AUTOMATED PERFORMANCE OPTIMISATION AND LAYOUT SYNTHESIS OF MEMS ACCELEROMETER WITH SIGMA-DELTA FORCE-FEEDBACK CONTROL LOOP

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Abstract

This contribution presents a novel methodology for automated optimal design of a MEMS accelerometer with Sigma-Delta force-feedback control loop from user defined high-level performance specifications and design constraints. The proposed approach is based on a simulation-based optimization technology using a genetic algorithm. The layout of the mechanical sensing element is generated simultaneously with the optimal design parameters of the Sigma-Delta control loop. As currently available implementations of AMS HDL languages are not suitable for complex mixed-technology system optimisation, the algorithm as well as aa fast dedicated sigma-delta accelerometer simulator have been implemented in C++. The underlying accelerometer model includes the sense finger dynamics described by a partial differential equation, which enables accurate performance prediction of the sensing element embedded in a in mixed-technology control loop.

1. Introduction

Usually, MEMS accelerometer design requires a significant amount of specialist human resources and time in the iterative trial-and-error design process to determine the crucial trade-offs in meeting the performance specifications. Some methodologies have proposed an automated approach to MEMS accelerometer synthesis [1] [2] [3] [4] where the design requirements are formulated as a numerical nonlinear constrained optimization problem, and solved with powerful optimization techniques. However, these approaches are constrained to open-loop or analogue feedback loop accelerometer design. Our goal is not only to automate the layout generation of the mechanical sensing element but also to obtain optimal system-level parameters in the Sigma-Delta force feedback control loop. In particular, our design methodology uses accurate modelling of the sense finger dynamics and a simulation-based optimization to improve the performance of a system-level digital accelerometer. Our approach is hollistic in the sense that the optimal design parameters and geometrical layout of the mechanical sensing element based are generated simultaneously.

Due to their relatively high resolution and low temperature sensitivity, capacitive accelerometers are widely used in various industrial applications. The commonly used lateral capacitive sensing element topology [5] [6] has been selected for this project and its layout and properties are discussed in the next section.

High-performance MEMS usually employ an electromechanical Sigma-Delta modulation feedback control to improve the performance and, in particular, to obtain a wider bandwidth, a larger dynamic range and also to convert the acceleration signals directly to a bit stream suitable for further digital processing [7] [8] [9] [10] [11].

It has been observed that the sense finger resonance, usually not included in standard models, affects the performance of the electromechanical Sigma-Delta feedback loop [12]. The conventional approach normally applied in simulations of such systems, where a 2nd order lumped Mass-Damper-Spring equation is used to model the mechanical sensing element, cannot capture the effect of the sense finger dynamics. In this contribution a distributed approach, where the sense fingers are modelled as cantilever beams whose motion can be described by Partial Differential Equations(PDEs), has been applied to reflect the effects of the sense finger dynamics.

2. Mixed-technology model of the system

A block diagram of the entire system is shown in Figure 1 and the layout of mechanical sensing element is shown in Figure 2. In the sensing element, the proof mass is suspended by two springs and it is equipped with sense and force comb fingers which are placed between fixed fingers to form a capacitive bridge. Such constructed mechanical sensing element can detect a differential change in the capacitance, which is caused by the displacement of sense fingers, and convert it to voltage (Vout). The electrostatic force acting on the force fingers is used as the feedback signal to pull the proof mass in the desired direction. V_{f1} and V_{f2} are the feedback voltages obtained from the DAC and $V_m(t)$ is a high frequency modulation voltage:

$$V_m(t) = V_{mA} \sin(2\pi f_m t) \quad (1)$$

where V_{mA} is the amplitude voltage and f_m is the modulation frequency. The lead compensator is required to ensure the stability of the control loop. A clocked 1-bit quantizer is used for oversampling and generating a pulse-density modulated digital output signal.

