

An Extended SVD-based Terminal and Model Order Reduction Algorithm



Pu Liu, Sheldon X.-D. Tan, Boyuan Yan, Bruce McGaughy*



Mixed-Signal Nanometer VLSI Research Lab.
Department of Electrical Engineering, UC Riverside
*Cadence Design Systems Inc.



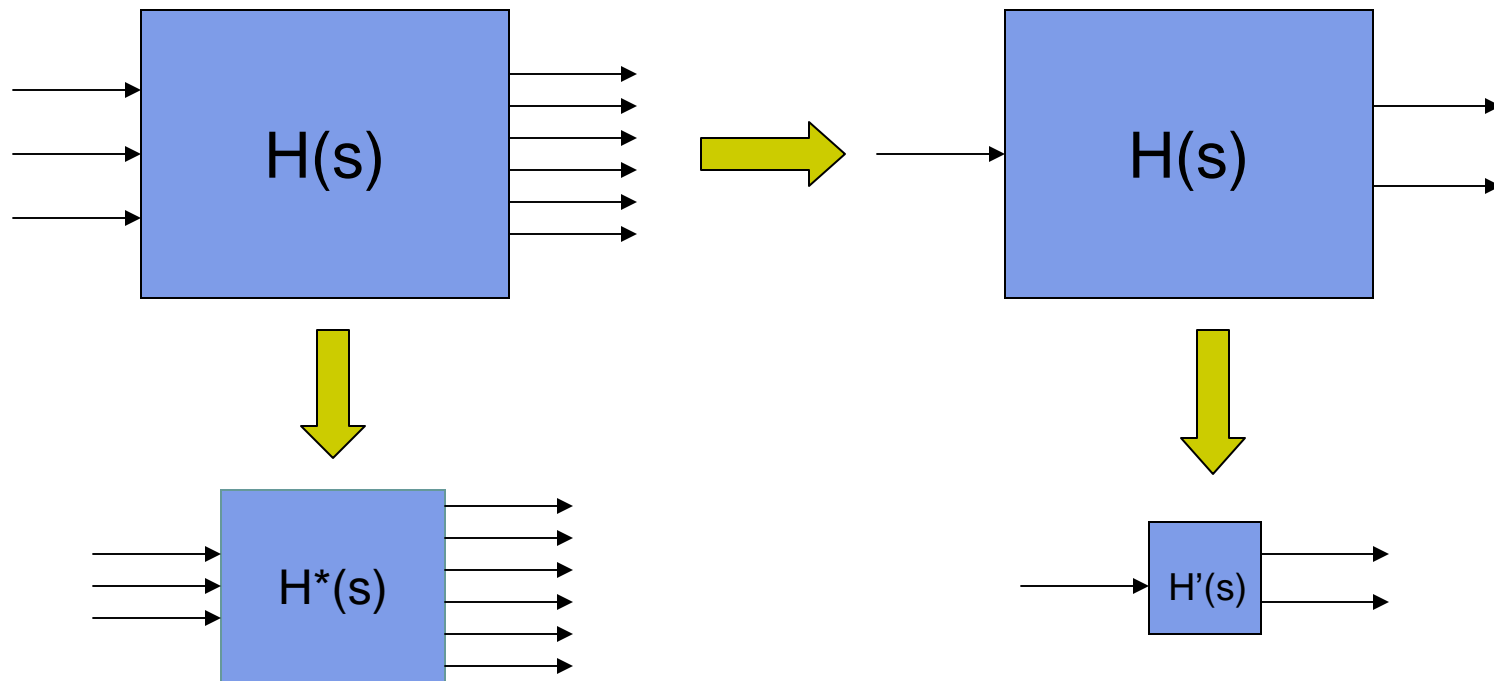


Outline

- Motivation of terminal and model order reduction
- Review of basic SVDMOR method
- The new extended-SVDMOR (ESVDMOR) algorithm
 - Input and output moment matrices
 - The SVD-based new terminal reduction method
 - ESVDMOR reduction flow
- Experimental results
- Summary



Why Terminal Reduction?



