Circuit Synthesizable Guaranteed Passive Modeling for Multiport Structures

Zohaib Mahmood, Luca Daniel
Massachusetts Institute of Technology

BMAS September-23, 2010

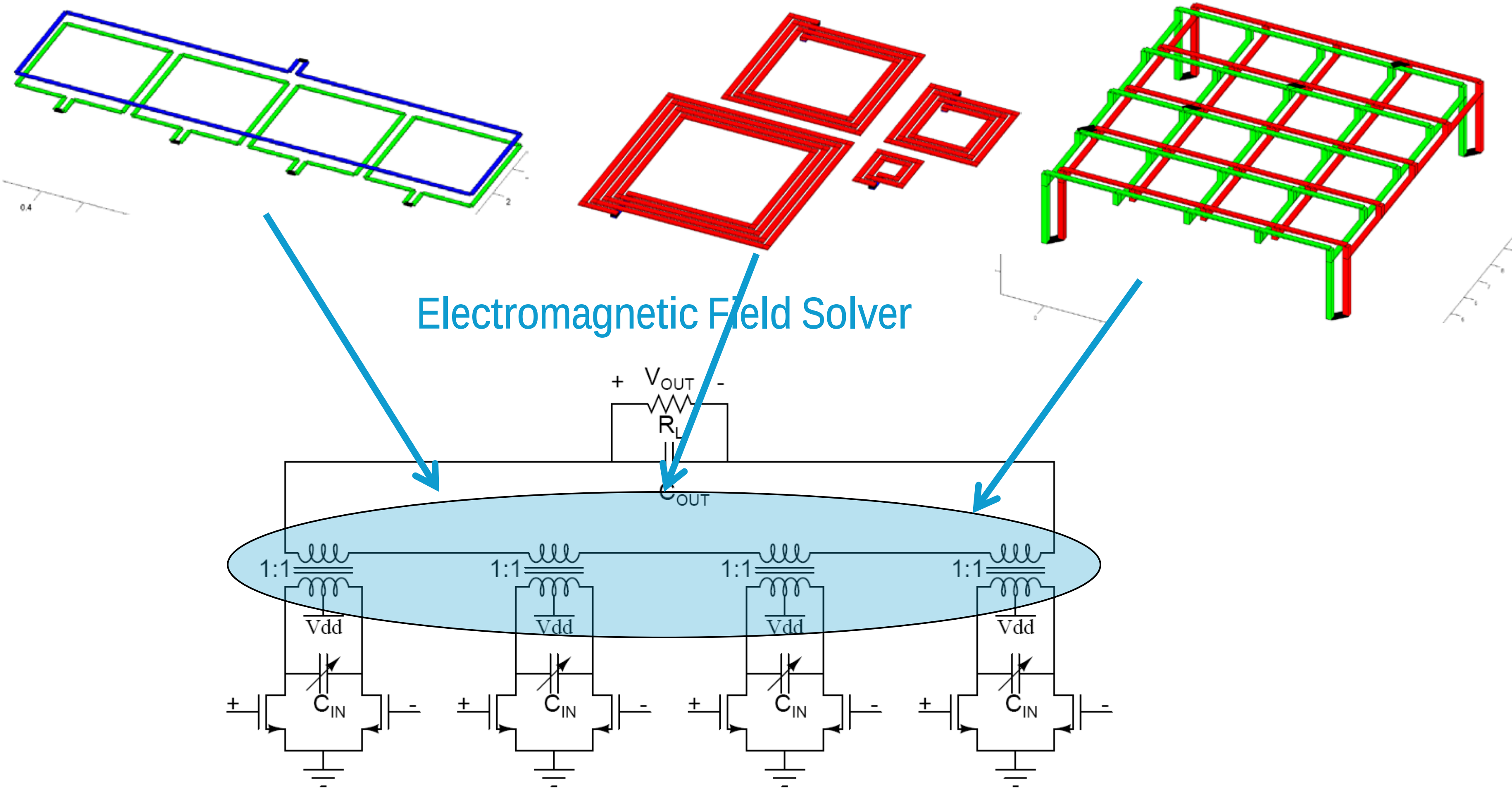
Outline

- Motivation for Compact Dynamical Passive Modeling
- What is Passivity?
- Existing Techniques
- Rational Fitting of Transfer Functions
- Results

Outline

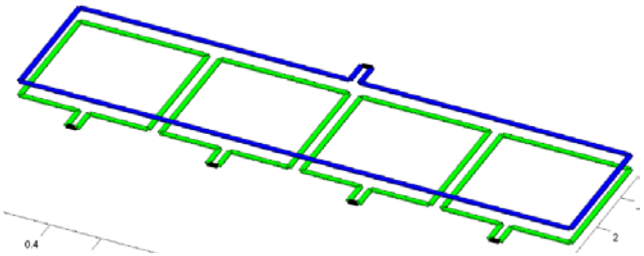
- **Motivation for Compact Dynamical Passive Modeling**
- **What is Passivity?**
- **Existing Techniques**
- **Rational Fitting of Transfer Functions**
- **Results**

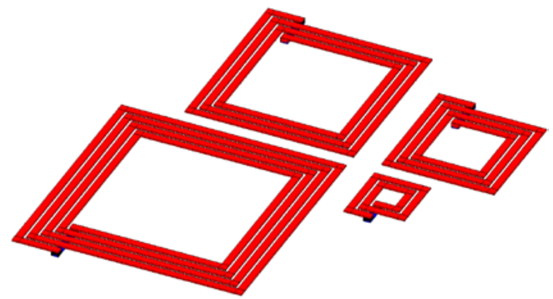
Motivation for Model Generation

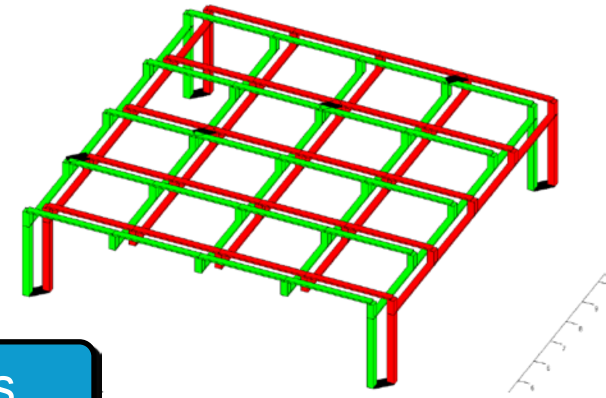


Circuit Simulator (Time Domain Simulations)

Motivation for Model Generation







Step 1: Field Solvers OR **Measurements**

↓

Frequency Response Samples *(H_i)*
S/Z-Parameters

↓

Step 2 : Samples ↦ Transfer Function Matrix

$$\hat{H}(s) = \begin{bmatrix} Z_{11}(s) & \dots & Z_{1N}(s) \\ . & . & . \\ Z_{N1}(s) & \dots & Z_{NN}(s) \end{bmatrix}$$

$\hat{H}(s)$ *must be* ***PASSIVE***

↓

Circuit Simulator (Time Domain Simulations)

Outline

- Motivation for Compact Dynamical Passive Modeling
- What is Passivity?
- Existing Techniques
- Rational Fitting of Transfer Functions
- Results

What is a Passive Network/Model?

DEFINITION

Passivity is the inability of a system (or model) to generate energy

- **All physical systems dissipate energy, and are therefore passive**
- **For numerical models of such systems, this is not guaranteed unless enforced**
- **Passivity for an impedance (or admittance) matrix is implied by 'positive realness'.**

Conditions for Passivity

Conditions for Passivity (*Hybrid Parameters*)

$\hat{H}(s)$ is passive iff:

$$\overline{\hat{H}(\overline{s})} = \hat{H}(s)$$

$$\hat{H}(s) \text{ is analytic in } \Re\{s\} > 0$$

$$\Psi(j\omega) = \hat{H}(j\omega) + \hat{H}(j\omega)^{\dagger} \pm 0 \;\; \forall \omega$$

Condition 1 – Conjugate Symmetry $\Leftrightarrow$ Real impulse response

Condition 2 – Stability $\Leftrightarrow$ All poles in left half plane

Condition 3 – Non-negativity $\Leftrightarrow$ Non-negative eigen values of $\Psi(j\omega)$

forall ω

Conditions for Passivity (*Hybrid Parameters*)

$\hat{H}(s)$ is passive iff:

$$\overline{\hat{H}(\overline{s})} = \hat{H}(s)$$

$$\hat{H}(s) \text{ is analytic in } \Re\{s\} > 0$$

$$\Psi(j\omega) = \hat{H}(j\omega) + \hat{H}(j\omega)^{\dagger} \succeq 0 \;\; \forall \omega$$

Condition 1 – Conjugate Symmetry $\Leftrightarrow$ Real impulse response

Condition 2 – Stability $\Leftrightarrow$ All poles in left half plane

Condition 3 – Non-negativity $\Leftrightarrow$ Non-negative eigen values of $\Psi(j\omega)$

forall ω

Conditions for Passivity

Conditions for Passivity (*Hybrid Parameters*)

$\hat{H}(s)$ is passive iff:

$$\overline{\hat{H}(\overline{s})} = \hat{H}(s)$$

$$\hat{H}(s) \text{ is analytic in } \Re\{s\} > 0$$

$$\Psi(j\omega) = \hat{H}(j\omega) + \hat{H}(j\omega)^{\dagger} \pm 0 \;\; \forall \omega$$