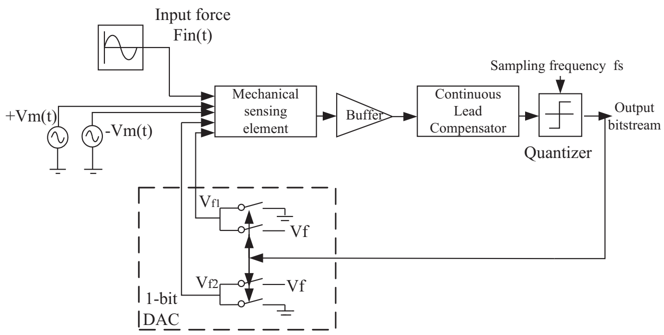


Figure 1. Electromechanical Sigma-Delta accelerometer

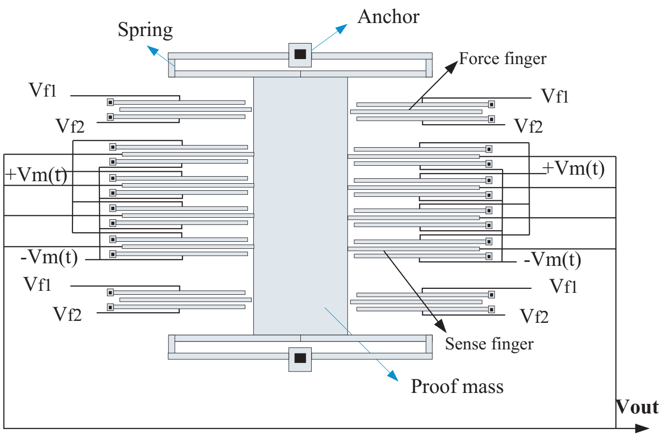


Figure 2. Lateral capacitive sensing element

Conventionally, the mechanical sensing element in figure 1 is a mass-damper-spring system modelled by a 2nd order differential equation:

$$f(t) = M\frac{d^2x}{dt^2} + D\frac{dx}{dt} + Kx \quad (2)$$

where $f(t)$ represents the input and feedback forces, M is the the proof mass, x is the deflection of the proof-mass, D and K are the damping coefficient and spring constant respectively.

In reality however, the sense element fingers may vibrate due to their own dynamics, thus rendering the feedback excitation ineffective, causing an incorrect output and a failure of the system [12]. This scenario cannot be reflected by the conventional model given by equation (2).

A more accurate, distributed mechanical sensing element model, which can exhibit higher resonant frequencies, is illustrated in Figure 3 where the electrostatic force acts along the length of the finger beam. C_1 and C_2 are the total distributed differential capacitances between the beam and the electrodes.

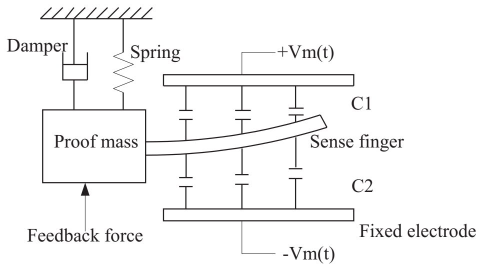


Figure 3. Sensing element distributed model.

The motion of the beam can be modelled by the following partial differential equation (PDE):

$$\rho A\frac{\partial^2 y(x,t)}{\partial t^2} + C_D I\frac{\partial^5 y(x,t)}{\partial x^4 \partial t} + EI\frac{\partial^4 y(x,t)}{\partial x^4} = F_e(x,t) \quad (3)$$

where $y(x,t)$, a function of time and position, represents the beam deflection, E, I, C_d, ρ, A are physical properties of the beam: ρ is the material density, A is the cross sectional area ($W_f * T$, where W_f and T are width and thickness of the beam), E is Young's modulus and I is the second moment of area which could be calculated as $I = TW_f^3/12$ and C_d is the internal damping modulus. The product EI is usually regarded as the stiffness .

$F_e(x,t)$ is the distributed electrostatic force along the beam:

$$F_e(x,t) = \frac{1}{2}\varepsilon L_f T\left[\frac{V_{mA}^2}{(d_0 - y(x,t))^2} - \frac{V_{mA}^2}{(d_0 + y(x,t))^2}\right] \quad (4)$$

where ε is the permittivity, L_f is the length of sense finger and d_0 is the initial gap between sense finger and fixed electrodes.

