VHDL-AMS Behavioural Modelling of a CMUT Element

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ABSTRACT

This paper reports the development, implementation and simulation of a behavioural model written in VHDL-AMS for a capacitive micromachined ultrasonic transducer (CMUT). Unlike previous behavioural models, this model incorporates the non-linear electro-mechanical relations of the CMUT. VHDL-AMS was chosen for the ease with which it can be used to implement these relations, and for its portability between software platforms. To the best of the authors' knowledge, this is the first reported VHDL-AMS model of a CMUT. The simulation results demonstrate that the model can be used to simulate a number of scenarios, including frequency responses, time responses and effects of DC bias voltage. Results are compared to a finite element method model, and show excellent agreement in resonant frequency and a 16 % error in pull-in voltage.

1. INTRODUCTION

Capacitive micromachined ultrasonic transducers (CMUT) are a relatively new transducer technology that has emerged from the ultrasonics and micro-electro-mechanical systems (MEMS) research communities. A CMUT element is made up of numerous cells, each of which consists of a thin membrane suspended above a substrate. An electrostatic force generated by a voltage applied between membrane and substrate perturbs the membrane from its resting position. By applying a suitable time-varying voltage superimposed on a fixed DC bias, the membrane can be made to vibrate at its resonance in the ultrasonic frequency range, thus emitting a pressure wave into the surrounding medium. Conversely, pressure waves from the medium incident on the membrane cause it to vibrate, which, if the fixed DC bias is maintained, generates a time-varying current that can be measured electronically. CMUT are therefore a viable substitute for piezoelectric transducers used currently in most ultrasound imaging systems.

CMUT have two primary advantages over piezoelectric transducers. Firstly, research literature (e.g. [1]) shows that CMUT can perform better than piezoelectric transducers in terms of bandwidth and sensitivity, giving them the ability to generate ultrasound images of greater resolution and thus discern smaller structures. This would be a fundamental improvement for ultrasonic imaging, and particularly beneficial for its use in cancer diagnosis and treatment. The second advantage lies in the ability of CMUT to be fabricated using technologies similar to those used to make integrated circuits (IC). This allows CMUT to be potentially fabricated together with CMOS driving, receiving and signal processing electronics on a single IC, and could greatly reduce the wiring needs between the ultrasound probe, containing the IC, and the rest of the ultrasound system. The wiring requirements of large 2D transducer arrays are currently a significant limitation in the development of high-resolution, real-time 3D ultrasonic imaging systems [2]. Using standard IC fabrication methods could also reduce the cost of fabricating ultrasonic imaging systems.

In this work, a behavioural model is developed for a CMUT element. Such a model allows the characteristics and performance of CMUT designs to be quickly tested and optimised, before performing lengthy simulation of finite-element method (FEM) models or investing in fabrication. It can also be incorporated into a macromodel along with models of the driving and receiving electronics. This is a particular advantage in the case where the CMUT and the electronics are to be fabricated on the same IC, as it allows the whole IC to be simulated together and gives the ability to optimise the parameters of the CMOS layout of the electronics along with those of the CMUT.

Previous behavioural models of CMUT [1, 3] have been based on a small-signal linearisation of the electro-mechanical relations about a DC bias point, allowing a 2-port electrical equivalent circuit to be formulated. Generally, in these models, a fixed capacitor represents the capacitance of the parallel electrodes of the CMUT, an ideal transformer performs the conversion of energy from the electrical to the mechanical domain, and a complex impedance models the mechanical impedance of the membrane. The work of Mason [4] is typically used to obtain an expression for this mechanical impedance. The approach was extended in [5] to account for the effects of mechanical interactions between the membrane and the gap beneath it. A PSpice (Cadence OrCAD, San Jose, CA, USA) implementation of this model was also developed [6].

The behavioural model developed in this work differs from previous models in that it does not rely on a linearised small-signal model. The inherently non-linear relations of the electro-mechanical coupling are preserved. The model is also bi-directional, allowing it to be used as a transmitter and a receiver. This is an advantage of the energy-domain modelling approach used in this work over signal-flow models such as those constructed in Simulink (The MathWorks, MA, USA). The model is implemented in VHDL-AMS (IEEE Std 1076.1-1999), and to the best of the authors' knowledge, this is the first reported model of a CMUT written in VHDL-AMS. The advantage of this implementation is that, being a well-recognised standard, it is compatible with various software platforms, allowing the CMUT to be simulated in different environments.