Terminal reduction leads to more compact models!



Projection Krylov Subspace

- A general linear circuit system

$$C\dot{x}_n = -Gx_n + Bu_m$$

$$i_m = Lx_n$$

- Transfer function in s domain

$$H(s) = L(G + sC)^{-1}B \quad \longrightarrow \quad H(s) = L(I - sA)^{-1}R$$
$$A = -G^{-1}C \quad R = G^{-1}B$$

- The block Krylov space $R \subseteq R^{n \times p}$

$$Kr(A, R, q) = \text{span}[R, AR, A^2R, A^3R, \dots]$$

$$P = \text{orthonomal}\{Kr(A, R, q)\}$$

Review of SVD MOR Algorithm



A time domain circuit equations

$$C\dot{x}(t) + Gx(t) = Bu(t)$$

$$y(t) = Lx(t)$$

Terminal reduction via SVD:

$$m_0 = U\Sigma V^T \approx U_k \Sigma_k V_k^T$$

$$B = B_b V_k^T \quad B_b = B V_k (V_k^T V_k)^{-1}$$

$$L^T = L_c U_k^T \quad L_c = L^T U_k (U_k^T U_k)^{-1}$$

Its transfer functions in s domain:

$$H(s) = L(G + sC)^{-1} B$$

$$m_0 = LG^{-1} B$$

$$m_1 = LG^{-1} C G^{-1} B$$

\vdots

$$m_i = L(G^{-1} C)^{-1} G^{-1} B$$

Only consider DC response (DC moment) !



SVD MOR method (cont.)

$$H(s) \approx U_k L_c^T (G + sC)^{-1} B_b V_k^T$$

$$H_r(s) = L_c^T (G + sC)^{-1} B_b$$

$$\hat{H}_r(s) = \hat{L}_c^T (\hat{G} + s\hat{C})^{-1} \hat{B}_b$$

$$H(s) = U_k \hat{L}_c^T (\hat{G} + s\hat{C})^{-1} \hat{B}_b V_k^T$$

where

$$\hat{G} = V^T G V; C = V^T C V;$$

$$\hat{B}_b = V^T B_b; \hat{L}_c^T = L_c^T V$$



SVDMOR Algorithm (cont.)

$$H = B^T \left(G + s C \right)^{-1} M$$

After SVD on boundary matrices

$$H = \begin{bmatrix} \text{---} \\ \text{---} \end{bmatrix} \left(G + s C \right)^{-1} \begin{bmatrix} \text{---} \\ \text{---} \end{bmatrix}$$

After projection MOR

$$H \sim \begin{bmatrix} \text{---} \\ \text{---} \end{bmatrix} \left(G^* + s C^* \right)^{-1} \begin{bmatrix} \text{---} \\ \text{---} \end{bmatrix}$$



Limitations of SVDMOR

- SVD is performed only on DC response or particular order of moments.
- Input and output terminals are reduced at the same time.
 - Mapping can lead to large errors due to different ranks of input and output matrices.
- The SVDMOR does not always lead to passive models.



Block Moment Details

- The $(i)^{\text{th}}$ moment block

$$m_i = \begin{bmatrix} m_{1,1}^i & m_{1,2}^i & \cdots & m_{1,p}^i \\ m_{2,1}^i & m_{2,2}^i & \cdots & m_{2,p}^i \\ \vdots & \vdots & \vdots & \vdots \\ m_{q,1}^i & m_{q,2}^i & \cdots & m_{q,p}^i \end{bmatrix}$$



