A Unified Electrical SPICE Model for Piezoelectric Transducers

Jean-Marc Gallière
University of Montpellier 2
Polytech'Montpellier, France
galliere@polytech.univ-montp2.fr

Philippe Papet
University of Montpellier 2
LPMC, France
papet@polytech.univ-montp2.fr

Laurent Latorre
University of Montpellier 2
LIRMM, France
latorre@lirmm.fr

ABSTRACT

The widespread use of piezoelectric materials in an increasing number of applications requires the development of advanced realistic electrical models. Until now, models were limited taking into account only one ceramic's operation mode, i.e., thickness or planar. On the other hand, it establishes that the robustness of the piezo-electronic systems is likely to weaken if the models making it possible to conceive do not improve, in particular by taking into account further real phenomena. This article proposes to merge, in a new electrical model, the two operation modes. It is demonstrated that the electrical behavior of the proposed model is in very good agreement with the real ceramic behavior.

1. INTRODUCTION

Nowadays, worldwide competition constraints the manufacturers to reduce the production cost in a drastic way. So, an important effort must be achieved by the designer to decrease the product development cycle time. During the design, the engineer's ambition is not only to validate design specifications but also to ensure the product's robustness. For this, designers make use of simulation tools to improve their efficiency. The result is a highly iterative and predictive engineering process that delivers innovative designs, higher quality products and reduced time-to-market. However, these data-processing tools are attractive only if faithful models of the components to be implemented are available. For several decades, engineers have attempted to create such powerful models. It is the same for the industry of piezoelectricity. Nevertheless, the modeling of piezoelectric ceramics, which are component located between two worlds - mechanical and electrical - is not an easy task.

An important issue when designing ultrasonic based systems is the knowledge of the ceramic behavior regarding two parts, the front/back mechanical side of the ceramic and the electrical part. Indeed, by definition the two fields closely interact. The electromechanical interaction, represented by electrical equivalent circuits, was first introduced by Mason [1]. Redwood [2] enhanced this electromechanical model by incorporating a transmission line, making possible to extract useful information on the temporal response of the piezoelectric component. Thus, it is possible to represent the propagation time for a mechanical wave from one side to another.

The piezoelectric crystal deforms in different ways at different frequencies. Those various deformations are called the vibration modes. Like most solid bodies, the vibration modes are a result of a system of standing waves. These modes can be expressed from a wave equation, in association with a series of overtone modes which are solutions of the same set of equations. A number of research works have been conducted in the past years dealing with the ceramic's behavior. Although these models perfectly match the electrical characteristics of the piezoelectric transducers, they suffer from a strong limitation: they cannot implement several vibration modes simultaneously. The objective of this paper is to present a unified electrical SPICE model permitting to handle together the planar and thickness bulk vibration modes of ceramic disks.

Among electrical simulators, SPICE [3] has a particular place. It is commonly used in electrical engineering where it became a standard. SPICE models are easy to share between various simulators, ensuring an easy diffusion. The SPICE software which we use for this study is the commercial version PSPICE Allegro AMS Simulator 15.7 from Cadence [4]. Our release works in a MS Windows® environment. The AMS Simulator's software allows users to create designs, control simulations and interpret the results in a single environment.

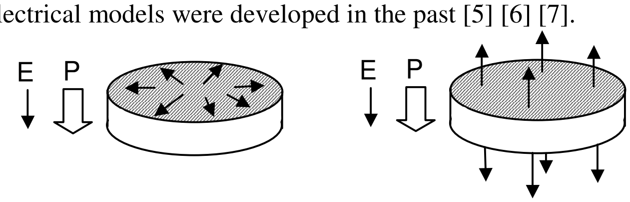
The paper is organized as follows. Section 2 recalls how the thickness and planar vibration models are electrically implemented in the literature. The third section introduces the new unified model that we propose. Section 4 compares simulation results obtained with our model with real ceramic measurements. Finally, we conclude in section 5.

