthogonal polynomials for statistical responses
  - Have not been applied to projection based model order reduction

# New contributions

- The contributions of proposed statistical spectrum model order reduction technique:
  - Applying statistical spectrum method to calculate moments
  - Modified Krylov subspace based model order reduction technique
  - Monte Carlo sampling to generate order reduced variational model

# Problem formulation

- In RC network, the modified nodal analysis (MNA) equation is $$Gv(t)+C\frac{dv(t)}{dt}=Bu(t)$$
- Variational elements exist in G and C matrices
  - The MNA equation becomes
    $$G(\xi)v(t)+C(\xi)\frac{dv(t)}{dt}=Bu(t)$$
  - Modeling intra-die Gaussian uncorrelated variations.
  - Variations caused by circuit parameters, such as metal wire, width, thickness, etc. and caused by transistor parameters, such as channel length, width, gate oxide thickness.

# Statistical Spectral Analysis Method (Ghanem'90)

- A random variable x (t,ξ) can be represented by a set of orthogonal polynomials $\gamma_i(\xi)$,

$$x(t,\xi) = \sum_{n=0}^{\infty} a_n(t)\gamma_n(\xi)$$

- Hermite Polynomials are the best expansion for normal random variables
  - An order p Hermit Polynomial is defined as

$$H_p(\{\alpha_1,\alpha_2,\ldots,\alpha_p\}) = (-1)^p e^{\frac{1}{2}\xi'\xi} \frac{\partial^p}{\partial\alpha_1\partial\alpha_2\ldots\partial\alpha_p} e^{\frac{1}{2}\xi'\xi}$$

$\{\alpha_i\}$ are any set of p variables chosen from $\{\xi_1, \xi_2, \ldots, \xi_n\}$ allowing repetition

  - For x (t,ξ), an order p Hermit Polynomial expansion is

$$x(t,\xi) = \sum_{i=0}^{N} a_i(t)H_i(\xi)$$

# Simulation Approach Based on Hermite PCs

- Solve $$G(\xi)v(t,\xi)+C(\xi)\frac{dv(t,\xi)}{dt}=Bu(t)$$
- For independent random variables, $\{\xi_1, \xi_2\}$, p=2
  - G(t,ξ), C(t,ξ) can be written in Hermite polynomial chaos (PC)

$$G(t,\xi)=G_0(t)+G_1(t)\xi_1+G_2(t)\xi_2+G_3(t)(\xi_1^2-1)+G_4(t)(\xi_2^2-1)+G_5(t)\xi_1\xi_2$$

$$C(t,\xi)=C_0(t)+C_1(t)\xi_1+C_2(t)\xi_2+C_3(t)(\xi_1^2-1)+C_4(t)(\xi_2^2-1)+C_5(t)\xi_1\xi_2$$

  - $\xi_1$, $\xi_2$ are independent with normal distribution
  - How to solve for $v_i(t)$ →→ through statistical moment computation

# Hermite polynomial chaos

- Use a series of orthogonal polynomials to decompose Guassian random processes.
- The basis of Hermite polynomial is expanded to second order in experiment. Higher order improves accuracy.
- Galerkin method (principle of orthogonality):

  → Error is minimized when the error is orthogonal to the approximation
- Calculate coefficients in deterministic form
- Mean and variance of random variables can be computed easily.

# Projection Krylov Subspace MOR - review

- A general linear circuit system

$$C\dot{x}_n = -Gx_n + Bu_m$$

$$i_m = Lx_n$$

- Transfer function in s domain

$$H(s) = L(G + sC)^{-1}B$$

$$A = -G^{-1}C \quad R = G^{-1}B$$

$$\Rightarrow \quad H(s) = L(I - sA)^{-1}R$$

- The block Krylov space

$$Kr(A, R, q) = span[R, AR, A^2R, A^3R, \dots]$$

$$V = orthonomal\{Kr(A, R, q)\}$$

# Standard Krylov Subspace MOR

- Using Arnoldi or Lanczos process to orthonormalize the Krylov subspace

$$Kr(A, R, q) = span[R, AR, A^2R, A^3R, \ldots]$$

  - ith block moment is computed and orthonormalized immediately after generation against all the existing orthonormalized block moments.
  - Congruence transformation to compute the reduced model

$$\hat{G} = V^T G V; \hat{C} = V^T C V; \hat{B} = V^T B$$

# Modified Krylov subspace MOR

- Compute all the block moments first.
- Orthonormalize moment vector to get projection matrix V by Gram-Schdmit method.