Moment Matrix Formulation

- Output moment matrix

$$M_o = \begin{bmatrix} m_0^T \\ m_1^T \\ \vdots \\ m_{r-1}^T \end{bmatrix}$$

- Each column represents a moment series of output due to all input's stimuli

- Input moment matrix

$$M_I = \begin{bmatrix} m_0 \\ m_1 \\ \vdots \\ m_{r-1} \end{bmatrix}$$

- Each column represents a moment series at all outputs due to an input stimulus



New Terminal Reduction

- Find projection subspace for terminal reduction

$$M_I = U_I \Sigma_I V_I^T \approx U_{I_{ki}} \Sigma_{I_{ki}} V_{I_{ki}}^T$$

$$M_O = U_O \Sigma_O V_O^T \approx U_{O_{ko}} \Sigma_{O_{ko}} V_{O_{ko}}^T$$

$$B = B_r V_{I_{ki}}^T$$

$$L = V_{O_{ko}} L_r$$

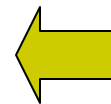
$$B_b = B V_{I_{ki}} (V_{I_{ki}}^T V_{I_{ki}})^{-1}$$

$$L_c = L^T U_{O_{ko}} (U_{O_{ko}}^T U_{O_{ko}})^{-1}$$

Notice that row in $V_{I_{ki}}, U_{O_{ko}}$ are orthogonal



$$B_b = B V_{I_{ki}}$$
$$L_c = L^T U_{O_{ko}}$$



$$V_{I_{ki}}^T V_{I_{ki}} = I, U_{O_{ko}}^T U_{O_{ko}} = I$$



New Terminal Reduction (cont'd)

- Terminal reduced subsystem

$$H(s) = V_{O_{ko}} L_r (G + sC)^{-1} B_r V_{I_{ki}}^T$$
$$H_r(s) = L_r (G + sC)^{-1} B_r$$

- Model order reduction and terminal recovery

$$\hat{H}_r(s) = \hat{L}_r (\hat{G} + s\hat{C})^{-1} \hat{B}_r$$

$$\hat{H}(s) = V_{O_{ko}} \hat{H}_r(s) V_{I_{ki}}^T$$

where

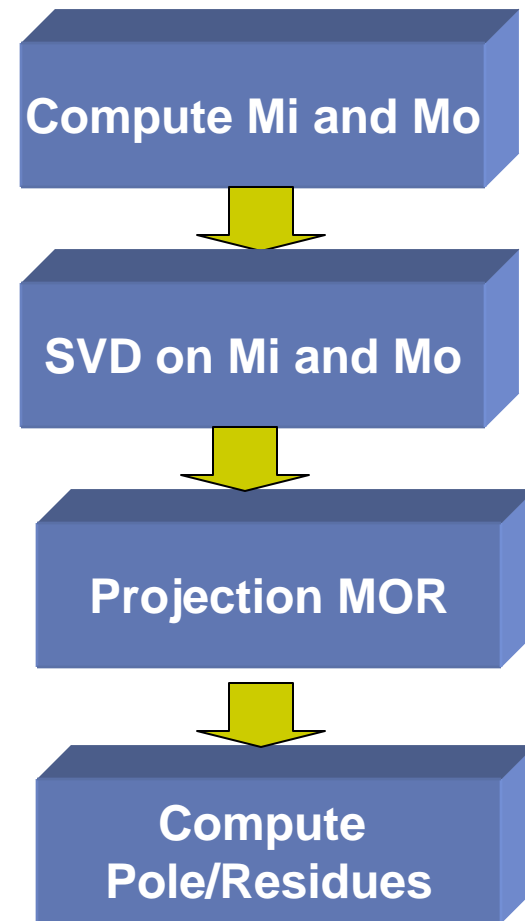
$$\hat{G} = V^T G V; C = V^T C V$$

$$\hat{B}_r = V^T B V_{I_{ki}}; \hat{L}_r = V_{O_{ko}}^T L B V$$



ESVDMOR Algorithm Flow

- i. Compute multiple block moments
- ii. Construct input and output moment matrices respectively
- iii. Perform SVD-based low rank approximation to input and output position matrices
- iv. Apply normal projection Krylov subspace based model order reduction on terminal reduced subsystem
- v. Computer the final transfer function of the reduced system





Passivity Discussion

- SVDMOR or ESVDMOR does **NOT** lead to passive models

$$C\dot{x}_n = -Gx_n + Bu_m$$

$$i_m = Lx_n$$

- Passive projection require $B^T = L$
- In SVDMOR-like methods, it becomes $B_r^T = L_r$
 - The moment matrix M or M_i/M_o must be symmetric
The interconnects are driven by the same source types
- When $B^T = L$, i.e. all the terminals are treated as both input and output terminals, the moment matrix typically are close to full rank.
 - We can't perform the terminal reduction at all.