The next section details the development of the CMUT model and its implementation in VHDL-AMS. This is followed by results of simulations of a CMUT design, with comparison to results from simulations of a FEM model of the same design.

2. MODELLING

2.1 CMUT Structure

The basic structure of the cross-section of the CMUT cell modelled in this work is shown in Figure 1. Assuming the membrane to be circular, *a* is its radius, and d_m, d_g and d_{in} are the thicknesses of the membrane, gap and insulating layer, respectively.

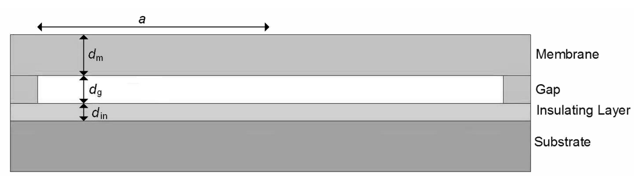

Figure 1: Basic cross-section of a CMUT cell.

The voltages used with the CMUT are applied between the membrane and substrate, although individual electrode layers may be used instead. The membrane is typically formed in polysilicon or silicon nitride. The gap may be open to air, or fully sealed, with the internal pressure depending on the pressure of the environment in which the gap was sealed. The insulating layer may or may not be present. Many different fabrication techniques and geometries have been reported in the literature, but the generalised structure of Figure 1 is used herein for modelling purposes. The numerous cells making up each CMUT element are electrically connected in parallel and are assumed to vibrate synchronously.

2.2 CMUT Model

To derive a behavioural model, the CMUT element is treated as a moveable plate capacitor with one degree-of-freedom. The moveable plate is represented by a mass attached to a mechanical spring and damper system, which represents the elasticity of the membrane and the acoustic impedance of the medium into which ultrasound is to be transmitted. Figure 2 shows a diagram of this system. Here, *x* is the displacement of the membrane from equilibrium and is assumed to be uniform over the whole membrane, resulting in piston-like movement of the membrane. The piston-like model of the CMUT reduces the model to one single mechanical degree-of-freedom, taken here as the displacement *x* of the centre of the membrane. This choice of the mechanical port influences the computation of the effective mass and effective spring constant of the model. $C(x)$ is the capacitance of the CMUT. The force exerted on the medium by the membrane is *f*, while *v* is the membrane velocity. m_o is the effective mass of the membrane, *k* is the effective spring constant, and *B* is the damping caused by the acoustic medium.

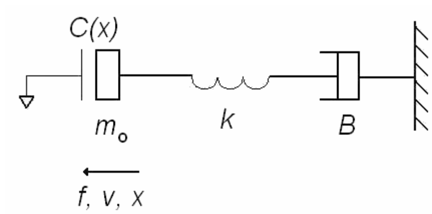

Figure 2: Diagram of the CMUT as a moveable plate capacitor attached to a mechanical spring-damper system.

The effective mass is a lumped parameter representing the inertial force of the membrane, and is given by

$$m_o = 0.613 \rho_m d_m A \,, \tag{1}$$

where $A = \pi a^2 N$ and ρ_m is the density of the membrane material. *N* is the number of cells per CMUT element. The factor of 0.613 accounts for the fact that the membrane will not move in a piston-like way, but in a bowl-like shape, with the centre deflecting more than points at the outer radius. It was determined by equating the fundamental resonant frequency of a mass-spring-damper system with that derived from consideration of harmonic mode shapes [7].

The following expression from [8] is used for the effective spring constant.

$$k = \frac{16 \pi Y_m d_m{}^3}{3a^2 (1 - \sigma_m{}^2)} N \,. \tag{2}$$

Here Y_m and σ_m are the Young's modulus and Poisson's ratio of the membrane material, respectively. The relation of the damping coefficient, *B*, to the characteristic acoustic impedance of the medium, Z_0, is

$$B = Z_0 A \,. \tag{3}$$

The variable capacitance, $C(x)$, is given by

$$C(x) = \frac{\varepsilon_0 A}{(d_0 - x)} \,, \tag{4}$$

where ε_0 is the permittivity of free space and

$$d_0 = d_g + \frac{d_{in}}{\varepsilon_{in}}, \tag{5}$$

with ε_{in} the relative permittivity of the insulating material.

The relationship between the electrical parameters – voltage u and current i – and the mechanical parameters, f and v, can be obtained by considering a two-port model for a moveable plate capacitor, as shown in Figure 3.