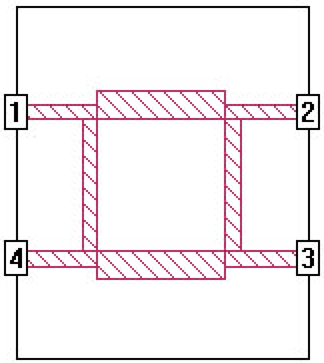
Condition 1 – Conjugate Symmetry $\Leftrightarrow$ Real impulse response

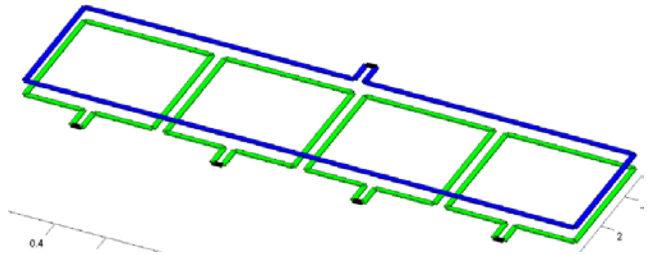
Condition 2 – Stability $\Leftrightarrow$ All poles in left half plane

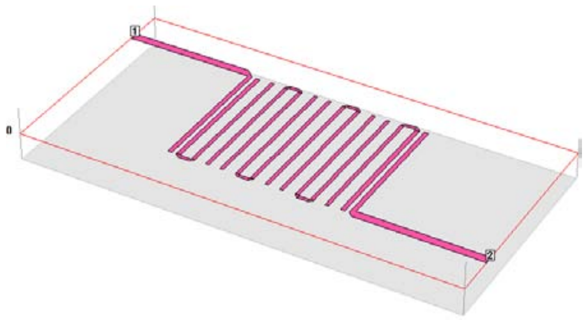
Condition 3 – Non-negativity $\Leftrightarrow$ Non-negative eigen values of $\Psi(j\omega)$ forall ω

Manifestation of Passivity

- **Multi Port Case**





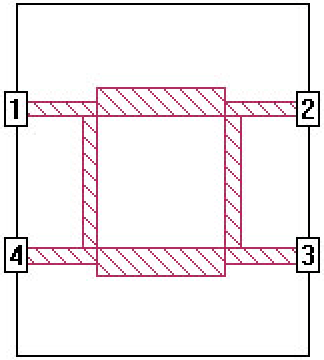


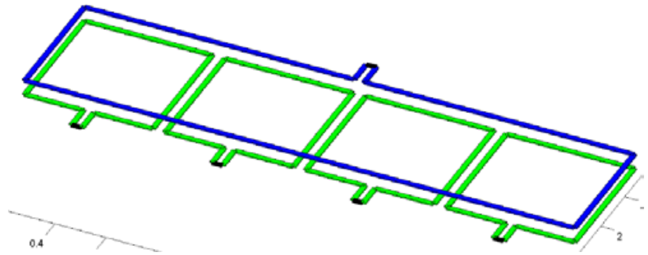
$$[Z(s)]_{n\times n} = [R(s)]_{n\times n} + j[X(s)]_{n\times n}$$

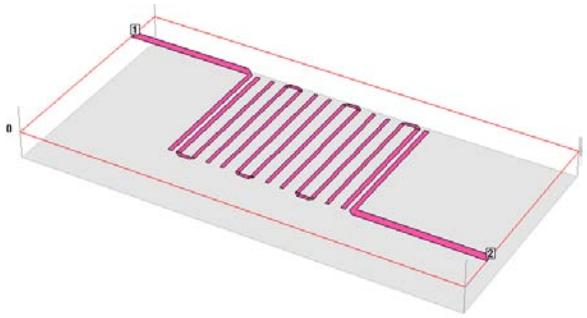
$$\begin{bmatrix} R_{1,1}(s) & \cdots & R_{1,n}(s) \\ \vdots & \ddots & \vdots \\ R_{n,1}(s) & \cdots & R_{n,n}(s) \end{bmatrix}$$

Manifestation of Passivity

- **Multi Port Case**







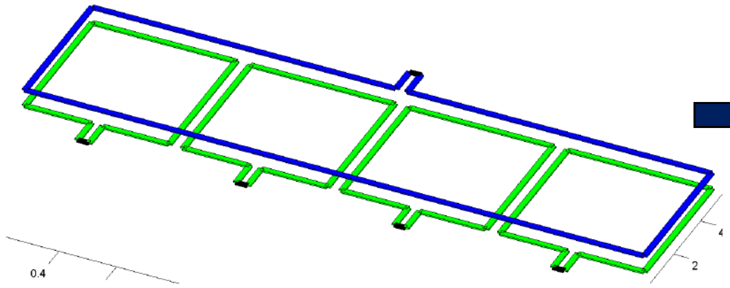
$$[Z(s)]_{n\times n} = [R(s)]_{n\times n} + j[X(s)]_{n\times n}$$

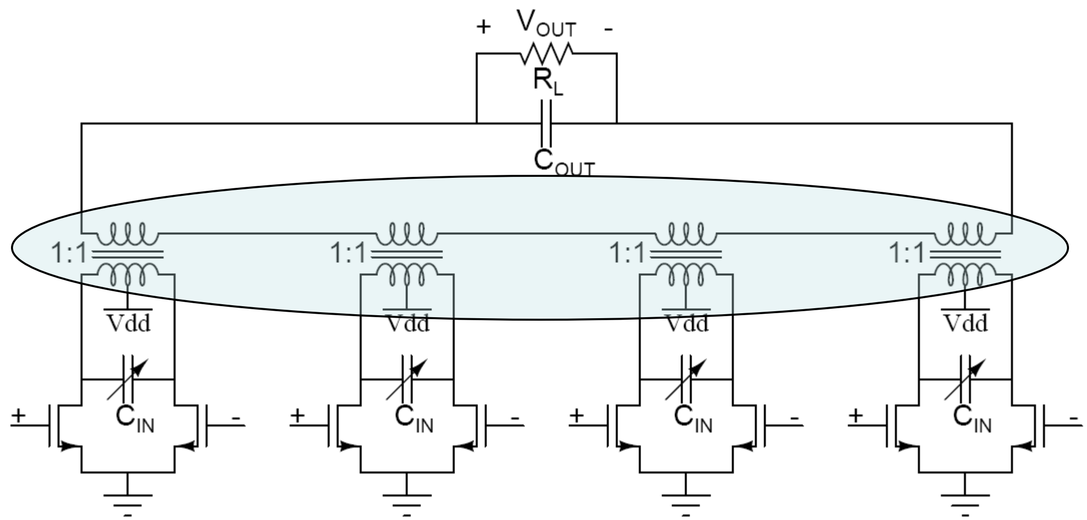
$$\begin{bmatrix} R_{1,1}(s) & \cdots & R_{1,n}(s) \\ \vdots & \ddots & \vdots \\ R_{n,1}(s) & \cdots & R_{n,n}(s) \end{bmatrix}$$

-Frequency dependent real matrix must be positive semidefinite for all frequencies

-Property of entire matrix, cannot enforce element-wise

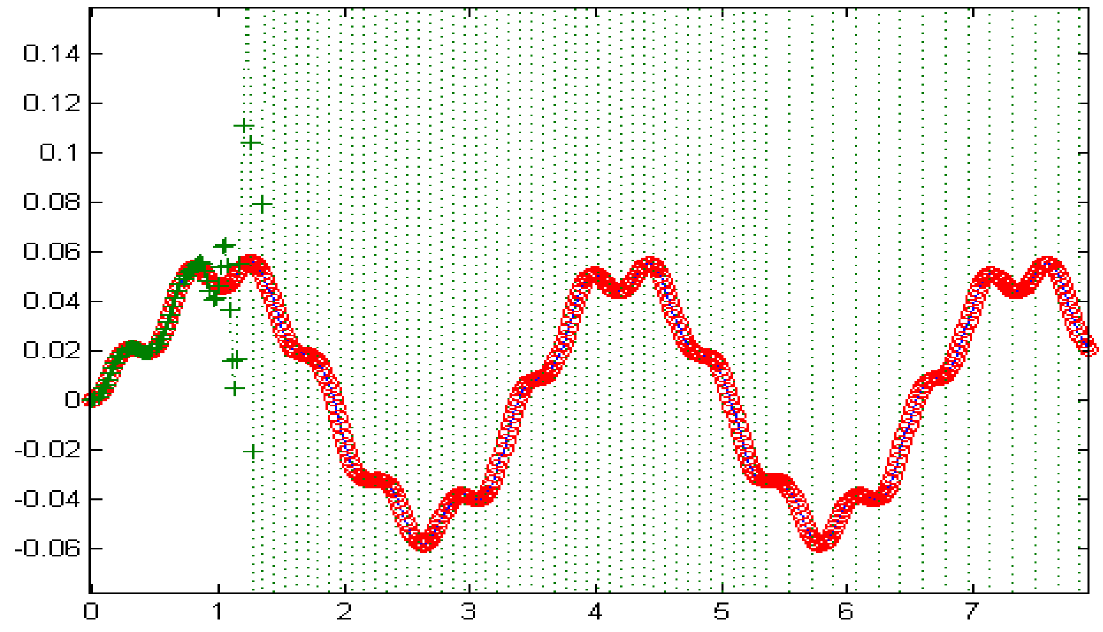
What if passivity is not preserved





-Circuit simulation may blow-up
-Simulator convergence issues
-Results may become completely non-physical.