2. THICKNESS AND PLANAR ELECTRICAL MODELS

The values of the piezoelectric properties of a material can be derived from the resonance behavior of suitably shaped samples subjected to a periodic electric field. This study will be limited to the cases of ceramic disks (probably the most convenient shape to fabricate) polarized (P) along the 3-axis (it is conventional to align the coordinate system with the poling directions) which is the axis of applied

978-1-4244-1567-0/07/$25.00 © 2007 IEEE

electric field (E). As a consequence the crystalline symmetry of the poled polycrystalline ceramics, which have ∞ - fold symmetry in a plane normal to the poling direction belongs to 6mm group in the hexagonal symmetry system. Therefore, for the analysis, a cylindrical coordinate system with its origin located at the center of the disc is most suitable. Due to the symmetry, only thickness and radial (planar) modes are excited (Fig. 1) and axes r and z are assumed to be pure mode propagation directions. For each of them, SPICE electrical models were developed in the past [5] [6] [7].



a) Planar Mode b) Thickness Mode

Fig. 1: Typical vibration modes of ceramic disks

Moreover, since biased surfaces are the two parallel surfaces of the disc, only the component Ez of the exciting electric field has to be considered. Taking into account these assumptions, a 3-D analytical model of piezoceramic disc has been developed in [13].

The wave equations in the radial and the thickness directions are:

$$\left[\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2}\right] = \rho\frac{\partial^2 u_r}{\partial t^2} \qquad \frac{\partial^2 u_z}{\partial z^2} = \rho\frac{\partial^2 u_z}{\partial t^2}$$

Where C^D_{11} and C^D_{33} are the elastic stiffness constants at constant electric displacement (they are not pure elastic constants due to the electromechanical coupling), u_r and u_z are the elastic displacements and ρ the density of the material.

2.1. Electrical study

From the equation of acoustic wave's propagation in piezoelectric materials, it is possible to write linear relations linking, on the one hand the mechanical magnitudes (force F and speed of particles u) which are preserved at an interface and, on the other hand electrical quantities (applied potential v_3 and intensity i_3 of the current). The fact of having an input vector of dimension 3 (i.e., two mechanicals and one electrical input), leads to an impedance matrix:

$$\begin{bmatrix} F1 \\ F2 \\ v_3 \end{bmatrix} = -j.\begin{bmatrix} Z/tan\left(\frac{\omega d}{u}\right) & Z/sin\left(\frac{\omega d}{u}\right) & h/\omega \\ Z/sin\left(\frac{\omega d}{u}\right) & Z/tan\left(\frac{\omega d}{u}\right) & h/\omega \\ h/\omega & h/\omega & 1/\omega C0 \end{bmatrix}.\begin{bmatrix} u_1 \\ u_2 \\ i_3 \end{bmatrix}$$

Where F1 and F2 (N) symbolize the forces, u_1 and u_2 (m/s) are the particle velocities inside the material, *w* the angular frequency (rad/s) and $h_{33} = e_{33}/\varepsilon^S$ (V/m) the piezoelectric constant with e_{33} (C/m^2) the piezoelectric coefficient. The mechanical impedance Z (rayl) is calculated knowing the ceramic density ρ (kg/m^3), the particle velocity u (m/s) and the area A (m^2) by using $Z = \rho.u.A$. The equivalent circuit of Fig. 2 can be easily derived from the previous piezoelectric impedance matrix [9].

The diagram of Fig. 2 explains the port definition for a thickness-mode transducer along with Redwood's version of Masson's equivalent circuit. The model consists of a capacitance C0, a negative capacitance –C0, an ideal transformer and a transmission line. C0 is the so called piezoceramic clamped capacity:

$$C0 = \varepsilon^S.\frac{A}{d}$$

Where ε^S (C^2/Nm^2) is the ceramic permittivity with zero or constant strain, A (m^2) is the area electrodes and d (m) his thickness.

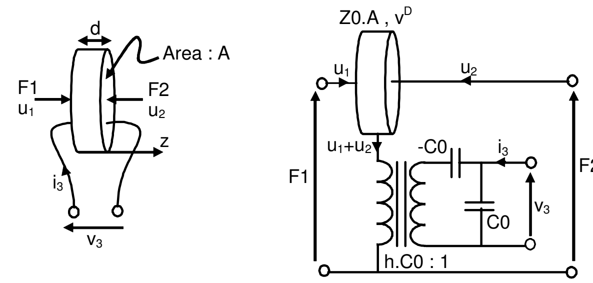


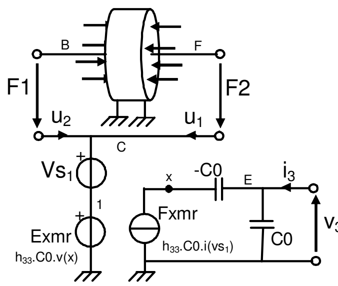
Fig. 2: Transducer and his equivalent circuit of Mason as adapted by Redwood