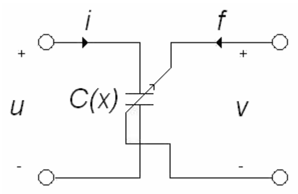

Figure 3: Two-port schematic of a moveable plate capacitor.

Using conservation of energy, the following two equations can be derived to describe the relations between the electrical and mechanical parameters.

$$i = C(x)\frac{du}{dt} + \frac{\varepsilon_0 A}{(d_0 - x)^2} vu\ . \tag{6}$$

$$f = -\frac{\varepsilon_0 A}{2(d_0 - x)^2} u^2\ . \tag{7}$$

Equations (6) and (7), along with the mechanical parameters of equations (1), (2) and (3), result in the equivalent circuit of Figure 4. In this circuit, C_m and L_k are the electrical equivalents of the mechanical parameters of Figure 2. Expressions for their values are shown. Note that force is the through variable (equivalent to current) while velocity is the across variable (equivalent to voltage). The circuit exhibits many non-linearities and irregular components, making it difficult to implement with standard SPICE electronic component models. Instead, VHDL-AMS was used, as equations (1)-(7) could be implemented directly. The model therefore retains the non-linearities of the electro-mechanical relationships of CMUT.

A FEM model of the CMUT was created using COMSOL Multiphysics (COMSOL AB, Stockholm, Sweden). It was used where possible to give some verification of the behavioural modelling results.

3. SIMULATION

3.1 CMUT Design

A CMUT design was created to allow simulation of a realistic structure. It was designed to be compatible with the PolyMUMPs (MEMSCAP, Inc., NC, USA) process, a precise and well-established fabrication technology. The design makes novel use of the PolyMUMPs technology to obtain the layer thicknesses and parameters given in Table 1.

Parameter	**Value**
a	32 µm
d_m	1.5 µm
d_g	0.75 µm
d_{in}	0.6 µm
Membrane material	Polysilicon
Insulation material	Silicon nitride
N	118

Table 1: Values for CMUT parameters used in simulations.

3.2 CMUT Simulation

The software package Simplorer (Ansoft, LLC, Pittsburgh, PA, USA) was used to perform simulations of the CMUT, as it allows VHDL-AMS models to be imported and integrated with electrical components from its standard libraries. The CMUT VHDL-AMS model was imported as a two-port component. In transmission simulations, a voltage source, representing the driving electronics, was connected to its electrical port, and a resistive load – representing the damping due to the acoustic medium – was connected to its mechanical port. The schematic is shown in Figure 5. Because force and velocity are analogous to current and voltage, respectively, the electrical load resistance, R_L, used in the schematic of Figure 5 is inversely related to the damping coefficient, B, and the characteristic acoustic impedance, Z_0, of the medium. Using (3), the relationship is therefore

$$R_L = \frac{v_{out}}{f_{out}} = \frac{1}{B} = \frac{1}{Z_0 A}\ . \tag{8}$$

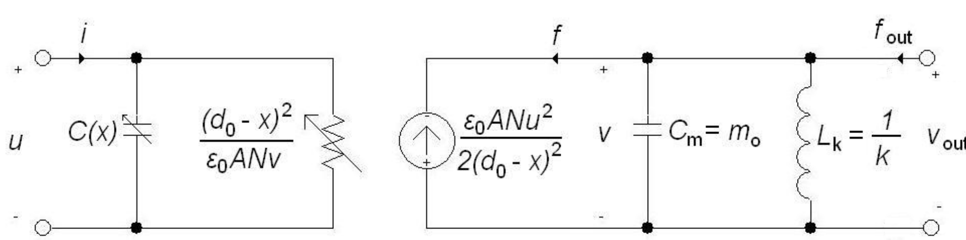

Figure 4: Full electrical equivalent circuit for CMUT.

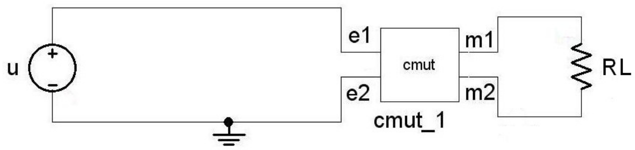

Figure 5: Simplorer circuit used for transmission simulations.

Firstly, the frequency response of the CMUT was determined using an AC analysis. The voltage source *u* was set to generate a sinusoid of amplitude 20 V superimposed on a 90 V DC bias. The load R_L was set to 6.27 kΩ and 1.76 Ω for air and water transmission simulations, respectively. The resulting output power density frequency responses are shown in Figure 6. In the air response, the behavioural model gives a sharp resonant peak at 5.78 MHz. The water response has a much wider bandwidth – approximately 115 MHz – without a distinct resonant peak. The peak power density is much lower when transmitting into water. The FEM model was used to perform an eigenfrequency analysis of the CMUT resonant modes. This gave a fundamental resonant frequency of 5.85 MHz.