o ... passive model
+... non-passive model
(time domain simulation)

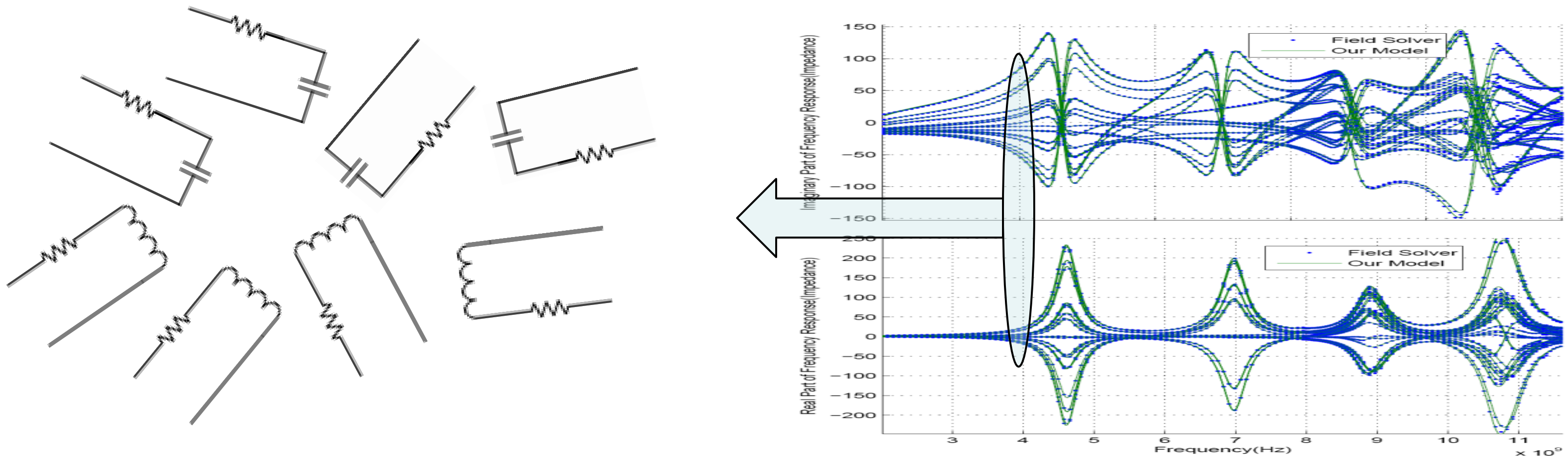


Outline

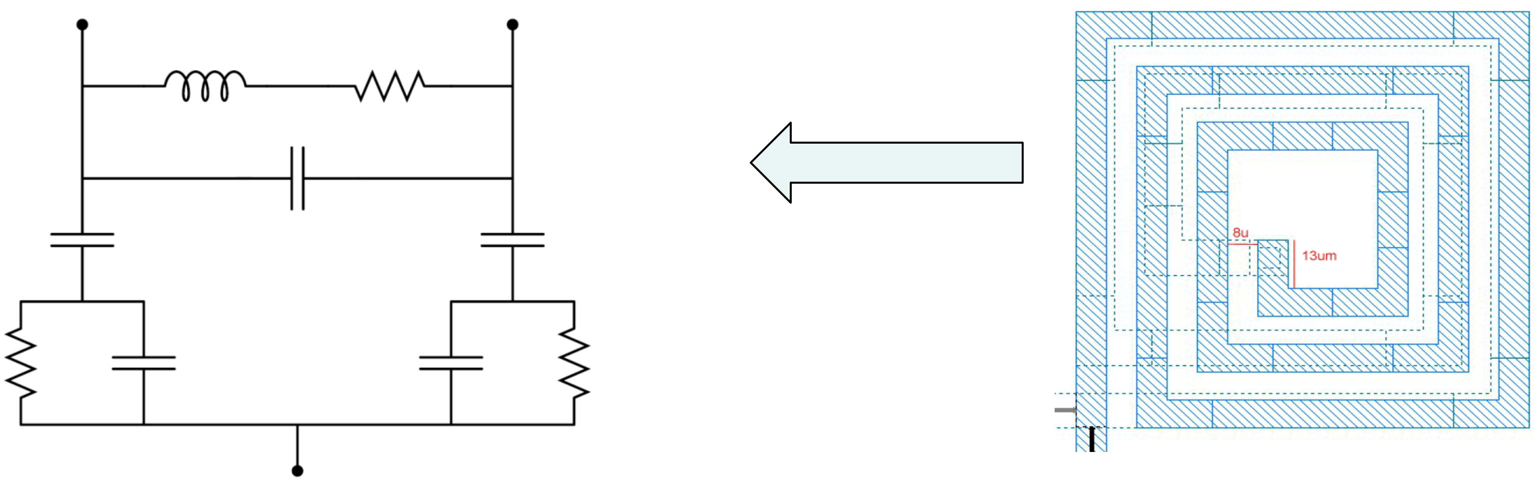
- Motivation for Compact Dynamical Passive Modeling
- What is Passivity?
- Existing Techniques
- Rational Fitting of Transfer Functions
- Results

Designers' way around -- Analytic / Intuitive Approaches

- **RL/RC Networks characterized at operating frequency**



- **Develop RLC Network from intuition**



Numerical Approaches

Technique	Pros	Cons
Projection approaches e.g. PRIMA [Odabasioglu 1997]	Passivity preserved	Does not work with frequency response data.
Vector Fitting [Gustavsen 1999]	Efficient, Robust	Passivity not preserved
Pole discarding approaches [Morsey 2001]	Passivity enforced	Highly restrictive, non-passive pole-residues are discarded
Perturbation based approaches [Talocia 2004, Gustavsen 2008]	Passivity enforced	Two step process. Final models may lose accuracy and optimality
Optimization based approaches [Suo 2008]	Passivity enforced	Computationally expensive, frequency dependent constraints

Our Approach: Enforce passivity <u>during</u> identification, using <u>efficient</u> optimization framework

Numerical Approaches

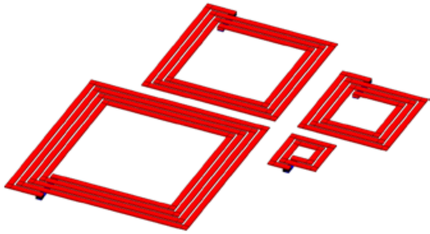
Technique	Pros	Cons
Projection approaches e.g. PRIMA [Odabasioglu 1997]	Passivity preserved	Does not work with frequency response data.
Vector Fitting [Gustavsen 1999]	Efficient, Robust	Passivity not preserved
Pole discarding approaches [Morsey 2001]	Passivity enforced	Highly restrictive, non-passive pole-residues are discarded
Perturbation based approaches [Talocia 2004, Gustavsen 2008]	Passivity enforced	Two step process. Final models may lose accuracy and optimality
Optimization based approaches [Suo 2008]	Passivity enforced	Computationally expensive, frequency dependent constraints

Our Approach: Enforce passivity <u>during</u> identification, using <u>efficient</u> optimization framework

Outline

- Motivation for Compact Dynamical Passive Modeling
- What is Passivity?
- Existing Techniques
- Rational Fitting of Transfer Functions
- Results

Problem Statement

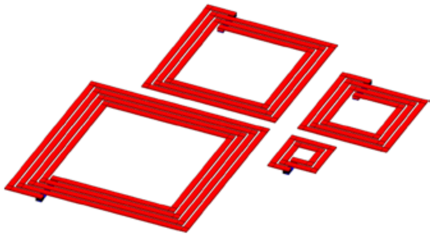


- Given frequency response samples $\{\omega_i, H_i\}$

Search for optimal passive rational approximation in the pole residue form

$$\hat{H}(s) = \sum_{k=1}^{\kappa} \frac{R_k}{s - a_k} + D$$

Problem Statement



- Given frequency response samples $\{\omega_i, H_i\}$

Search for optimal passive rational approximation in the pole residue form

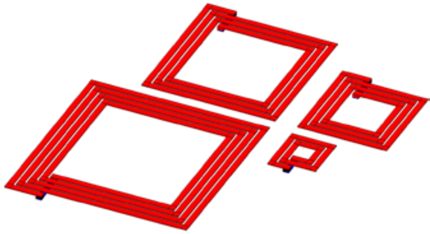
$$\hat{H}(s) = \sum_{k=1}^{\kappa} \frac{R_k}{s - a_k} + D$$

- Formulate as optimization problem

$$L_2 : \min_{p,q} \sum_i \left| H_i - \hat{H}(s) \right|^2 \qquad L_\infty : \min_{p,q} \max_i \left| H_i - \hat{H}(s) \right|$$

Subject to: $\hat{H}(s)\ PASSIVE$

Problem Statement



- Given frequency response samples $\{\omega_i, H_i\}$

Search for optimal passive rational approximation in the pole residue form

$$\hat{H}(s) = \sum_{k=1}^{\kappa} \frac{R_k}{s - a_k} + D$$

- Formulate as optimization problem

$$L_2 : \min_{p,q} \sum_i \left| H_i - \hat{H}(s) \right|^2 \qquad L_\infty : \min_{p,q} \max_i \left| H_i - \hat{H}(s) \right|$$