Example

- Treat all terminals of net1026 as bi-direction

#	m_0	m_1
1	0.58970	7.5175
2	0.55093	0.33515
3	0.50215	0.30440
4	0.41875	0.28897
-	---	---
-	---	---
260	0.00059548	0.0099117
261	0.0001499	0.0098567
262	2.9535e-17	0.0096795



Example 1

- Net27 (14 inputs, 118 outputs)

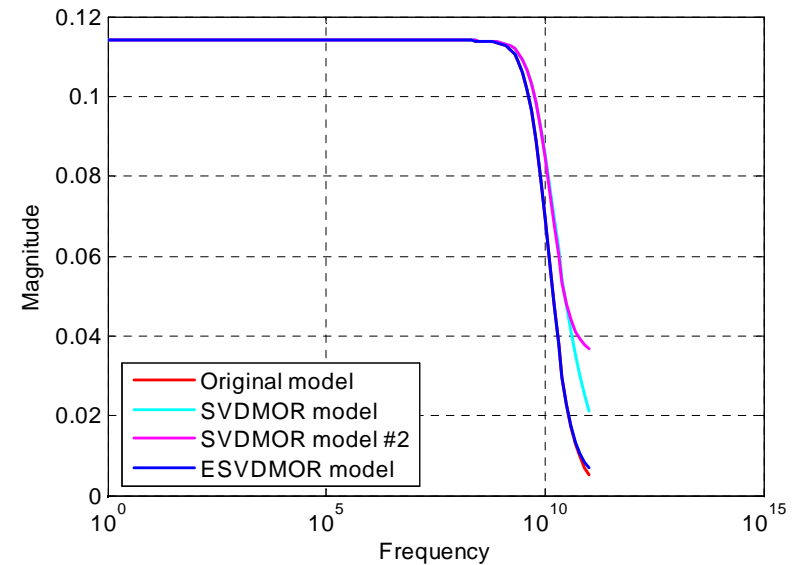
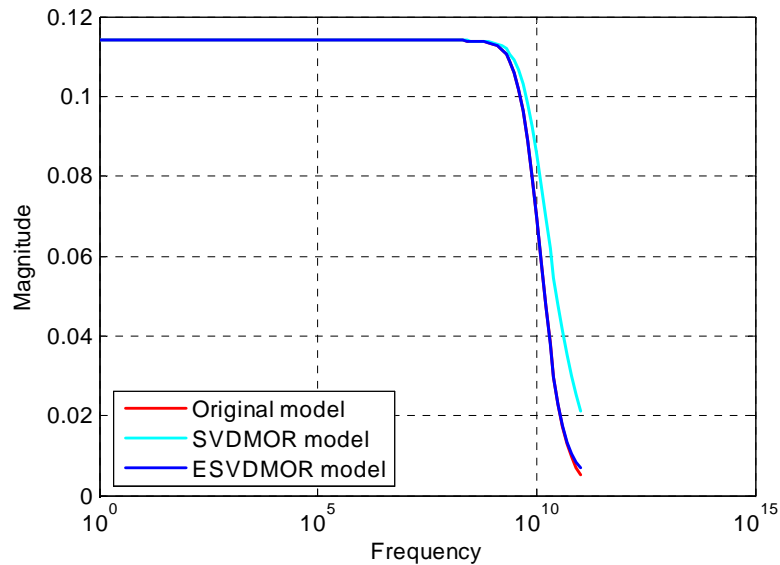
#	m_0	M_I	M_O
1	5.1587	5.1587	19.828
2	3.9883e-14	3.9883e-14	4.4677
3	1.6681e-14	1.6681e-14	1.6517
4	---	---	0.3045
5	---	---	0.0348
6	---	---	2.4611e-3
7	---	---	1.6134e-4

Table 1: The singular values of DC moment, Input moment matrix and Output moment matrix of the circuit net27.