The mechanical part of the piezoelectric transducer is easily represented into SPICE using a transmission line model. This class of component is well known and modeled; in addition, it fits perfectly in this context. In first approximation, two parameters are sufficient to entirely define the mechanical part of the transducer, i.e., the impedance Z and the sound propagation delay *td* through the transducer. If more accuracy is required and/or when acoustic losses are significant, e.g. transducers with low mechanical Q-factor, losses in the piezoelectric material can be added. In a previous work [10], Püttmer et al. define a SPICE model of the transducer taking into account the losses with a transmission line modeled as lumped ladders.

2.2. Thickness-Mode Transducer model

Among the thickness-mode transducer's models, we chose, because of its simplicity of implementation, the Morris's model [5] (Fig. 3). This model is a direct transcription of the Redwood's model [2].

In this model, the ideal transformer of Mason's model is replaced by linear dependent sources Exmr and Fxmr. Compared to Morris [5], the unusual negative capacitance –C0 is now well interpreted by the SPICE simulator. Vs_1 is a zero valued source used by dependent source Fxmr as required by SPICE and have no effect on the circuit operation.




```
.subckt Thickness E F B C

c0 E 0 {C0}
cneg E x {-C0}
r1 x 0 100meg
fxmr 0 x vs1 {h33*C0}
exmr 1 0 x 0 {h33*C0}
vs1 C 1
tech F 0 B 0 z0={ZpztT} td={tdT}

.ends
```

Fig. 3: Morris's SPICE thickness mode transducer diagram and his related SPICE netlist

The resistance r1 (not include in the schematic) is needed to fulfill the SPICE requirement that every node have a DC path to ground to perform the mandatory bias computation. Furthermore, in first approximation, the r1 resistance can be selected to reflect dielectric losses if desired.

The impedance ZpztT and the travel time tdT of the transmission line according to the thickness d are calculated from physical parameters with the following relations:

ZpztT = A.Vpzt.ρ (rayl) with Vpzt = $(C_{33}^{D}/\rho)^{1/2}$ (m/s)

tdT = d/Vpzt (s)

In Fig. 4, a comparison is made between the electrical impedance Z = v_3/i_3 obtained with the Morris's model by simulation and a ceramic measurement performed with an Agilent 4294A precision impedance analyzer from [11]. The Ferroperm [12] commercial transducer used in this study is a circular PZ26 of 16mm diameter by 2mm thick. The experimental set-up is composed of this analyzer with his impedance test kit and a spring-clip fixture which applies very little mechanical loading in such a way that the sample is under free piezoelectric resonator condition.

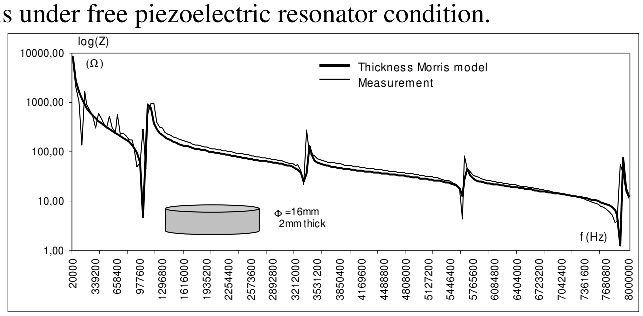


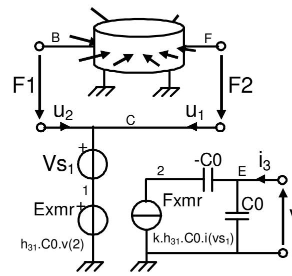
Fig 4: Impedance comparison between experiment and simulation for Thickness-mode Morris model

As one can observe in fig. 4, the simulated harmonic resonances appear at the desired frequencies, i.e., ~1MHz, ~3MHz, ~5MHz… However, since model does not take into account the two kinds of losses: mechanical and dielectric, the peaks appear sharper in the simulation. This can be corrected by modeling the losses starting from the model of the transmission line as explain by Püttmer et al. in [10].

For lower frequencies (<1MHz), the simulated curve differ more significantly from the real one. In fact, the peaks which emerge in the measured curve embody the planar resonance not managed by the Morris's model.