Subject to: $\hat{H}(s)$ $PASS$

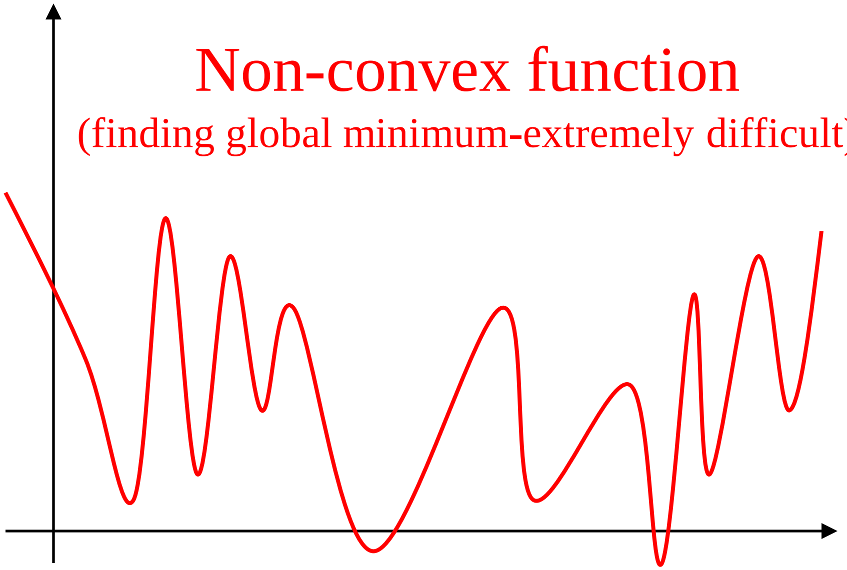
NON-CONVEX!!

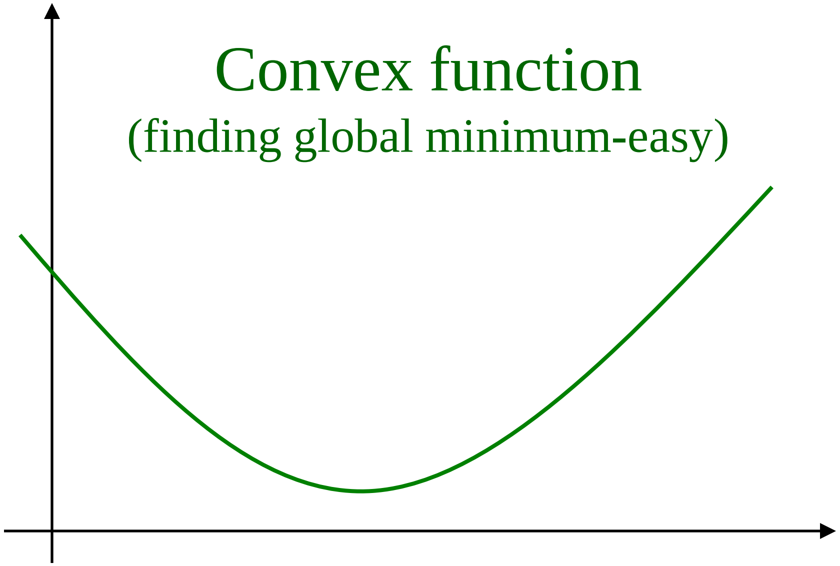
Convex Optimization Problems

- Non-convex problems difficult to solve

$$\min_{p,q} \sum_i \left| H_i - \hat{H}(s) \right|^2$$

Subject to: $\hat{H}(s):\ \ \text{PASSIVE}$





- **Must reformulate as convex optimization problem**
  - **Convex objective function**
  - **Convex constraints**

Modeling Flow

Step 1: Identify a COMMON set of STABLE *poles*

-Vector Fitting Algorithm [Gustavsen 1999]

- Optimization Framework [Suo 2008]

Step 2: Use POLES from step 1 to identify passive model

$$\min_{p,q} \sum_i \left| H_i - \hat{H}(s) \right|^2$$

Subject to: $\hat{H}(s): \ \text{PASSIVE}$

Problem Formulation

$$\hat{H}(j\omega) = \sum_{k=1}^{\kappa} \frac{\mathbf{R_k}}{j\omega - a_k} + \mathbf{D}$$

$$= \sum_{k=1}^{\kappa_r} \hat{H}_k^r(j\omega) + \sum_{k=1}^{\kappa_c/2} \hat{H}_k^c(j\omega) + \mathbf{D}$$

$$= \sum_{k=1}^{\kappa_r} \frac{\mathbf{R_k^r}}{j\omega - a_k^r} + \sum_{k=1}^{\kappa_c/2} \left\{ \frac{\Re\mathbf{R_k^c} + j\Im\mathbf{R_k^c}}{j\omega - \Re a_k^c - j\Im a_k^c} + \frac{\Re\mathbf{R_k^c} - j\Im\mathbf{R_k^c}}{j\omega - \Re a_k^c + j\Im a_k^c} \right\} + \mathbf{D}$$

Problem Formulation

$$\hat{H}(j\omega) = \sum_{k=1}^{\kappa} \frac{\mathbf{R_k}}{j\omega - a_k} + \mathbf{D}$$

$$= \sum_{k=1}^{\kappa_r} \hat{H}_k^r(j\omega) + \sum_{k=1}^{\kappa_c/2} \hat{H}_k^c(j\omega) + \mathbf{D}$$

$$= \sum_{k=1}^{\kappa_r} \frac{\mathbf{R_k^r}}{j\omega - a_k^r} + \sum_{k=1}^{\kappa_c/2} \left\{ \frac{\Re \mathbf{R_k^c} + j\Im \mathbf{R_k^c}}{j\omega - \Re a_k^c - j\Im a_k^c} + \frac{\Re \mathbf{R_k^c} - j\Im \mathbf{R_k^c}}{j\omega - \Re a_k^c + j\Im a_k^c} \right\} + \mathbf{D}$$

series/parallel interconnection of **first** order networks

series/parallel interconnection of **second** order networks

resistive/conductive network

Problem Formulation

$$\hat{H}(j\omega) = \sum_{k=1}^{\kappa} \frac{\mathbf{R_k}}{j\omega - a_k} + \mathbf{D}$$

$$= \sum_{k=1}^{\kappa_r} \hat{H}_k^r(j\omega) + \sum_{k=1}^{\kappa_c/2} \hat{H}_k^c(j\omega) + \mathbf{D}$$

$$= \sum_{k=1}^{\kappa_r} \frac{\mathbf{R_k^r}}{j\omega - a_k^r} + \sum_{k=1}^{\kappa_c/2} \left\{ \frac{\Re\mathbf{R_k^c} + j\Im\mathbf{R_k^c}}{j\omega - \Re a_k^c - j\Im a_k^c} + \frac{\Re\mathbf{R_k^c} - j\Im\mathbf{R_k^c}}{j\omega - \Re a_k^c + j\Im a_k^c} \right\} + \mathbf{D}$$

series/parallel interconnection of **first** order networks

series/parallel interconnection of **second** order networks

resistive/conductive network

Passivity Conditions:

Condition 1 – *Conjugate Symmetry*: Enforced by construction

Condition 2 – *Stability*: Enforced during pole-identification

Condition 3 – *Non-negativity*: Enforced on the building blocks

Passivity Conditions

$$\hat{H}(j\omega) = \sum_{k=1}^{\kappa_r} \hat{H}_k^r(j\omega) + \sum_{k=1}^{\kappa_c/2} \hat{H}_k^c(j\omega) + \mathbf{D}$$

- **A *sufficient* condition for passivity:**

$$\hat{H}_k^r(j\omega), \hat{H}_k^c(j\omega), \mathbf{D} \ \text{passive} \ \forall k \Rightarrow \hat{H}(j\omega) \ \text{passive}$$

$$\Re\hat{H}_k^r(j\omega) \pm 0, \Re\hat{H}_k^c(j\omega) \pm 0, \mathbf{D} \pm 0 \ \forall k \Rightarrow \Re\hat{H}(j\omega) \pm 0$$

Passivity Conditions

$$\hat{H}(j\omega) = \sum_{k=1}^{\kappa_r} \hat{H}_k^r(j\omega) + \sum_{k=1}^{\kappa_c/2} \hat{H}_k^c(j\omega) + \mathbf{D}$$

- **Real-only poles**

$$\hat{H}_k^r(j\omega) = \frac{\mathbf{R_k^r}}{j\omega - a_k^r}$$

$$\hat{H}_k^r(j\omega) = -\frac{a_k^r \mathbf{R_k^r}}{\omega^2 + a_k^{r\,2}} - j\frac{\omega \mathbf{R_k^r}}{\omega^2 + a_k^{r\,2}}$$

$$\Re \hat{H}_k^r(j\omega) = -\frac{a_k^r \mathbf{R_k^r}}{\omega^2 + a_k^{r\,2}} \pm 0 \Rightarrow \boxed{\mathbf{R_k^r} \pm 0}$$

Passivity Conditions

$$\hat{H}(j\omega) = \sum_{k=1}^{\kappa_r} \hat{H}_k^r(j\omega) + \sum_{k=1}^{\kappa_c/2} \hat{H}_k^c(j\omega) + \mathbf{D}$$