Example 1 (cont.)

- Net27 (14 inputs, 118 outputs)



SVD MOR with 1 input 1 output; ESVD MOR with 1 input 5 output; SVD MOR #2 with 5 input 5 output.



Example 2

- Net1026 (6 inputs, 256 outputs)

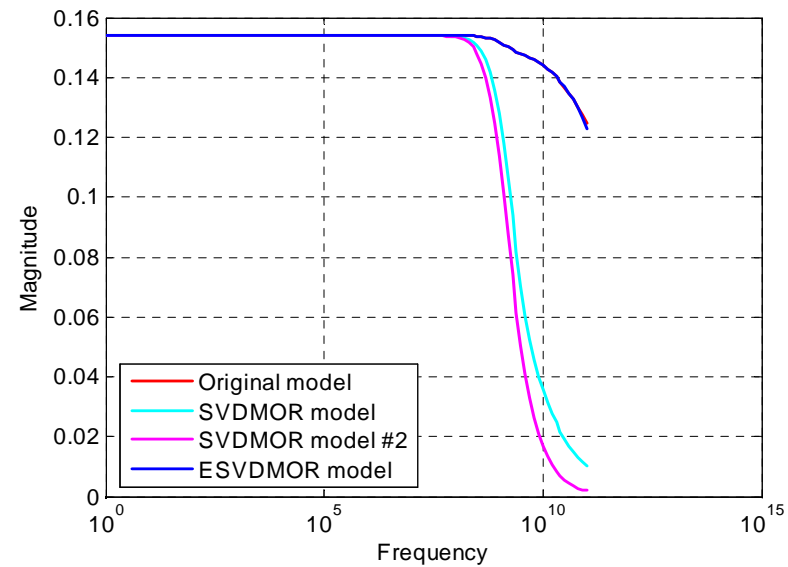
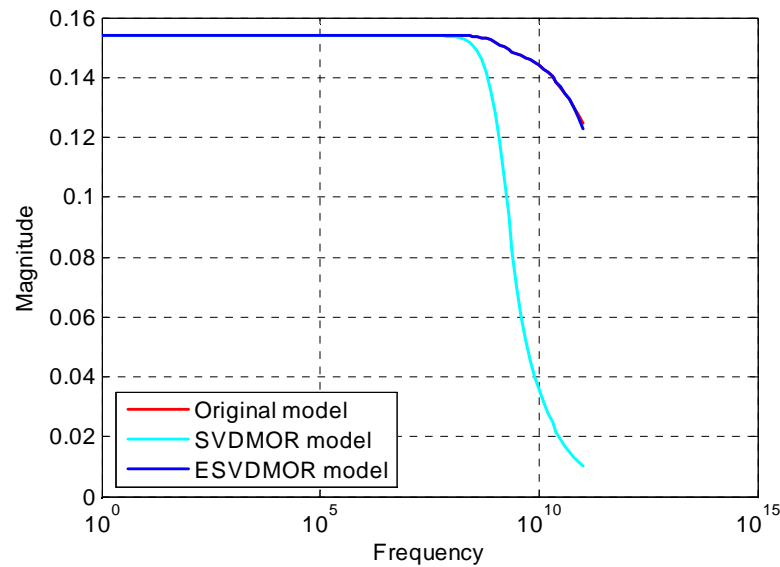
#	m_0	M_I	M_O
1	7.5175	7.5175	2097.6
2	2.7376e-15	2.7376e-15	10.501
3	7.5742e-16	7.5742e-16	1.6625
4	---	---	0.12577
5	---	---	0.00134
6	---	---	8.278e-6
7	---	---	3.9216e-8

Table 2: The singular values of DC moment, Input moment matrix and Output moment matrix of the circuit net1026.