2.3. Planar-Mode Transducer model

To consider the planar vibratory effects, the model must be adapted. In this case the electric field becomes perpendicular to the acoustic waves spread. From the previous work of Iula et al. [13] the electro-mechanical part of the model can be established (Fig. 5).




```
.subckt Planar E F B C

c0 E 0 {C0}
cneg E 2 {-C0}
r1 2 0 100meg
fxmr 0 E vs1 {k*h31*C0}
exmr 1 0 2 0 {h31*C0}
vs1 C 1
tech F 0 B 0 z0={ZpztP} td={tdP}

.ends
```

Fig. 5: SPICE planar mode transducer diagram and his corresponding SPICE netlist

The other important change to consider is the modification of the transmission line parameters. The anisotropic properties of piezoelectric materials imply a change in the characteristic impedance as well as in the propagation time through the ceramics. It comes: (r represents the disk radius)

ZpztP = 2πr.d.VpztP.ρ (rayl) with VpztP = $(C_{11}^{D}/\rho)^{1/2}$ (m/s)

tdP =2r/VpztP (s)

The electrical impedance Z = v_3/i_3 computed with the Planar's model is compared with a measurement (Fig. 6). The transducers employed for this comparison is a circular PZ26 of 16mm diameter by 1mm thick.

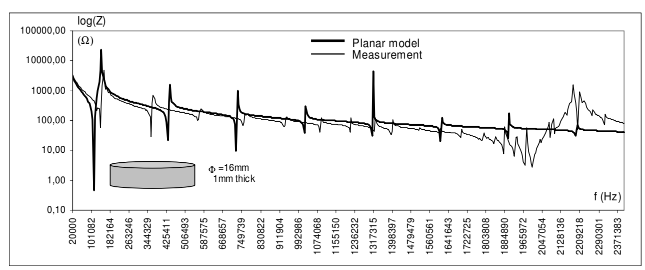


Fig 6: Impedance comparison between experiment and simulation for Planar-mode model

This time, the planar mode is well taken into account by the model for the first planar resonance frequency. As far as the high order planar modes are concerned, the resonance frequencies obtained with the model are higher than those obtained by measurement. On the other hand, it very clearly appears a vanishing of fundamental (and harmonics) oscillations of the thickness mode.

From the preceding study, it emerges that mismatches exist between real measurement and simulation. In fact, the models are valid only for a limited part of the frequency spectrum (fig. 4 and fig. 6). But what happen if the two modes are mixed? It could be the case when ceramics' thicknesses increase regarding the diameter. For this reason, in the next section a new model is presented. This model will be able to manage, together, the two modes of operation on the entire frequency spectrum of ceramic's use.

3. THE NEW UNIFIED MODEL

As we have just seen it, the two modes of vibration are independently considered in two distinct models, each one of them treating only part of the real component operation. If one wants to simulate the real behavior of ceramics, it is necessary to integrate in a one and only model both the planar mode and the thickness mode. The proposed model must be able to take into account both the coupling between the planar and thickness modes and the mechanical interactions of the disk with the media on major faces and on curved surface. To this end, Iula et al. [13] demonstrate that the voltage v across a disk ceramic is expressed by the equation (1):

$$v = \frac{h_{33}}{jw}(u_1 + u_2) + \frac{kh_{31}}{jw}(u_3 + u_4) + \frac{i}{jwC0} \qquad (1)$$

Where:

$$k = \frac{2d}{r}; \qquad u_{1,2} = \sqrt{\frac{C_{33}^D}{\rho}}; \qquad u_{3,4} = \sqrt{\frac{C_{11}^D}{\rho}}$$

The equation (1) can be written:

$$i = jwC0u - h_{33}C0(u_1 + u_2) - kh_{31}C0(u_3 + u_4) \quad (2)$$

Equation (2) along with the Kirchoff's current law allows an equivalent electrical representation (Fig. 7). This is achieved by association of one capacitor C0 and two current-controlled current sources i_1 and i_2.