- **Real-only poles**

$$\hat{H}_k^r(j\omega) = \frac{\mathbf{R_k^r}}{j\omega - a_k^r}$$

$$\hat{H}_k^r(j\omega) = -\frac{a_k^r \mathbf{R_k^r}}{\omega^2 + a_k^{r\,2}} - j\frac{\omega \mathbf{R_k^r}}{\omega^2 + a_k^{r\,2}}$$

$$\Re \hat{H}_k^r(j\omega) = -\frac{a_k^r \mathbf{R_k^r}}{\omega^2 + a_k^{r\,2}} \pm 0 \Rightarrow \mathbf{R_k^r} \pm 0$$

- Direct Matrix

$$D \pm 0$$

Passivity Conditions

$$\hat{H}(j\omega) = \sum_{k=1}^{\kappa_r} \hat{H}_k^r(j\omega) + \sum_{k=1}^{\kappa_c/2} \hat{H}_k^c(j\omega) + \mathbf{D}$$

- **Real-only poles**

$$\hat{H}_k^r(j\omega) = \frac{\mathbf{R_k^r}}{j\omega - a_k^r}$$

$$\hat{H}_k^r(j\omega) = -\frac{a_k^r \mathbf{R_k^r}}{\omega^2 + a_k^{r\,2}} - j\frac{\omega \mathbf{R_k^r}}{\omega^2 + a_k^{r\,2}}$$

$$\Re \hat{H}_k^r(j\omega) = -\frac{a_k^r \mathbf{R_k^r}}{\omega^2 + a_k^{r\,2}} \pm 0 \Rightarrow \mathbf{R_k^r} \pm 0$$

- Direct Matrix

$$D \pm 0$$

- **Complex poles**

$$\Re \hat{H}_k^c(j\omega) \pm 0. \Rightarrow$$

$$.CPR(\omega) = \left(-\Re a_k^c \Re \mathbf{R_k^c} - \Im a_k^c \Im \mathbf{R_k^c}\right) + \omega^2 \left(-\Re a_k^c \Re \mathbf{R_k^c} + \Im a_k^c \Im \mathbf{R_k^c}\right) \pm 0 \,, \forall \omega$$

$$. \Leftrightarrow$$

$$.\lim_{\omega \to 0} CPR(\omega) \Rightarrow -\Re a_k^c \Re \mathbf{R_k^c} - \Im a_k^c \Im \mathbf{R_k^c} \pm 0$$

$$.\lim_{\omega \to \infty} CPR(\omega) \Rightarrow -\Re a_k^c \Re \mathbf{R_k^c} + \Im a_k^c \Im \mathbf{R_k^c} \pm 0$$

Passivity Conditions

$$\hat{H}(j\omega) = \sum_{k=1}^{\kappa_r} \hat{H}_k^r(j\omega) + \sum_{k=1}^{\kappa_c/2} \hat{H}_k^c(j\omega) + \mathbf{D}$$

- **Real-only poles**

$$\hat{H}_k^r(j\omega) = \frac{\mathbf{R_k^r}}{j\omega - a_k^r}$$

$$\hat{H}_k^r(j\omega) = -\frac{a_k^r \mathbf{R_k^r}}{\omega^2 + a_k^{r2}} - j\frac{\omega \mathbf{R_k^r}}{\omega^2 + a_k^{r2}}$$

$$\Re \hat{H}_k^r(j\omega) = -\frac{a_k^r \mathbf{R_k^r}}{\omega^2 + a_k^{r2}} \pm 0 \Rightarrow \mathbf{R_k^r} \pm 0$$

- Direct Matrix

$$D \pm 0$$

Linear Matrix Inequalities (*Extremely* efficient)

- **Complex poles**

$$\Re \hat{H}_k^c(j\omega) \pm 0. \Rightarrow$$

$$.CPR(\omega) = \left(-\Re a_k^c \Re \mathbf{R_k^c} - \Im a_k^c \Im \mathbf{R_k^c}\right) + \omega^2 \left(-\Re a_k^c \Re \mathbf{R_k^c} + \Im a_k^c \Im \mathbf{R_k^c}\right) \pm 0 \text{ , } \forall \omega$$

$$. \Leftrightarrow$$

$$.\lim_{\omega \to 0} CPR(\omega) \Rightarrow -\Re a_k^c \Re \mathbf{R_k^c} - \Im a_k^c \Im \mathbf{R_k^c} \pm 0$$

$$.\lim_{\omega \to \infty} CPR(\omega) \Rightarrow -\Re a_k^c \Re \mathbf{R_k^c} + \Im a_k^c \Im \mathbf{R_k^c} \pm 0$$

Unknowns

Convex formulation

$$\underset{\mathbf{R_k^r},\mathbf{R_k^c},\mathbf{D}}{\text{minimize}} \sum_i \left|\Re H_i - \Re \hat{H}(j\omega_i)\right|^2 + \sum_i \left|\Im H_i - \Im \hat{H}(j\omega_i)\right|^2$$

subject to

$$\mathbf{D} \pm 0$$

$$\mathbf{R_k^r} \pm 0 \; \forall k = 1,\ldots,\kappa_r$$

$$-\Re a_k^c \Re \mathbf{R_k^c} + \Im a_k^c \Im \mathbf{R_k^c} \pm 0 \; \forall k = 1,\ldots,\kappa_c$$

$$-\Re a_k^c \Re \mathbf{R_k^c} - \Im a_k^c \Im \mathbf{R_k^c} \pm 0 \; \forall k = 1,\ldots,\kappa_c$$

Linear Matrix Inequalities
(*Extremely* efficient)

where

$$\hat{H}(j\omega) = \sum_{k=1}^{\kappa_r} \frac{\mathbf{R_k^r}}{j\omega - a_k^r} + \sum_{k=1}^{\kappa_c} \frac{\mathbf{R_k^c}}{j\omega - a_k^c} + \mathbf{D}$$

Final Optimization Problem

Samples $\{\omega_i, H_i\}$, Desired number of poles (N)

$$\textit{Find N Stable poles } a_k$$

a_k

$$\underset{\mathbf{R_k^r},\mathbf{R_k^c},\mathbf{D}}{\text{minimize}} \sum_i \left|\Re H_i - \Re \hat{H}(j\omega_i)\right|^2 + \sum_i \left|\Im H_i - \Im \hat{H}(j\omega_i)\right|^2$$

$$\text{subject to: } \mathbf{D} \pm 0,\ \mathbf{R_k^r} \pm 0 \ \forall k = 1,\ldots,\kappa_r$$

$$-\Re a_k^c \Re \mathbf{R_k^c} + \Im a_k^c \Im \mathbf{R_k^c} \pm 0 \ \forall k = 1,\ldots,\kappa_c$$

$$-\Re a_k^c \Re \mathbf{R_k^c} - \Im a_k^c \Im \mathbf{R_k^c} \pm 0 \ \forall k = 1,\ldots,\kappa_c$$

Convex Optimization Problem

$a_k, \mathbf{R_k^r}, \mathbf{R_k^c}, \mathbf{D}$

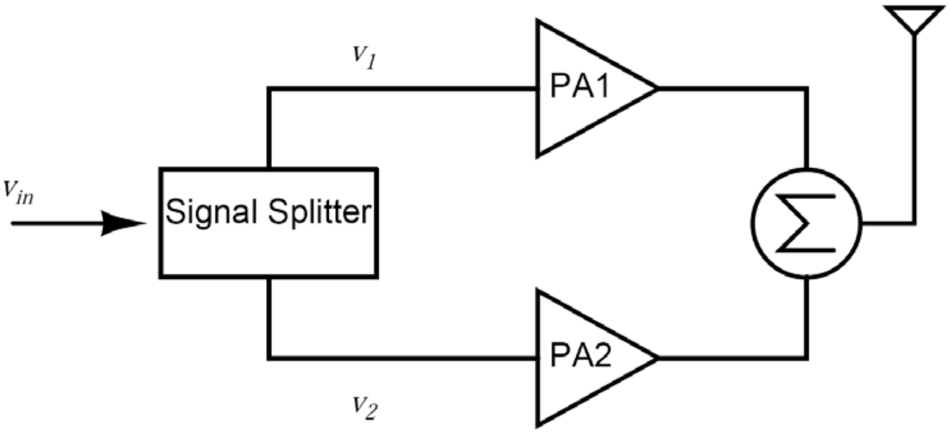
$$\hat{H}(j\omega) = \sum_{k=1}^{\kappa_r} \frac{\mathbf{R_k^r}}{j\omega - a_k^r} + \sum_{k=1}^{\kappa_c} \frac{\mathbf{R_k^c}}{j\omega - a_k^c} + \mathbf{D}$$

Circuit Module (Netlist/ VerilogA – *Dynamical Model*)