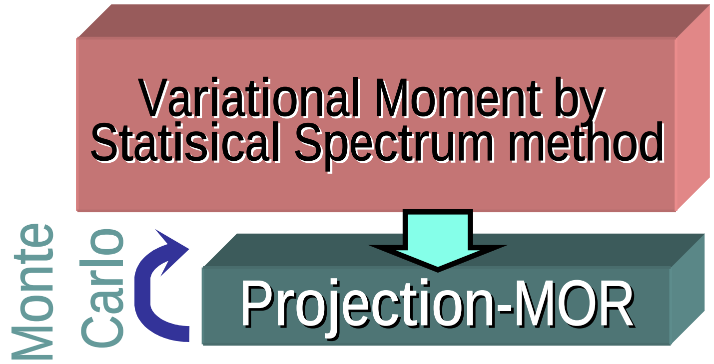
$$m_0 = G^{-1}B;$$
$$m_1 = -G^{-1}Cm_0;$$
$$\dots$$
$$m_i = -G^{-1}Cm_{i-1}; \forall i > 0$$

- Congruence transform $\hat{G} = V^T GV; \hat{C} = V^T CV; \hat{B} = V^T B$

Reason: To divide the statisitcal modeling process between moment computation and model generations

# New statisitcal MOR method

- Compute all the moments using statistical spectrum method. (as moments can be computed by solving Ax = b)
- Compute all the reduced models using Monte Carlo methods from variational moments
  - Orthonormalization
  - Congruence transformation
  - Pole/residues computations



# Galerkin method in Hermit PCs

- In Galerkin method, the best approximation of v is obtained when the error is orthogonal to the approximation

$$< e(t,\xi), H_k(\xi) >= 0$$

$$k = 0,1,\ldots, p$$

Where P is the order of Hermit PC

# Example: one random variable

- Expand Hermite PC to first order (p=1), variational G and C becomes

$$G = g_0 + g_1\xi; C = c_0 + c_1\xi$$

- Expand Hermite PC to second order (p =2) on moments

$$m_0 = [a_{m0}, a_{m1}, a_{m2}][1, \xi, \xi^2 - 1]^T;$$

$$m_{2q} = [a_0, a_1, a_2][1, \xi, \xi^2 - 1]^T;$$

$$m_{2q-1} = [b_0, b_1, b_2][1, \xi, \xi^2 - 1]^T;$$

  - [am0,am1,am2], [a0,a1,a2], and [b0,b1,b2] are coefficients for m0, m2q, and m2q-1 with respect to Hermite PC

# Solution for one variable

- Applying Galerkin method, and the equalities in Gaussian distributions; the 0th moment is given by:

$$\begin{bmatrix} g_0 & g_1 & 0 \\ g_1 & g_0 & 2g_1 \\ 0 & 4g_1 & 4g_0 \end{bmatrix} \begin{bmatrix} a_{m0} \\ a_{m1} \\ a_{m2} \end{bmatrix} - \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} = 0$$

- Likewise, the higher order moments are computed recursively

$$\begin{bmatrix} g_0 & g_1 & 0 \\ g_1 & g_0 & 2g_1 \\ 0 & 2g_1 & 2g_0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} - \begin{bmatrix} c_0 & c_1 & 0 \\ c_1 & c_0 & 2c_1 \\ 0 & 2c_1 & 2c_0 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} = 0$$

# Solution for n random variables

- Similarly, the 0th moment can be expressed as:

$$\begin{bmatrix} g_0 & g_1 & g_2 & \dots & g_i & \dots & g_n \\ g_1 & g_0 & 0 & \dots & 0 & \dots & 0 \\ g_2 & 0 & g_0 & \dots & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ g_i & 0 & 0 & \dots & g_0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ g_n & 0 & 0 & \dots & \dots & \dots & g_0 \end{bmatrix} \begin{bmatrix} a_{m0} \\ a_{m1} \\ a_{m2} \\ \dots \\ a_{mi} \\ \dots \\ a_{mn} \end{bmatrix} - \begin{bmatrix} B \\ 0 \\ 0 \\ \dots \\ 0 \\ \dots \\ 0 \end{bmatrix} = 0$$

- The recursive moments can be computed as:

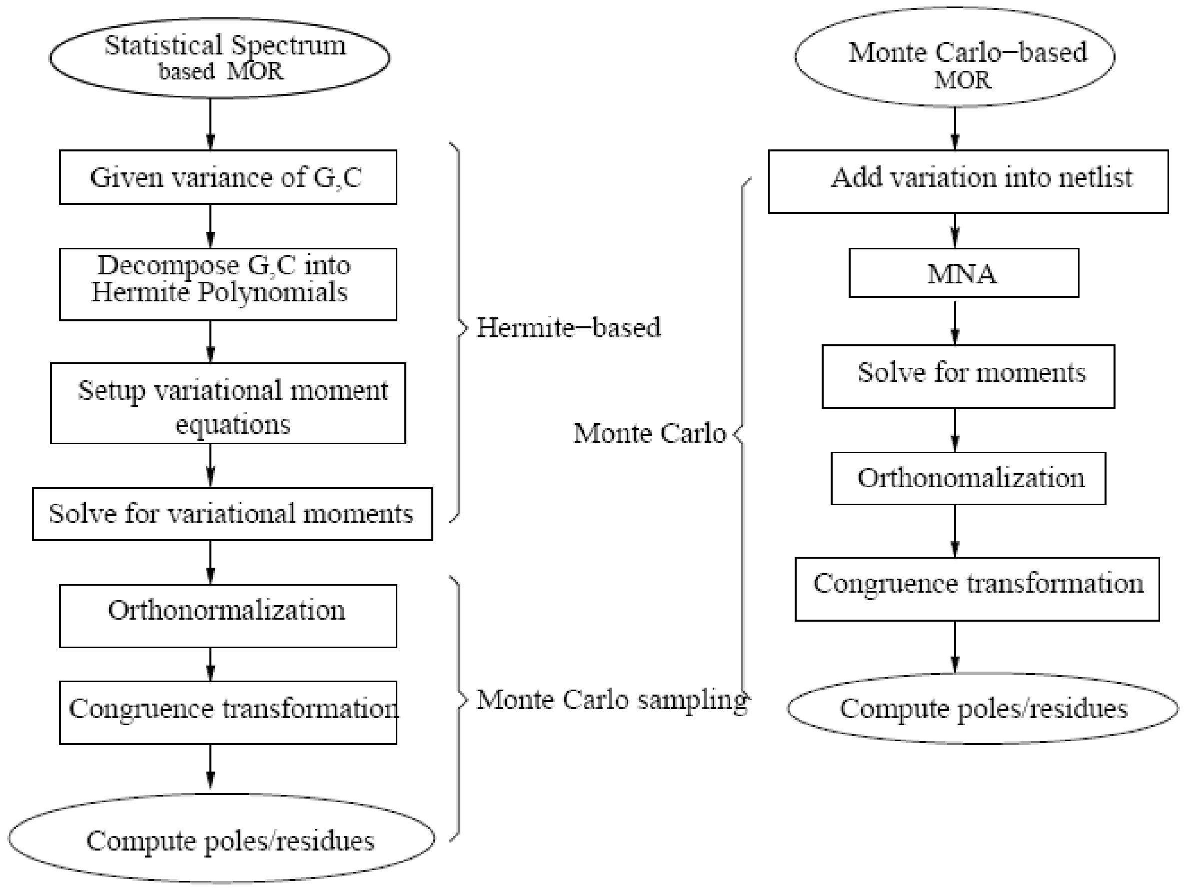
$$\begin{bmatrix} g_0 & g_1 & g_2 & \dots & g_i & \dots & g_n \\ g_1 & g_0 & 0 & \dots & 0 & \dots & 0 \\ g_2 & 0 & g_0 & \dots & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ g_i & 0 & 0 & \dots & g_0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ g_n & 0 & 0 & \dots & \dots & \dots & g_0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \dots \\ a_i \\ \dots \\ a_n \end{bmatrix} + \begin{bmatrix} c_0 & c_1 & c_2 & \dots & c_i & \dots & c_n \\ c_1 & c_0 & 0 & \dots & 0 & \dots & 0 \\ c_2 & 0 & c_0 & \dots & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ c_i & 0 & 0 & \dots & c_0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ c_n & 0 & 0 & \dots & \dots & \dots & c_0 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \dots \\ b_i \\ \dots \\ b_n \end{bmatrix} = 0$$

# Statistical Spectrum Model Order Reduction (SSMOR)

- Modified Krylov subspace model order reduction
- In each sampling, orthonormalize moment vectors thru Gram-Schmidt algorithm
- SSMOR computes variational block moments, then switch to Monte Carlo to generate reduced models
- Congruence transformation takes G and C into dimension-reduced matrices → guarantee passivity
- Use projection-based method to obtain poles and residues