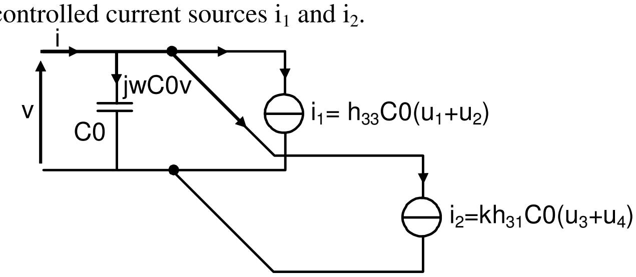


Fig. 7: Electrical part of the unified model

The overall current i which is flowing through the ceramic's electrodes is related to the particle velocity as described in (2) by u_1, u_2, u_3 and u_4. The force applied in each ceramic's surface is linked to the current by the equation f=h.i/jw (3) where h, the piezoelectric constant, takes the value of h_{33} or h_{31} depending of which mode is involved.

In the case of thickness mode, substituting equation (2) in (3):

$$f_{thickness} = h_{33}C0\left[v - \frac{h_{33}}{jw}(u_1 + u_2) - \frac{kh_{31}}{jw}(u_3 + u_4)\right] \quad (4)$$

Equation (4) is reorganized to reveal the negative capacitor –C0:

$$f_{thickness} = h_{33}C0\left[v - \frac{1}{jwC0}h_{33}C0(u_1 + u_2) - \frac{1}{jwC0}kh_{31}C0(u_3 + u_4)\right] \quad (5)$$

Likewise, for the planar mode the force can be expressed by:

$$f_{planar} = h_{31}C0\left[v - \frac{1}{jwC0}kh_{31}C0(u_3 + u_4) - \frac{1}{jwC0}h_{33}C0(u_1 + u_2)\right] \quad (6)$$

Where $x = -\frac{1}{jwC0}kh_{31}C0(u_3 + u_4)$ and

$y = -\frac{1}{jwC0}h_{33}C0(u_1 + u_2)$ are the potential across the negative capacitance -C0 respectively.

According to (5) and (6), the electro-mechanical part of the model is achieved. In Fig. 8, the two previously defined current-controlled current sources i_1 and i_2 are still present and two new voltage-controlled voltage sources e_1 and e_2 are included, each one driving a vibration mode.

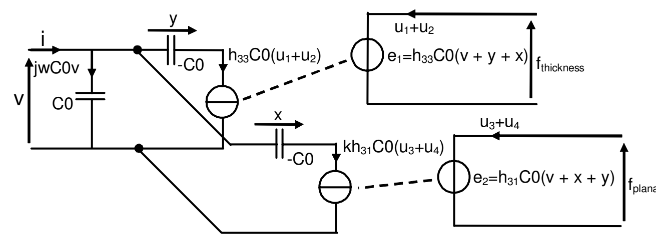


Fig. 8: Electro-mechanical part of the unified model

Finally, two transmission lines are added to symbolize the mechanical part of the ceramic (Fig. 9). Each one of it takes into account the acoustic wave propagation according to a privileged direction, planar or thickness. Consequently, it is possible to connect 4 independent acoustic forces to each surfaces of the transducer, i.e., F1, F2, F3 and F4.

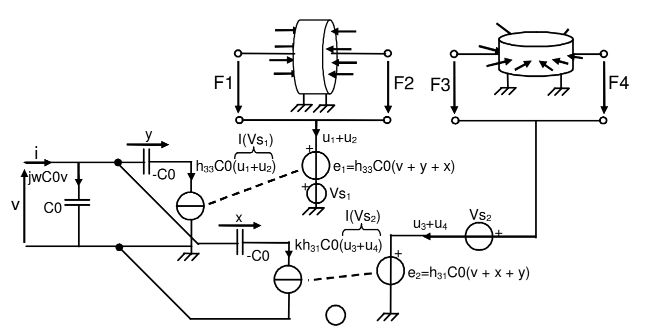


Fig. 9: The unified model

Like previous models, two zero valued sources are added Vs_1 and Vs_2 in the schematic. They are used to compute the "current" values (u_1+u_2) and (u_3+u_4) and have no effect on the circuit operation.

4. EXPERIMENT VERSUS SIMULATION

In order to obtain an experimental validation of the proposed model, the same impedance measurement than those performed previously for the thickness and planar modes is achieved. Fig. 10 shows the measured and simulated amplitude of the input impedance for a 2mm thick ceramic disk. A wide frequency spectrum is chosen to demonstrate the good agreement between the model and the dual intrinsic resonance of the ceramic.