Outline

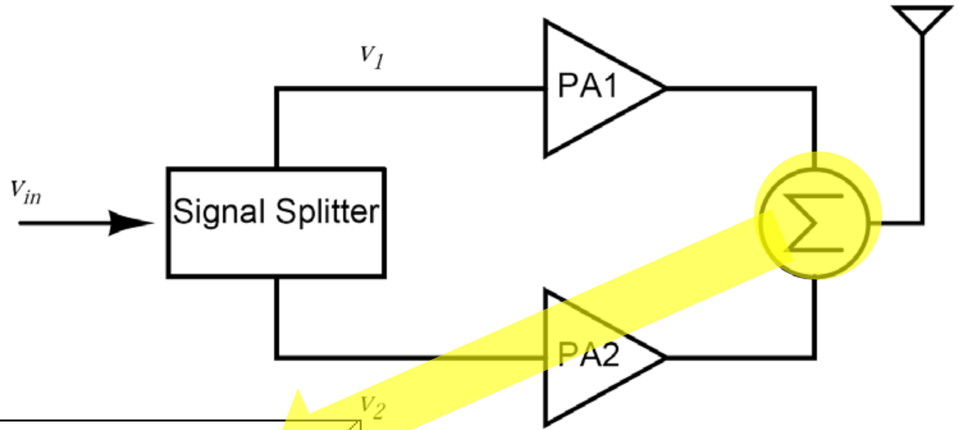
- Motivation for Compact Dynamical Passive Modeling
- What is Passivity?
- Existing Techniques
- Rational Fitting of Transfer Functions
- Results

Results: LINC Power Amplifier



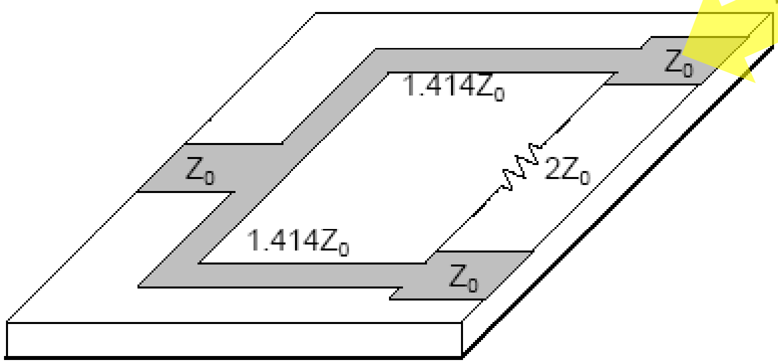
Block diagram of the LINC power amplifier architecture

Results: LINC Power Amplifier

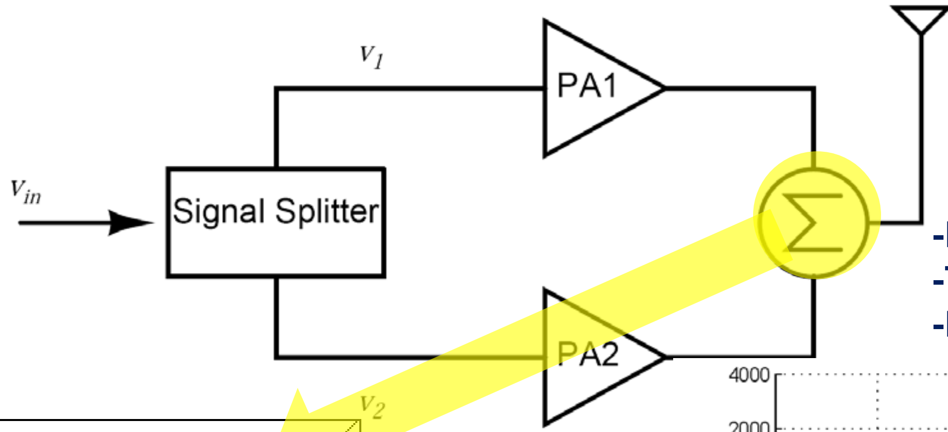


Block diagram of the LINC power amplifier architecture

Layout of the Wilkinson Combiner



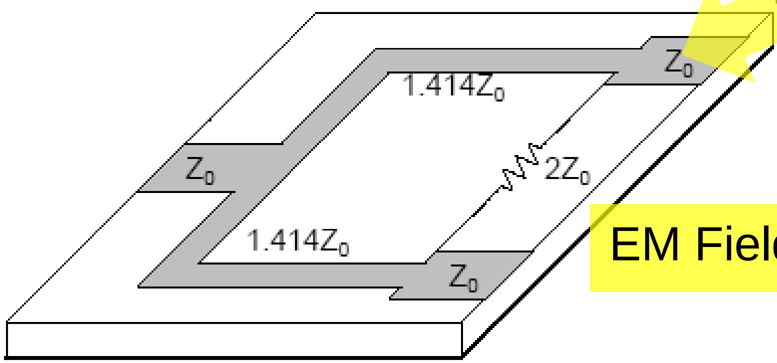
Results: LINC Power Amplifier



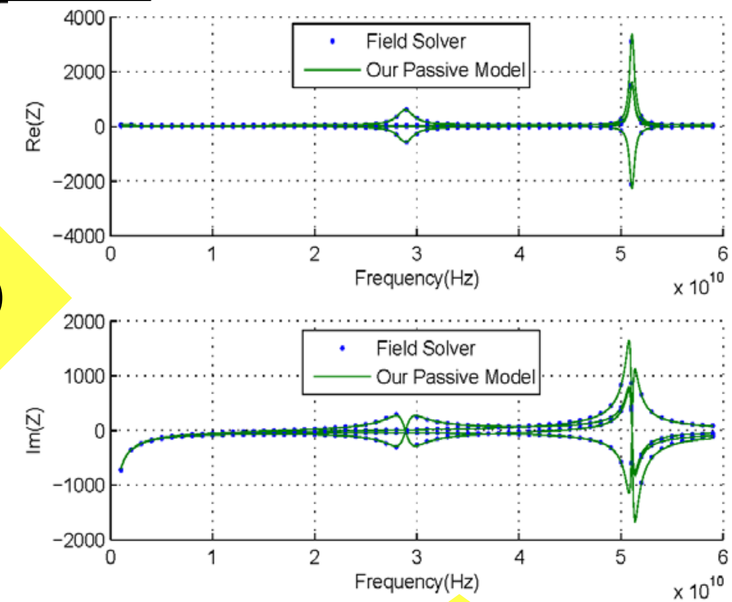
Block diagram of the LINC power amplifier architecture

-Number of poles – 10
-Time [Matlab] – 2 seconds
-Error < 0.7%

Layout of the Wilkinson Combiner



EM Field Solver (dots)



Comparing real and imaginary parts of the impedance parameters from EM field solver (dots) and our passive model (solid lines)

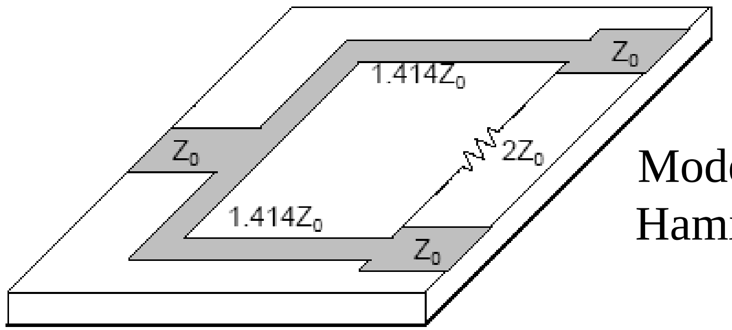
Transfer Function Matrix

$$\hat{H}(s) = \begin{bmatrix} Z_{11}(s) & \dots & Z_{1N}(s) \\ \vdots & \vdots & \vdots \\ Z_{N1}(s) & \dots & Z_{NN}(s) \end{bmatrix}$$

Passive Modeling Algorithm (solid lines)

Results: LINC Power Amplifier

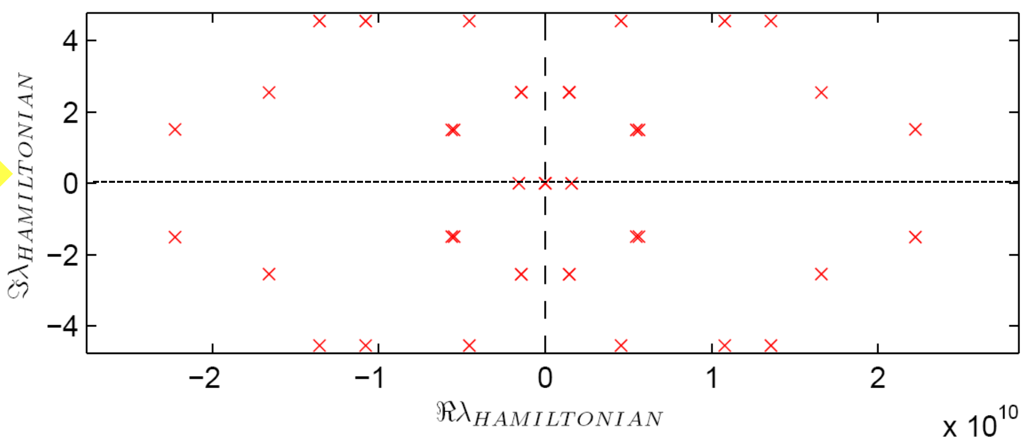
Hamiltonian Matrix Based Passivity Test



Model is passive if the associated Hamiltonian matrix has no purely imaginary eigen value

Transfer Function Matrix

$$\widehat{H}(s) = \begin{bmatrix} Z_{11}(s) & \ldots & Z_{1N}(s) \\ . & . & . \\ Z_{N1}(s) & \ldots & Z_{NN}(s) \end{bmatrix}$$