# SSMOR flowchart

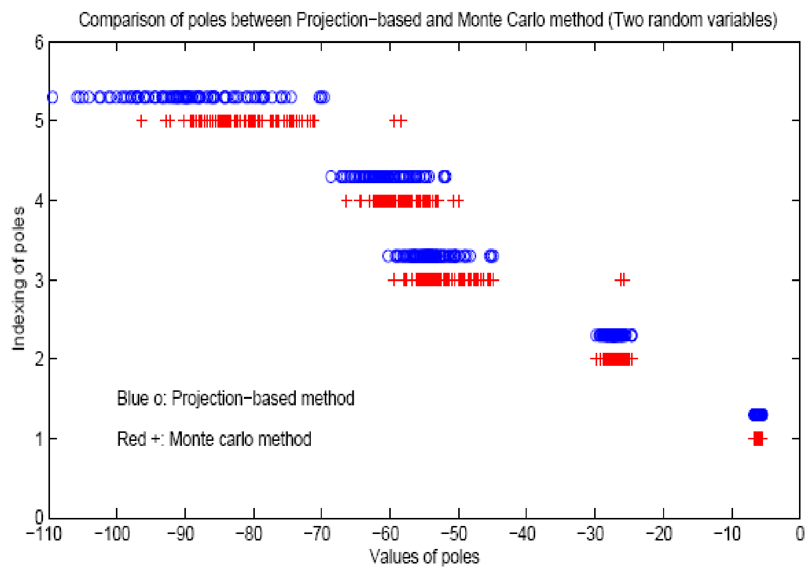
**Monte Carlo sampling on reduced models →→ much faster!**

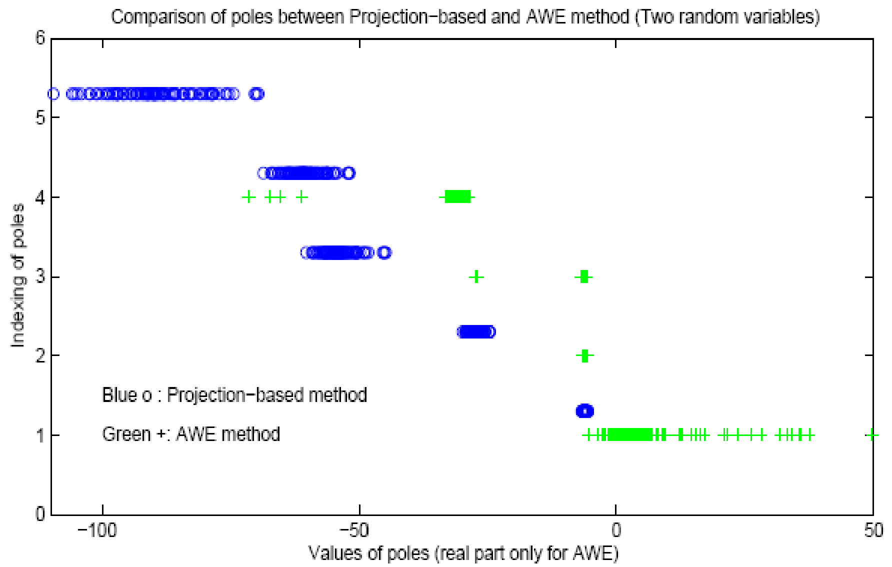


# Experiment

- First q moments are obtained (q=10), to obtain 5 poles and residues.
- Compare accuracy between explicit moment matching, AWE, and modified Krylov subspace projection based method
- Smaller size for fast simulation, and test scalability in larger sized circuits
- The sample size is 2000 for Monte Carlo to guarantee roughly 99% confidence level
- Piecewise linear responses are obtained to compare the percentage of errors for deterministic and stochastic stimulus.