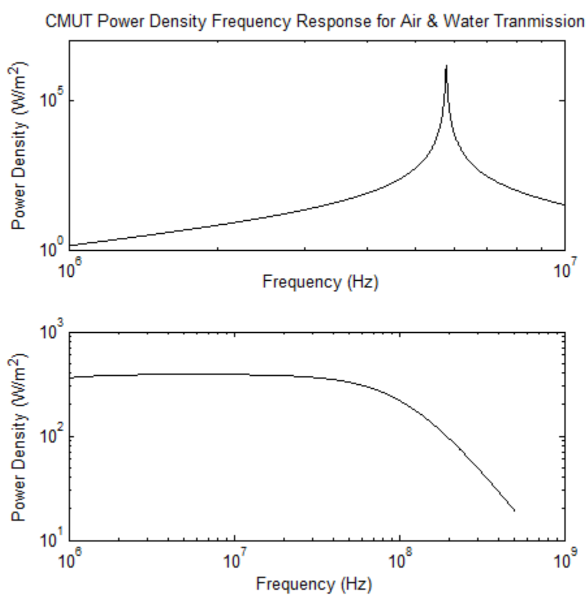

Figure 6: Power density frequency responses of CMUT for transmission into air (top) and water (bottom).

The FEM model was also used to obtain the resonant mode shapes. These mode shapes, the first two of which are shown in Figure 7, illustrate the higher-order vibrational resonances of the membrane. The behavioural model, however, is only valid for the fundamental mode.

Because of its non-linear formulation, the behavioural model can be used to observe the displacement of the membrane due to the DC bias voltage, and to investigate the pull-in phenomenon. Pull-in occurs when the electrostatic force due to the applied voltage outweighs the elastic

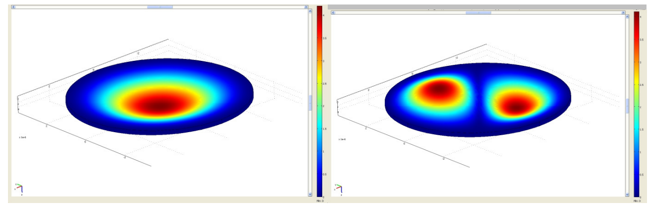

Figure 7: First two resonant modes of the CMUT membrane. The colour bar shows small (blue) to large (red) displacement.

restoring forces, causing the membrane to collapse onto the substrate. The displacement of the membrane with increasing DC bias voltage was simulated in Simplorer, and the result is shown in Figure 8, along with the FEM result. For the behavioural model, the large discontinuity indicates the pull-in voltage occurs at approximately 230 V. At higher voltages, the given displacement exceeds the value of d_g, indicating an invalid solution. The pull-in displacement is approximately 0.265 μm. In comparison, the FEM model plot gives a pull-in voltage of 275 V and a pull-in displacement of 0.315 μm. Note that the FEM software terminates the simulation when an invalid result is generated at pull-in, and so the plot simply stops here.

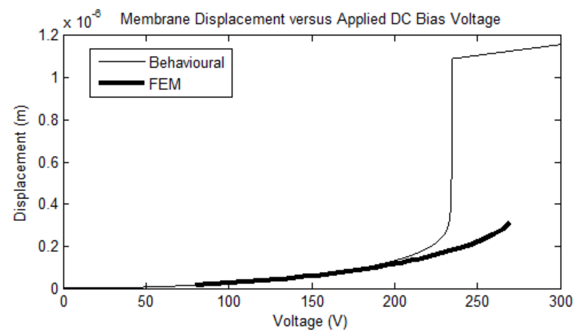

Figure 8: Membrane displacement versus DC bias voltage showing pull-in phenomena for behavioural and FEM models.

To illustrate that the membrane does not in reality deform in a piston-like fashion, the FEM model was used to plot a cross-section of the membrane shape for various DC bias voltages. As can be seen in Figure 9, the membrane shape is curved, with a large displacement at its centre and zero displacement at the outer edges. Larger DC bias voltages cause greater membrane curvature and displacement.