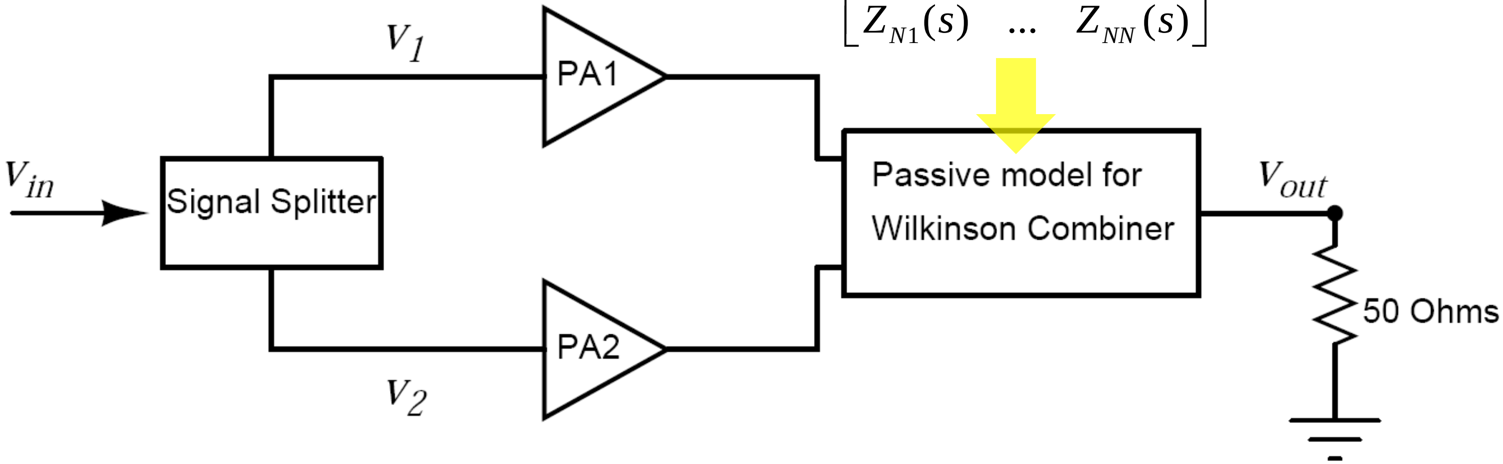
Zoomed-in eigen values of the associated Hamiltonian matrix for the identified model of Wilkinson combiner

Results: LINC Power Amplifier

Block diagram of the LINC power amplifier as **simulated** inside the circuit simulator

$$\widehat{H}(s) = \begin{bmatrix} Z_{11}(s) & \dots & Z_{1N}(s) \\ . & . & . \\ Z_{N1}(s) & \dots & Z_{NN}(s) \end{bmatrix}$$

Passive Transfer Matrix

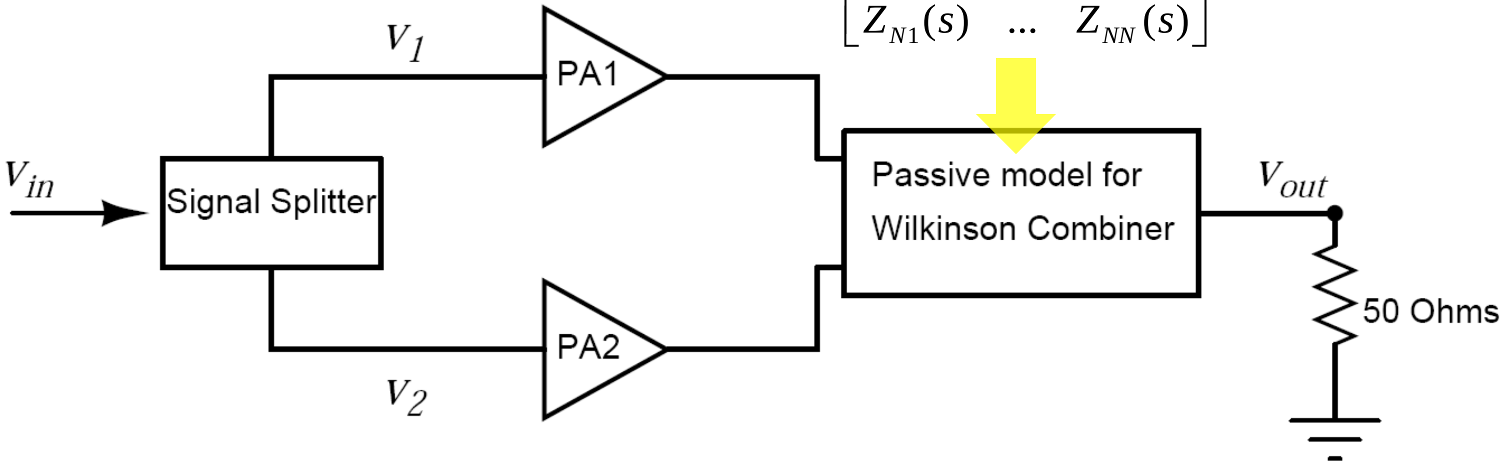


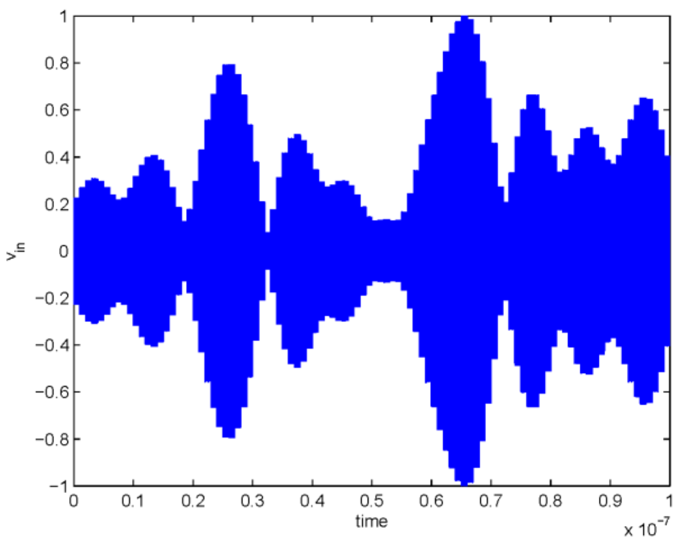
Results: LINC Power Amplifier

Block diagram of the LINC power amplifier as **simulated** inside the circuit simulator

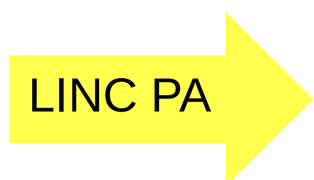
$$\hat{H}(s) = \begin{bmatrix} Z_{11}(s) & \dots & Z_{1N}(s) \\ . & . & . \\ Z_{N1}(s) & \dots & Z_{NN}(s) \end{bmatrix}$$

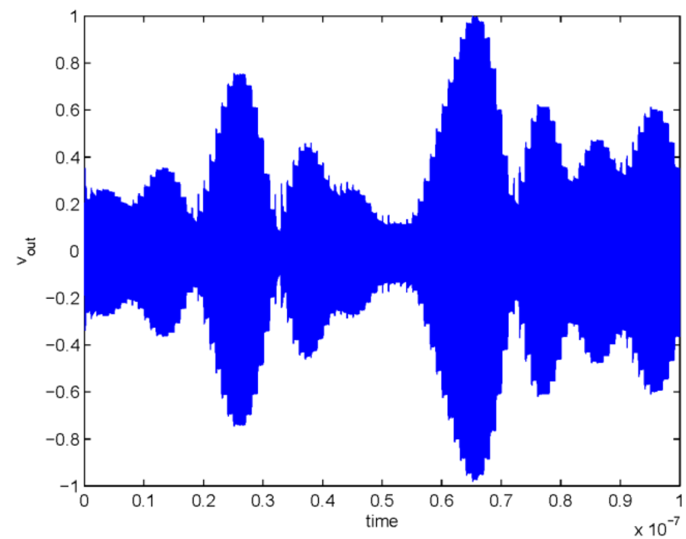
Passive Transfer Matrix





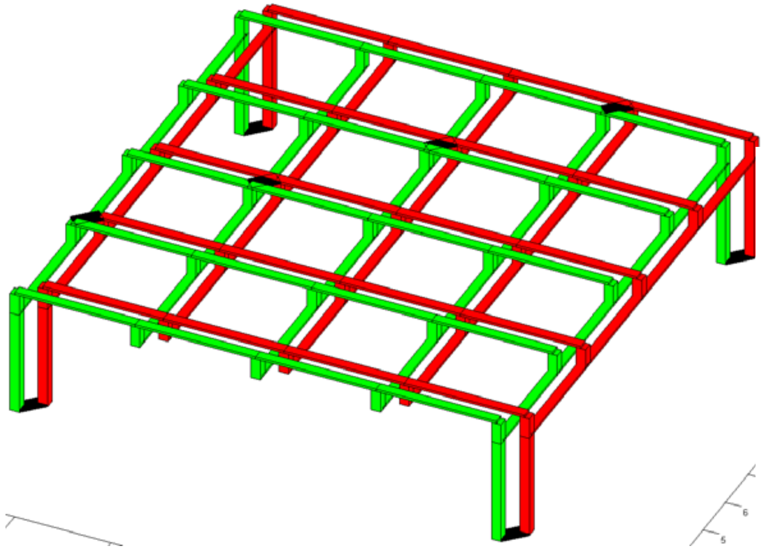
Normalized Input Voltage (v_{in})