The root ends of the sense fingers move with the lumped proof mass whose deflection could be modelled by the 2nd order differential equation similar to equation (2):

$$M\frac{d^2z(t)}{dt^2} + D\frac{dz(t)}{dt} + Kz(t) = F_f(t) + F_{in}(t) \quad (5)$$

The total distributed sense capacitance between the sense fingers and electrodes is:

$$C_1(t) = N_s \varepsilon T \int_0^{L_f} \frac{1}{d_0 - y(x,t)} dx \quad (6)$$

$$C_2(t) = N_s \varepsilon T \int_0^{L_f} \frac{1}{d_0 + y(x,t)} dx \quad (7)$$

Where N_s is the number of sense fingers. The output voltage can be calculated as:

$$V_{out}(t) = \frac{C_1 - C_2}{C_1 + C_2} V_m(t) \quad (8)$$

Like in the conventional model, the lower resonant mode is caused by the dynamics of the structure mass, when the sense finger and lumped mass move together. The resonant frequency is approximately $\omega_0 = \sqrt{K/M}$ where the K is the suspension spring constant and M is the total mass of the lumped mass and sense fingers. The higher resonant mode is related to the sense finger resonance. If it occurs, the fingers bend significantly while the lumped mass has a small deflection.

3. Performance optimisation algorithm and experimental results

The proposed automated design approach explores the design according to user defined specifications and optimises the structural parameters of the sensing element and the Sigma-Delta control loop parameters. The design variables are listed in table 1. For the mechanical sensing element, proof mass, comb fingers and springs have the same thickness(T). The numbers of sense(Ns) and force(Nf) rotor fingers represent the structural parameters.

3.1 Initial design

The initial design parameters are shown in table 2. The sense finger resonance may affect the performance of the Sigma-Delta control loop as fingers might bend significantly. The first two resonant frequencies could be calculated as those of the cantilever beam [12] :

$$\omega_i = \alpha_i^2 \frac{W_f}{L_f^2} \sqrt{\frac{E}{12\rho}} \quad \alpha 1 = 1.875, \alpha 2 = 4.694 \quad (9)$$

Here we perform parameter sweeps to analyze the effect of the sense finger resonance. Figure 5 shows the result of sweeping the length of sense finger from 100um to 230um. SNR changes with the length of fingers and a failure of the $\Sigma\Delta$ control loop is captured when the length is above 210um. Figure 4 presents the power spectral density (PSD) of the output bitstream. However, this effect cannot be captured in conventional model.

Design Variables of sensing element		***range***
Wpm	Width of proof mass	50um-150um
Lpm	Length of proof mass	300um-700um
Ls	Length of spring	200um-300um
Ws	Width of spring	1um-5um
Lf	Length of fingers	50um-200um
Wf	Width of fingers	0.8um-2um
d0	Initial gap	1um-3um
T	Thickness of sensing element	2um-8um
Structural parameters		***range***
Ns	Number of sense fingers	10-30
Nf	Number of force fingers	5-20
Sigma-Delta control loop parameters		***range***
V_{mA}	Amplitude of modulation voltage	1V-5V
ZERO	Zero of lead compensator	0.01-10
POLE	Pole of lead compensator	100-20000
Vf	Feedback force voltage	1V-10V

Table 1. Parameters of Sigma-Delta accelerometer

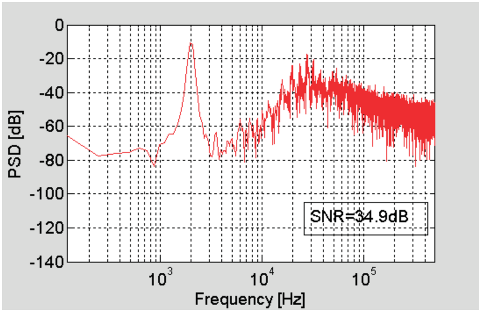
3.2 Optimization algorithm

The automated optimal design process is shown in figure 6. Genetic algorithm is used in combination with a dedicated behavioral MEMS accelerometer model to optimise the objective function which in these experiments is selected to either maximize the signal to noise ration (SNR) or the static sensitivity. The input specifications are the geometrical constraints of the sensing element, control loop parameter ranges and performance specifications. The algorithm generates an optimized layout of the sensing element and parameters of the $\Sigma\Delta$control loop.