# Projection based vs. AWE




**Projection-based method matches well with Monte Carlo**

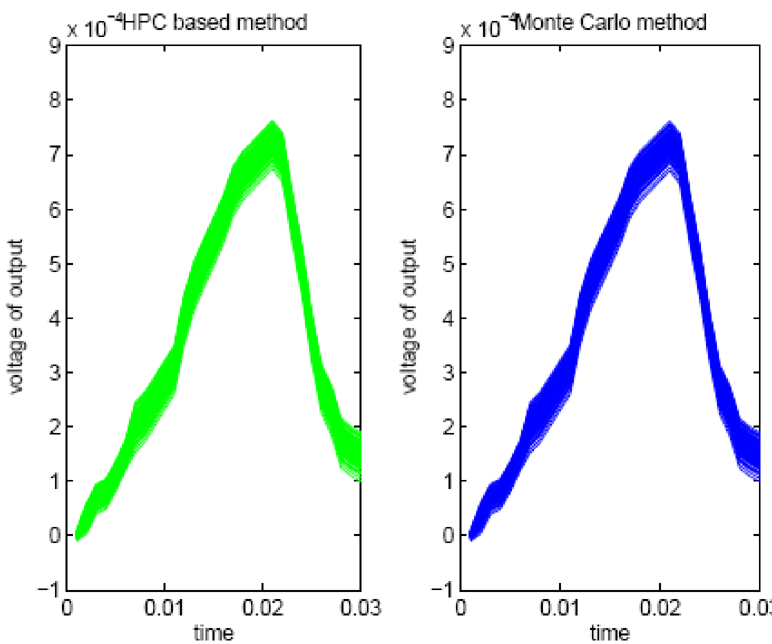
**SSMOR is projection-based!**

**Pole placement does NOT converge!**

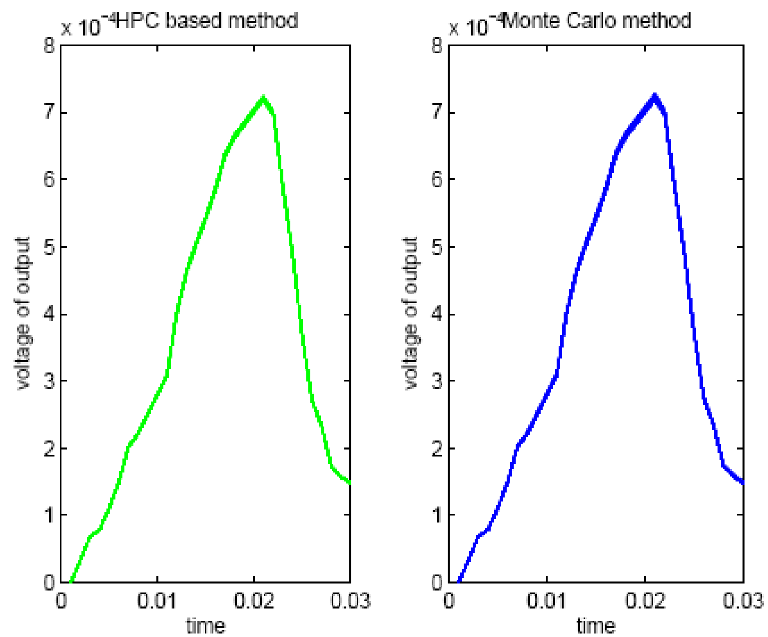
**AWE is numerically unstable**

# Piecewise linear responses (SSMOR)

Stochastic stimulus




Deterministic stimulus




Responses are almost identical in comparison to Monte Carlo!

# SSMOR vs. Monte Carlo

**Comparison of voltage response →→ less than 1% of errors**

| Time instance (e-3) s | SSMOR | | MC | | % error | |
|---|---|---|---|---|---|---|
| | mean (e-5) | std (e-6) | mean (e-5) | std (e-6) | mean % | std % |
| 3 | 6.852 | 9.518 | 6.835 | 9.487 | 0.25 | 0.327 |
| 5 | 11.11 | 9.489 | 11.078 | 9.454 | 0.253 | 0.369 |
| 20 | 70.27 | 15.56 | 70.1 | 15.45 | 0.232 | 0.732 |

| | #node | SSMOR | MC | Speedup |
|---|---|---|---|---|
| Ckt1 | 33 | 1 | 37.03 | 37 times |
| Ckt2 | 553 | 1 | 162.16 | 162 times |
| Ckt3 | 1720 | 1 | 118.03 | 118 times |

**SSMOR method is much faster without losing accuracy!**

**Runtime comparison between → → 100X speedup**

# Conclusion

- Statistical Spectrum Model Order Reduction (SSMOR) is suitable to consider inter-die and intra-die variations
- Fast analysis on interconnect, such as power grid or clock tree networks, with stochastic stimulus without losing accuracy
- Order of faster than Monte Carlo methods with reasonable accuracy.
- AWE is numerically unstable; projection-based method is used in SSMOR and was shown to be very robust.
- Need to consider spatial correlations in the future.