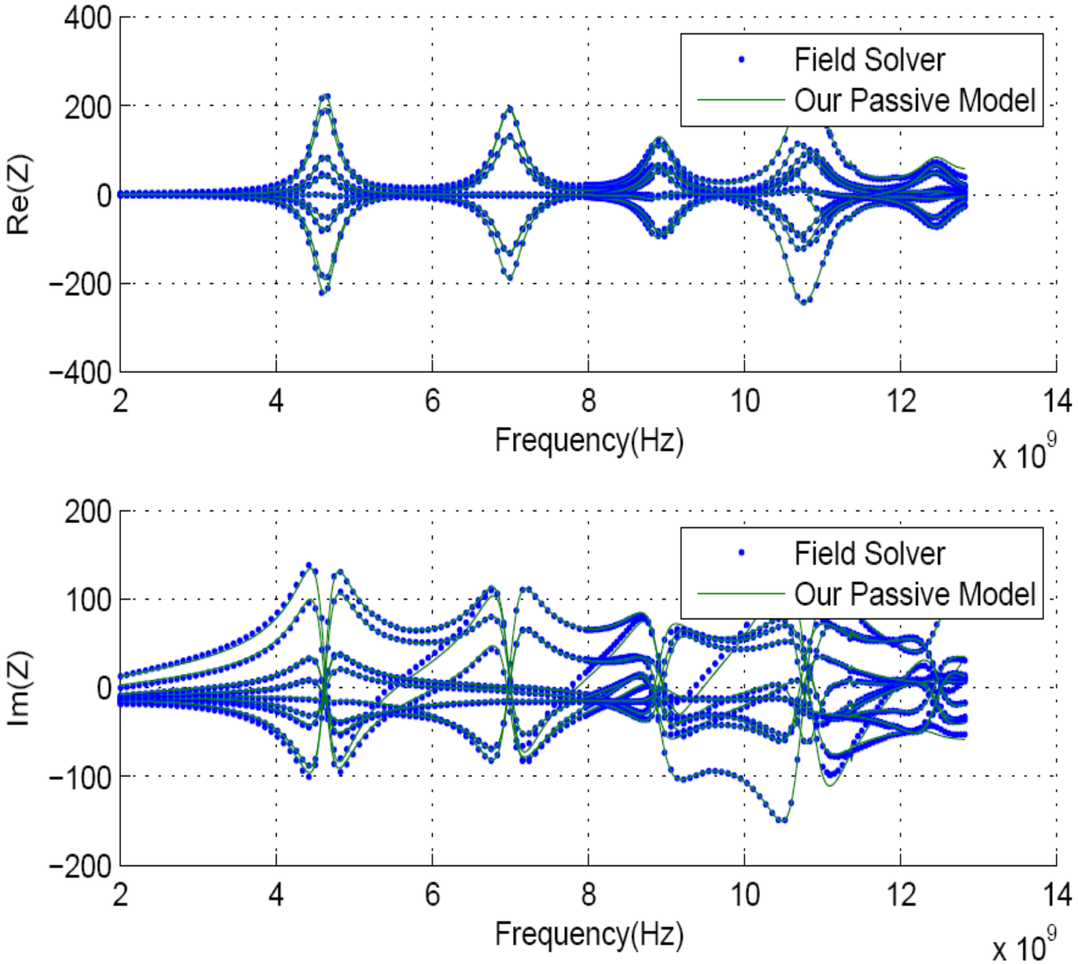


Normalized Output Voltage (v_{out})

Example: 8-Port Power/Gnd Distribution Grid



QUALITY CHECK (Impedance matrix)

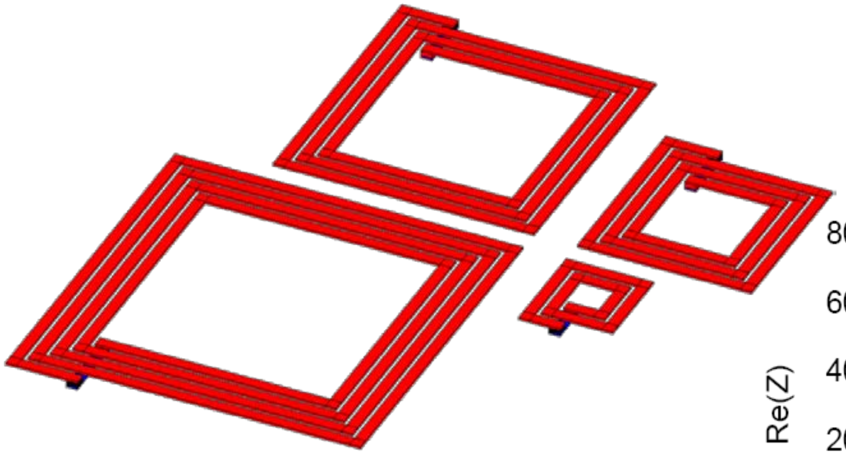


- Hamiltonian Matrix Based Passivity Test – Passed

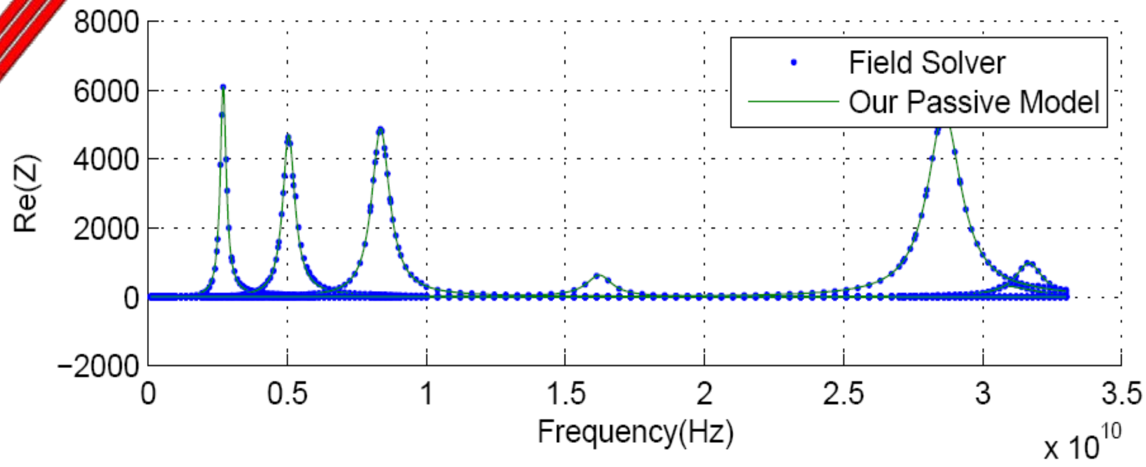
-Number of poles –20

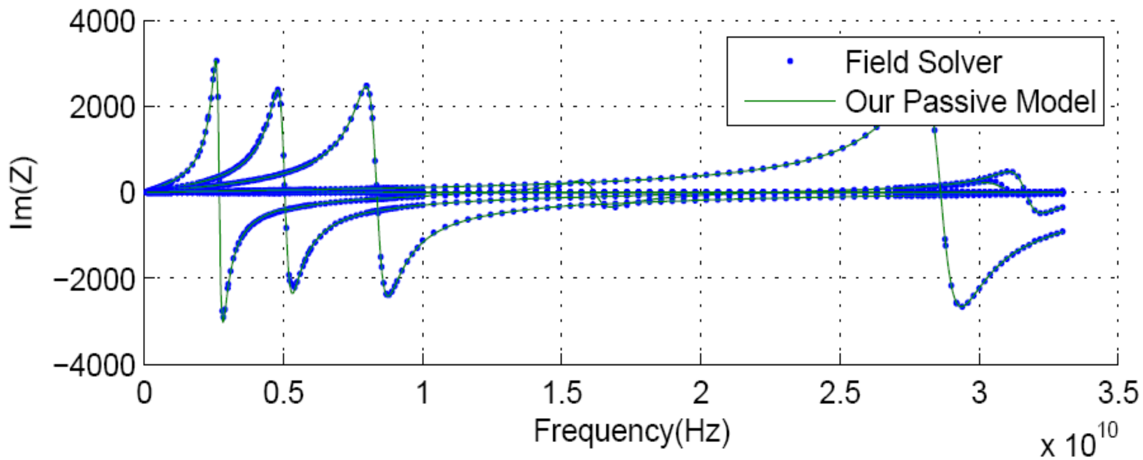
-Time [Matlab] –74 seconds

Example: 4-Port Inductor Array



QUALITY CHECK
(Impedance matrix)





- Hamiltonian Matrix Based Passivity Test – Passed
-Number of poles –23
-Time [Matlab] –72 seconds

Comparison

Structure	Number of Ports	Number of Poles	Time[1] (seconds)	
			[Suo 2008]	This Work [Matlab]
Wilkinson Combiner	3	10	83	2
Power Distribution Grid	8	20	-NA-	74
Coupled RF inductors	4	23	-NA-	72

[1]Laptop: Core2Duo 2.1GHz, 3GB, Windows 7

Conclusions

- Summarized how to develop models from freq. response
- Proposed a Convex Optimization based modeling algorithm
- Demonstrated orders of magnitude improvement over similar techniques
- Presented an example demonstrating system level simulation of analog circuits

THANK YOU