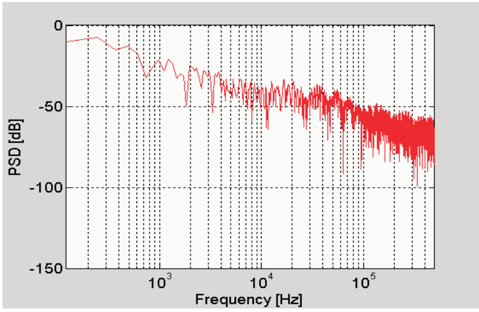
3.3 Fitness Function and experimental results

The proposed approach is demonstrated by two experiments. In Experiment 1 the fitness function (optimisation goal) is to maximise the SNR, and in Experiment 2 - the static sensitivity. In both experiments the input performance constraints are:

1) *Signal-to-noise ratio: SNR>30dB*



(a) Power spectrum density of output bitstream (sense finger length=120um)



(b) Power spectrum density of output bitstream (sense finger length=210um)

Figure 4. Power spectrum density analysis

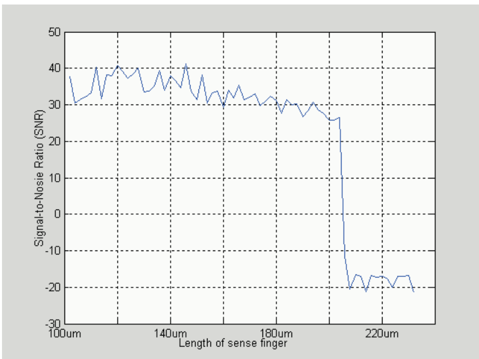


Figure 5. Resonance of sense fingers affect the performance of $\Sigma\Delta$ loop. $\Sigma\Delta$ control loop failure is captured when the sense finger length exceeds 210um

It is the ratio of the total power in the signal bins to the total power in the noise bins [13]:

$$SNR = \frac{Pw_{signal}}{Pw_{noise}} \tag{10}$$

where Pw_{signal} and Pw_{noise} are the signal and noise power in band respectively.

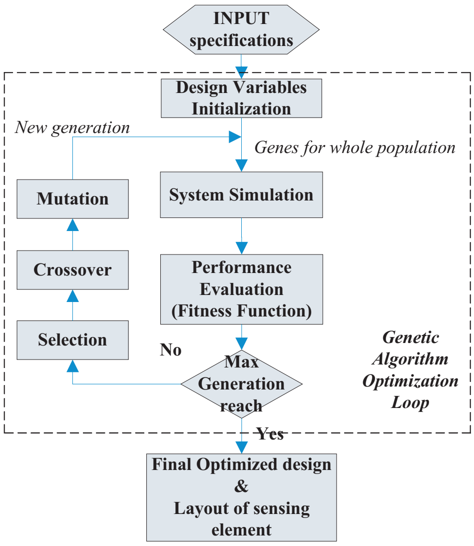


Figure 6. Synthesis process of proposed approach

2)*Static sensitivity of mechanical sensing element: S>1.5fF/g*

$$\Delta C = C_0\left(\frac{d_0}{d_0 - \Delta X} - \frac{d_0}{d_0 + \Delta X}\right) \tag{11}$$

$$C_0 = N_s \frac{\varepsilon T L_f}{d_0} \tag{12}$$

Where C_0 is the static capacitance and ΔX is the displacement of the sense fingers when applying 1g ($1g = 9.8m/s^2$) acceleration.

The search for a solution is guided by an optimization objective. The objective fitness function is in the following format:

$$F_{fit} = K * \frac{Objective}{Objective_r} \tag{13}$$

where K is a constant whose value depends on whether the performance constraints are met. K is set to 100 if these constraints both are met, otherwise K is 0.01. Objective is static sensitivity or SNR obtained from each simulation while $Objective_r$ are the user defined reference value. SNR_r and S_r are the reference SNR and static sensitivity got from user defined constraints.

Experiment 1 : maximum SNR

Fitness functions:

$$F_{fitSNR} = K * \frac{SNR}{SNR_r} \tag{14}$$

where the reference SNR:

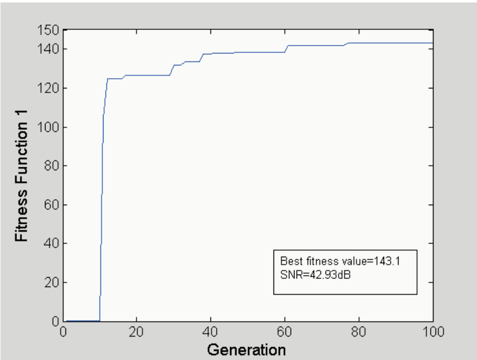
$$SNR_r = 30dB \tag{15}$$

Experiment 2 : maximum static sensitivity
Fitness functions:

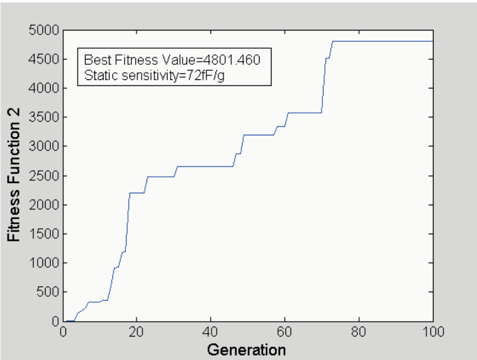
$$F_{fitS} = K * \frac{S}{S_r} \tag{16}$$

where the reference static sensitivity:

$$S_r = 1.5fF/g \tag{17}$$



(a) Experiment 1: maximum SNR



(b) Experiment 2: maximum static sensitivity

Figure 7. Fitness improvement

Optimizations were carried out using the following design parameters:
1) Oversampling ratio: OSR=64
2) Sampling frequency: f_s=1MHz
3) Input force:

$$F_{in}(t) = F_{Amp} \sin(2\pi f_{input} t) \tag{18}$$

where F_{Amp}=100g*M ($1g = 9.8m/s^2$) and f_{input}=2KHz.

The fitness functions for both experiments are shown in figure 7 and optimisation results in table 2. As expected, the structure optimised for maximum sensitivity has more and longer sense fingers. The layout of the mechanical sensing element, which is also generated by the system, is shown in figure 8.

There are many crucial trade-offs in the MEMS accelerometer design. For example, static sensitivity is dependent on the length and number of the sense fingers. However, the performance of sigma-delta modulation may be severely affected by the length of sense fingers to the extent that a complete failure of the sigma-delta control may occur when the fingers are too long. The maximum number of fingers is also limited by the length of proof mass. To maintain the same resonant frequency, the finger width should be reduced if the length of the proof mass increases. This results in a sensitivity decrease. The presented optimization-based approach deals with these trade-offs effectively for a given choice of the design objectives.

4. Conclusion

This paper presents an effective simulation-based approach for synthesis of MEMS accelerometer in Sigma-Delta control loop. Due to the complex nature of the optimisation process, the algorithm has been implemented in C++ to overcome limitations of the available AMS HDL tools. While these tools are extremely well suited for complex modelling, implementation of post-processing of simulation results and optimisation algortihms is difficult. Future work will extend the digital MEMS accelerometer design to multi-objective optimisation and an automated synthesis of the control loop where higher order Sigma-Delta systems will be used to maximise the performance.

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Design Variables	***Initial Design***	***Static sensitivity optimized design***	***SNR optimized Design***	***Design Variables***	***Initial Design***	***Static sensitivity optimized design***	***SNR optimized Design***
T	6um	7.93um	7.66um	d0	1um	1um	1um
Wpm	136um	148.6um	91um	Ns	30	30	26
Lpm	468um	641.2um	433.6um	Nf	20	18	12
Ls	300um	298.6um	267um	Vm	4V	2.5V	3.7V
Ws	2um	1um	1.38um	ZERO	3.4	4.26	1.0
Lf	130um	186.7um	107.15um	POLE	777	1.69e+4	597
Wf	1.85um	1.85um	1.3um	Vf	4V	5V	5.4V
Device Parameters				***Device Parameters***			
Spring constant	0.881N/M	0.177N/M	0.511N/M	Damping coefficient	45.7 uN(m/s)	142.5 uN(m/s)	69.27 uN(m/s)
Mass	0.899ug	1.76ug	0.7046ug	SNR	34.95dB	30.4dB	42.9dB
Static sensitivity	3.84fF/g	72fF/g	5.65fF/g	Static capacitance	207fF	393fF	189fF
F_r	5KHz	1.6KHz	4.29KHz				

Table 2. Automated optimized design results

(a) Initial Design
SNR=34.95dB
Static Sensitivity=3.84fF/g

(b) Optimization design of experiment 1
(Static sensitivity optimized design)
SNR=30.4dBdB
Static Sensitivity=72fF/g

(c) Optimization design of experiment 2
(SNR optimized design)
SNR=42.9dBdB
Static Sensitivity=5.65fF/g

Figure 8. Synthesized layouts. Comparing with SNR optimized design, sensitivity optimized design contains more and longer fingers.

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