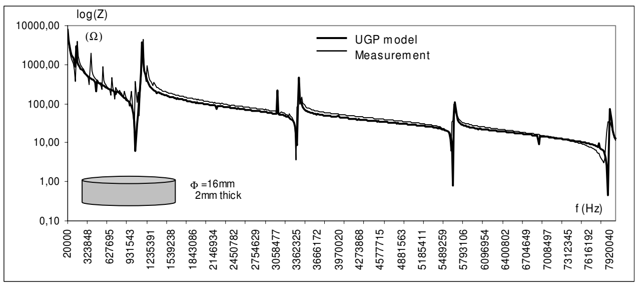


Fig. 10: Impedance comparison between experiment and simulation for the unified model (2mm thick)

Again, Fig. 11 shows the same comparison than the previous one but for a thinner ceramic disk. Here also a good agreement is observed (better for the first planar than for the first thickness resonance frequency). The small difference found for the first thickness harmonic come from the fact that the two electrical Ni/Au contact layers are not represented in the model. To this end, the model can be easily enhanced with the adjunction of two appropriated transmission lines instead of the conducting layers [7].

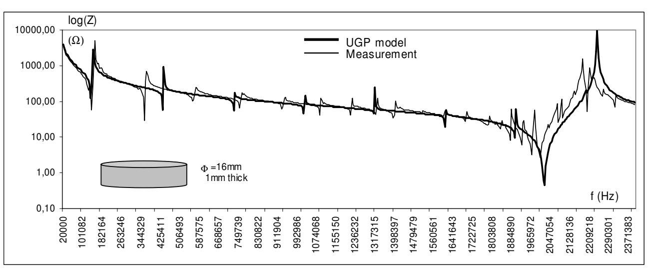


Fig. 11: Impedance comparison between experiment and simulation for the unified model (1mm thick)

5. CONCLUSION

In this work, we have developed a comprehensive method for the development of a new Spice model of cylinder shaped piezoceramic elements, based on the analysis of the electromechanical behavior of the PZT ceramic. This model is valid for any diameter to thickness ratio. It describes the electromechanical coupling between thickness and planar modes by coupling electrically and mechanically in explicit form the two dimensional vibration. In order to validate this unified spice model and to determine the accuracy of results, an experimental validation of the model has been carried out. Calculated impedance versus frequency is then successfully compared with measured values.

REFERENCES

[1] W. P. Mason, "Electromechanical Transducers and Wave Filters," Princeton, NJ: Van Nostrand, 1948, pp. 201-209, 399-404

[2] M. Redwood, "Transient Performance of a Piezoelectric Transducer," J. Acoust. Soc. Amer., vol. 33, no 4, April 1961, pp527-536

[3] http://bwrc.eecs.berkeley.edu/Classes/IcBook/SPICE/

[4] http://www.cadence.com/

[5] S. A. Morris and C. G. Hutchens, "Implementation of Mason's Model on Circuit Analysis Programs," IEEE Trans. On Ultrasonics, Ferroelectrics and Frequency Control, vol. UFFC-33 no 3, May 1986, pp295-298

[6] W. M. Leach, "Controlled-Source Analogous Circuits and SPICE Models for Piezoelectric Transducers," IEEE Trans. On Ultrasonics, Ferroelectrics and Frequency Control, vol. 41 no 1, January 1994, pp60-66

[7] J. Van Deventer and T. Löfqvist, "PSPICE Simulation of Ultrasonic Systems," IEEE Trans. On Ultrasonics, Ferroelectrics and Frequency Control, vol. 47 no 4, July 2000, pp1014-1024

[8] IEEE Standard on Piezoelectricity, IEEE, New York, 1987

[9] W. P. Mason, "Physical Acoustics," New York, Academic Press, vol. 1 part A, 1964

[10] A. Püttmer et al., "SPICE Model for Lossy Piezoceramic Transducers," IEEE Trans. On Ultrasonics, Ferroelectrics and Frequency Control, vol. 44 no 1, January 1997, pp60-65

[11] http://cp.literature.agilent.com/litweb/pdf/5968-3809E.pdf

[12] http://www.ferroperm-piezo.com

[13] A. Iula, N. Lamberti and M. Pappalardo, "An Approximated 3-D Model of Cylinder-Shaped Piezoceramic Element for Transducer Design," IEEE Trans. On Ultrasonics, Ferroelectrics and Frequency Control, vol. 45 no 4, July 1998, pp1056-1064