Example 2 (cont.)

- Net1026 (6 inputs, 256 outputs)



SVD MOR with 1 input 1 output; ESVD MOR with 1 input 5 output; SVD MOR#2 with 5 input 5 output.



Example 2 (cont.)

- Treat all terminals of net1026 as bi-direction

#	m_0	m_1
1	0.58970	7.5175
2	0.55093	0.33515
3	0.50215	0.30440
4	0.41875	0.28897
-	---	---
-	---	---
260	0.00059548	0.0099117
261	0.0001499	0.0098567
262	2.9535e-17	0.0096795



Example 3

- Net55 (59 inputs, 31 outputs)

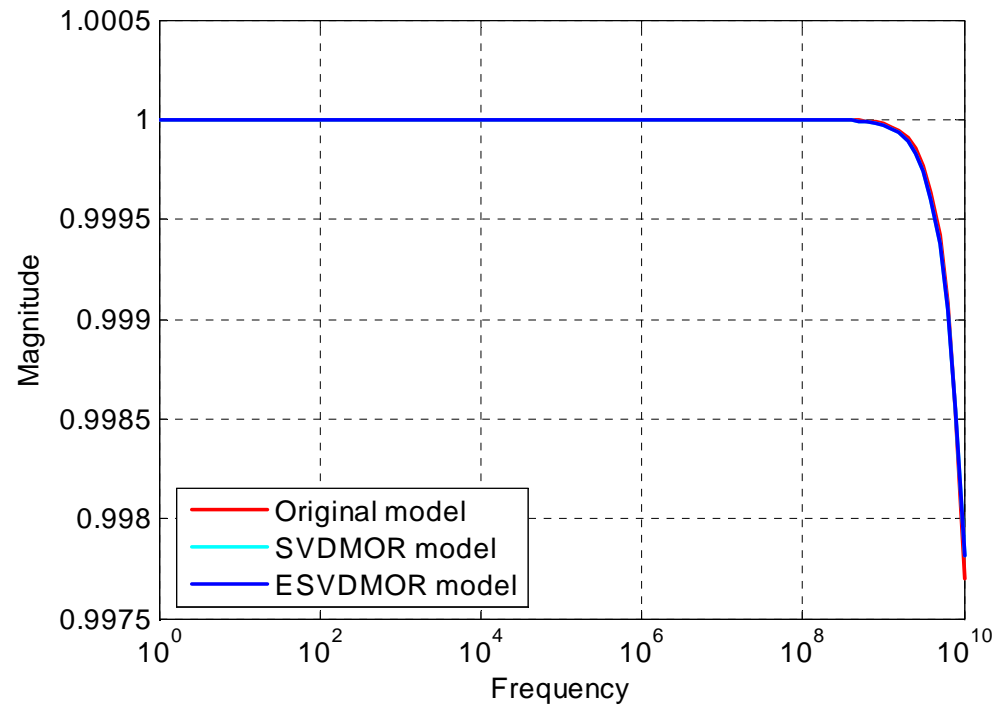
#	m_0	M_I	M_O
1	4.224	4.224	4.224
2	2.4495	2.4495	2.4495
3	0.4126	0.4126	0.4126
4	5.5185e-16	3.1765e-13	5.5185e-16

Table 4: The singular value of DC moment, Input moment matrix and Output moment matrix of the circuit net55.



Example 3 (cont.)

- Net55 (59 inputs, 31 outputs)

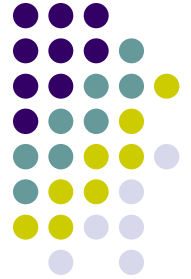


SVDMOR, ESVDMOR, and SVDMOR#2 are all with 3 input 3 output.



Summary

- SVD MOR only consider DC moment.
- Moment matrices include more information.
- ESVD MOR use high order moments to exploit the appropriate number of independent input and output terminals separately.
- SVD based terminal reduction methods are ineffective when passivity is considered.



Thanks & Questions?