The effect of the DC bias voltage on the electro-mechanical energy conversion efficiency (output mechanical power over input electrical power) of the CMUT was also simulated. A 20 V, 6 MHz sinusoid was superimposed on the DC bias, which was swept from zero to the pull-in voltage. The mechanical load was set to $R_L = 1.9\ \Omega$, to represent fatty tissue. The result of Figure 10 shows that the efficiency increases with increasing DC bias voltage. The efficiency is generally very low, but a large relative increase is obtained by operating near the pull-in voltage.

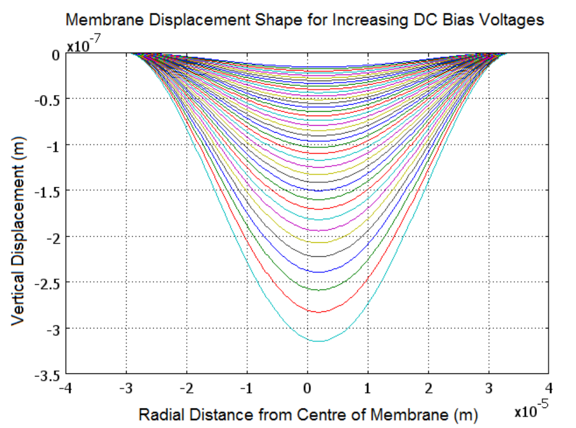

Figure 9: Shape of membrane cross-section for various DC bias voltages. The curves progress from top to bottom in steps of 5 V from 80-270 V.

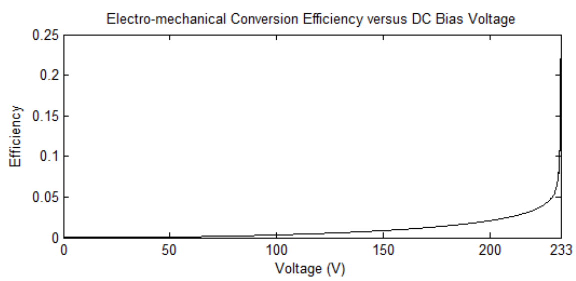

Figure 10: CMUT efficiency versus DC bias voltage.

The previous results are all oriented around the use of a CMUT as a transmitter, but in practice it is also used as a receiver. The behavioural model developed is bi-directional, allowing simulation of both transmit and receive behaviour. To test this ability, a time-domain simulation was performed using the Simplorer circuit shown in Figure 11. Here, two instances of the CMUT model are used, one to transmit an ultrasonic pulse and one to receive it after it passes through an intervening medium. Both CMUT have the parameters of Table 1 and are biased with 90 V. The medium is modelled in VHDL-AMS. It simply imposes a time delay and attenuation on the input pulse depending on its thickness and acoustic properties. This model does not account for all the complexities of ultrasound-tissue interaction, but nonetheless allows the transmit and receive abilities of the CMUT model to be tested in a somewhat realistic manner. For simulation purposes, the medium was assumed to be fatty tissue, with a propagation speed of 1479 m·s^{-1} and a thickness of 0.1 m. A 20 V, 100 ns pulse was input to the transmitting CMUT using the "Pulse" block of Figure 11, and the resulting

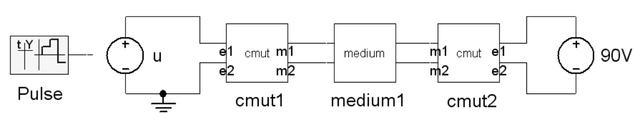

Figure 11: Simplorer circuit used for transmit/receive simulations.

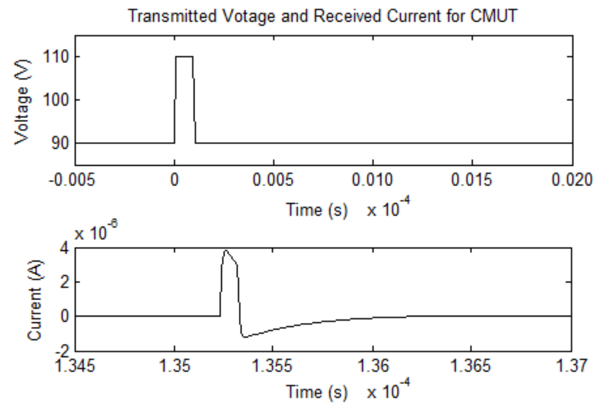

Figure 12: Time-domain simulations showing the delayed output current response of a receiving CMUT (bottom) due to the transmitted voltage pulse (top).

current at the output of the receiving CMUT was measured as a function of time. The input voltage and output current are plotted together in Figure 12. These results show that the 20 V input pulse generates a 4 µA output pulse, which is delayed by approximately 135 µs.

4. DISCUSSION

The results of Section 3 demonstrate the versatility and accuracy of the behavioural VHDL-AMS model developed for a CMUT element in this work. Excellent agreement was observed in the resonant frequency of the CMUT predicted by behavioural and FEM models, with an error of only 1.2 %. The use of (1) to give an effective mass ensures the accuracy of the fundamental resonant frequency, but it should be noted that the model cannot currently provide the resonant frequencies of higher order harmonics shown in Figure 7. The results of Figure 6 show that the bandwidth of the CMUT increases dramatically when used with media of high characteristic acoustic impedance, Z_0. This is consistent with CMUT literature (e.g. [1]), and one of the primary reasons CMUT currently receive a lot of attention from researchers and companies interested in their applicability to ultrasonic imaging.

The behavioural and FEM model pull-in simulations of Figure 8 show good agreement for all voltages except those near pull-in. The error in the behavioural model value for pull-in voltage is approximately 16 %. This can be attributed to the piston-like movement assumption in the derivation of (4)-(7). This assumption essentially attempts to model the membrane deformations shown in Figure 9 with horizontal lines tangential to the minima of the curves. At bias-voltages well below pull-in voltage, this is a fair approximation, as the range of displacements of the curved membrane is relatively small. However, the approximation fails near pull-in, as a greater surface area is near the substrate for a piston-like membrane than for the equivalent curved membrane, resulting in a larger electrostatic force that collapses the piston membrane at a lower bias voltage. The error of 16 % is a significant improvement on previously reported errors of 30-40 % for a parallel plate approximation [9]. The error could be further improved by using a polynomial approximation of the membrane shape to derive new expressions for (4)-(7). The pull-in voltage and displacement of the behavioural model (230 V and 0.265 μm) agree very well with the theoretical values for a moveable plate capacitor [10]: 227 V and 0.271 μm, respectively, for the design used in this work. These results demonstrate the non-linear nature of the behavioural VHDL-AMS model and its ability to function independently of the DC bias voltage. These qualities give it more accuracy and flexibility than previously reported models, which rely on a fixed bias voltage to determine critical model parameters and are accurate only for relatively small variations about that voltage.

The plot of transmission efficiency versus DC bias voltage in Figure 10 illustrates that the nearer the CMUT is biased to pull-in, the more efficient its operation. This agrees with previously reported results [11], and shows the bias voltage is a crucial parameter in determining CMUT efficiency.

The behavioural model is also able to be used without modification in time-domain simulations and as a receiving transducer. This was demonstrated in Figure 12, where a voltage pulse input to a transmitting CMUT passes through an acoustic medium and results in a delayed current pulse at the output of a receiving CMUT. This simulation took only 5.8 s on a 2 GHz desktop PC. An equivalent time-domain simulation using FEM models would take many hours. The speed of simulation is clearly an advantage of the behavioural model.

Despite some inaccuracies in the behavioural CMUT model, its speed and flexibility allow it to be simulated in a number of different scenarios. It can be used to quickly explore the effects of changing the parameters of the CMUT, the transmit and receive electronics, and the acoustic medium. Further, the portability of the VHDL-AMS implementation allows the model to be integrated into other software packages.

5. CONCLUSION

A behavioural model for a CMUT element was developed in this work. The model was based on a moveable plate capacitor attached to a mass-spring-damper system, and preserved the non-linear electro-mechanical interactions of the CMUT. It was implemented in VHDL-AMS, and then imported to the schematic simulation software, Simplorer. A number of results were obtained, including transmit frequency response, membrane pull-in and transmit efficiency. Where possible, the results were compared with a FEM model of the same CMUT design. Excellent agreement was shown for the value of the fundamental resonant frequency. The behavioural model gave a pull-in voltage with approximately 16 % error compared to the value of the FEM model. The error was likely due to the piston-like movement assumption made in the derivation of the model. Further results demonstrated the ability of the behavioural model to be used in time response simulation, and to act as both a transmitter and receiver of ultrasound.

Planned future improvements for the behavioural model include better modelling of the CMUT membrane shape, to remove the piston assumption; incorporation of the effect of the interaction between the membrane and the gap beneath it; and improved modelling of the acoustic domain. The CMUT design of Section 3.1 is to be fabricated in the near future. This will allow experimental validation of the results presented in this work.

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